ADVANCED ARITHMETIC

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PREFACE

The following have been controlling ideas in the preparation of this book:

1. In sequence of topics, to follow the plan adopted in the author's Primary and Intermediate Arithmetics, that of recognizing the value of the various courses of study in use in different parts of the country. Whatever originality may be demanded and legitimately shown in the preparation of a text-book, an author is bound to recognize the consensus of opinion as to topics and sequence. For example, modern courses invariably suggest the repetition of the most important portions of arithmetic from time to time, but they favor a somewhat exhaustive treatment of each subject whenever it is under discussion. The extreme spiral system, in which no topic is ever thoroughly treated at one time, but each is repeated until the pupil wearies of it, is psychologically too unwarranted to be considered seriously. On the other hand, the old-time plan of presenting important chapters but once is equally unscientific. Between these extremes lies the mean of the modern courses of study.

2. In arrangement by grades, to recognize the prevailing courses of study in the country, and to outline the work usually covered in the seventh and eighth school years, the author's Intermediate Arithmetic having covered the work of the fifth and sixth years.

3. In the selection of problems, to touch the actual life of this country at this time; to give correct ideas of the business customs of to-day; to embody the mathematical principles in interesting and instructive groups of problems;

to touch the genuine interests of pupils in the story of our national resources and industries rather than to dwell upon the technicalities of minor trades in which they have no immediate or prospective concern; and to come in contact with human life rather than with those phases of science which are quite as foreign to the interests of boys and girls as are the mere abstract problems of numbers.

4. In the matter of abstract drill work, to recognize the fact that a large number of "problems without content" are necessary to concentrate the attention on the operations and to impart the computing habit. The numbers selected have been those demanded by the conditions of the present day, the fractions and compound numbers being those in common use rather than those never met in business, and the integers being the ordinary ones of daily life. Very large numbers have generally been used only in such applied problems as represent the real conditions that the children meet in their geography, their elementary science, and their newspaper reading.

The necessity for a frequent review of the fundamental operations is recognized by all teachers, and hence the book opens with such a review, presenting the subject in a slightly more scientific manner and introducing such short processes as are really usable in business.

In fine, the book is written for the use of those teachers who wish to preserve the best that was in the old-style arithmetic, with its topical system and its abundant drill, while giving to it a modern arrangement and securing "mental discipline" through problems of to-day rather than through the tiresome, meaningless, unreal inheritances of the past.

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ADVANCED ARITHMETIC

CHAPTER I

I. A GENERAL REVIEW OF ARITHMETIC

WRITING NUMBERS

1. Reason for this review. This class has now studied most of the important subjects of Arithmetic. Soon we shall take the more advanced business applications. As an aid to this advanced work we should review the foundations of all work in Arithmetic, the common operations.

Where such a review is not thought to be advisable, it will of course be omitted.

2. Origin of our numerals. Our numerals, 1, 2, and so on to 9, originated in India about 2000 years ago. The zero (0) was added more than 1200 years ago, thus making the system the excellent one that we now know. Without the 0 it was but little better than the Roman system. The numerals were learned by the Arabs more than 1100 years ago, and by A.D. 1200 became somewhat known in Europe. About 400 years ago they became well known in schools and in business, and they are now used in most of the civilized world.

3. The great feature of this system. It is the *place value* that makes the Arabic or Hindoo system better than all others. The Roman VI means five + one, while 51 means five *tens* + one, the 5 having not only the value *five*, but the place value *tens*.

4. Decimal system. Because the places or orders increase tenfold to the left, and decrease by tenths to the right, we call our system a *decimal system*, from the Latin *decem*, meaning ten.

ORAL EXERCISE

1. How many different figures are used in the Arabic system? How many in the Roman system for numbers below a hundred?

2. If I write *ten five* in figures in the two systems, 105 and XV, what values are indicated? Why do the two numbers have different values? What is the use of the 0?

3. In the number 1,904,526,738, give the place value of each figure, as 8 units, 3 tens, and so on. Give the name of each period, as 738 units, 526 thousands, and so on.

4. What do you mean by the words *separatrix*, *order*, *period*, *decimal*, *naught*, *place value*, *billion?* Do not try to repeat definitions, but answer in your own way.

5. Why is our common system, the Arabic, better than the old Roman system? You might illustrate by trying to multiply one number by another.

6. Read the following numbers taken from book chapters: XXVIII, XLI, LXI, LXXXIX, XCIV, CLXVI.

WRITTEN EXERCISE

1. Write in the Arabic system: MDCLXIX, MCCCXLIV.

2. Write in the Roman system: 49, 79, 94, 96, 99, 146.

3. Write in the Roman system the number of the present year; of last year; of fifty years ago.

4. Write in the Arabic system the number twenty-one million, four hundred seventy-five.

5. Write in common figures the number one billion, one million, one thousand one, and one ten-thousandth.

6. Write in common figures the number of Ex. 5 decreased by 42,675; also by 12.245; also by 295,001.127.

5. Uniform scale. Because in our system of writing numbers each place has uniformly ten times the value of the place at its right, we are said to use a *uniform scale.* Because this is a scale of ten it is called a *decimal scale.*

6. Varying scale. A scale that is not uniform is called a *varying scale.*

Uniform Scale	*Varying Scale*
10 units = 1 ten.	2 pints = 1 quart.
10 tens = 1 hundred.	8 quarts = 1 peck.
10 hundreds = 1 thousand.	4 pecks = 1 bushel.

ORAL EXERCISE

1. What is an even number? Illustrate.

2. What name is given to numbers that are not even?

3. How can you tell whether or not a number is even?

4. What is meant by the word *integer?* the word *fraction?* the expression *decimal system?*

5. Give three illustrations of a varying scale. What kind of scale is that of United States money?

6. Tell some reasons why the decimal fraction is better for practical use than the common fraction, except in a few cases.

WRITTEN EXERCISE

1. Write in common figures the number one hundred forty-two thousand, eight hundred fifty-seven.

2. Multiply the number in Ex. 1 by 2, by 3, by 4, by 5, and by 6, and write the results in words.

The results in Ex. 2 are peculiar, all having the same figures as the number multiplied, but differently arranged.

3. Count by 3's from 3 to 27 and write the results. Multiply 37 by each of these numbers and write the nine products.

In Ex. 2 one of the interesting numbers of Arithmetic was considered, and another of the curiosities of number is seen in this example. With a view to further interest, several other curious properties are given in the following examples.

4. Count by 9's from 9 to 81 and write the results. Multiply 12,345,679 by each of these numbers and write the nine products.

5. Multiply 142,857 by 326,451 and tell the peculiarities of the partial products.

6. Multiply the following numbers by 7: 15,873, 31,746, 47,619, 63,492, 79,365, 95,238, 111,111, 126,984, 142,857, each number being 15,873 more than its predecessor.

7. Perform the multiplications indicated: 9×9, 99×99, 999×999, 9999×9999. Write a rule for such products, and apply it to writing the product of 99,999 by 99,999.

8. Perform the multiplications indicated: 11×11, 111×111, 1111×1111. Write a rule for such products, and apply it to writing the product of 111,111 by 111,111.

9. Multiply 143 by 7; by 11×7; by 135×7; by 234×7. From these results tell what the result would be of multiplying by 345×7; by 789×7.

10. Write the values of the following:

$0 \times 9 + 1$	$0 \times 9 + 8$
$1 \times 9 + 2$	$9 \times 9 + 7$
$12 \times 9 + 3$	$98 \times 9 + 6$
$123 \times 9 + 4$	$987 \times 9 + 5$
$1234 \times 9 + 5$	$9876 \times 9 + 4$

7. Units. We usually speak of things counted as *units*. If we are counting dollars, $1 is the unit; if cents, 1 ct. is the unit; if dozens, 1 doz. is the unit; if thousands, 1000 is the unit. If we are counting fifths, $\frac{1}{5}$ is the unit, and we count $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and so on. A unit may, therefore, be one object, or a group of objects, or a part of some object.

ORAL EXERCISE

1. Count from $\frac{1}{8}$ to 1, using $\frac{1}{8}$ as the unit.

2. Count from 0.1 to 1, using 0.1 as the unit.

3. State the most common units of length and tell where we might use each.

4. State the most common units of area and tell where we might use each.

5. State the most common units of capacity (and volume), including liquid, dry, and cubic measures.

6. State some units suggested by these numbers:

$\frac{22}{23}$ 0.125 325 ft. 263 bu. 5000 300

7. In the fraction $\frac{21}{25}$, what is the fractional unit? What term of the fraction tells you this? What term tells how many such units are taken?

WRITTEN EXERCISE

1. Express 233 units, each unit being $\frac{1}{427}$; 1 ft.; 0.001.

2. Express in terms of the next smaller unit in the table: 37.5 ft., 48 gal., 23 mi., 48 yd., 75 lb., 6 T., 81 sq. ft.

3. Express in terms of the next larger unit in the table: 37 in., 42 ft., 36 oz., 960 rd., 4000 lb., 100 qt., 1728 in.

4. Express in terms of the unit 1 in. the following: 9 rd., 7 yd., 6 mi., 4 ft., 3 yd., 7 ft., $3\frac{1}{2}$ ft., $7\frac{1}{2}$ yd., $6\frac{1}{2}$ rd., $2\frac{1}{2}$mi.

ORAL EXERCISE

1. In the common fraction $\frac{3}{4}$, name the terms and tell what each expresses about the fraction.

2. The fraction $\frac{3}{4}$ may be thought of as meaning 3 of what parts of a unit? Or it may be thought of as $\frac{1}{4}$ of how many units?

3. If a yard is called *one*, a foot will be called what fraction? An inch will be called what fraction? A rod will be called what mixed number?

4. How is a fraction reduced to lower terms? When is it said to be in its lowest terms? Why do you ever need to reduce fractions to lowest terms?

5. How do you proceed to add one fraction to another? to subtract one fraction from another? Why is this?

6. How do you know that $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$? Illustrate at the board by taking $\frac{1}{2}$ of $\frac{1}{3}$ of a foot. Show also that $\frac{1}{3}$ of $\frac{1}{2}$ has the same value as $\frac{1}{2}$ of $\frac{1}{3}$.

WRITTEN EXERCISE

1. Draw a rectangle 1 in. long. Find $\frac{1}{2}$ of $\frac{1}{4}$ of it; also $\frac{1}{4}$ of $\frac{1}{2}$ of it. Show that $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{4}$ of $\frac{1}{2}$.

2. In the same way show by a line that 2 times $\frac{1}{3}$ in. = $\frac{2}{3}$ in., and that $\frac{1}{3}$ of 2 in. = $\frac{2}{3}$ in.

3. Find the value of $\frac{1}{5}$ of $\frac{2}{3}$ in., representing the work by lines. Also find the value of $\frac{2}{3}$ of $\frac{1}{5}$ in.

4. In the same way show that $\frac{2}{5}$ of $\frac{2}{3}$ in. = $\frac{4}{15}$ in.; also that $\frac{2}{3}$ of $\frac{2}{5}$ in. = $\frac{4}{15}$ in.; also that $\frac{2}{5}$ of $\frac{3}{4} = \frac{3}{4}$ of $\frac{2}{5}$.

5. Mark an inch line into halves, and also into thirds. Show from it that the ratio of $\frac{1}{2}$ to $\frac{1}{3}$ is 3 : 2, or that $\frac{1}{2} \div \frac{1}{3} = \frac{3}{2}$. Show also that $\frac{1}{3} \div \frac{1}{2} = \frac{2}{3}$.

ORAL EXERCISE

1. Read: $2\frac{3}{8}$, 2 ft. 3 in. Explain the difference in scales.

2. Read: 5.8, 5 lb. 8 oz., 5 hr. 8 min. State the scales.

3. Recite the tables of length, square measure, cubic measure, dry and liquid measure, weight, United States money, and time; also the table of dozens and gross.

4. Express as inches: 2 ft. 3 in., 5 ft. 8 in., 100 ft. 9 in., 1 yd., 1 yd. 2 in., 1 yd. 1 ft., 1 yd. 10 in.

5. Express as feet: 3 yd., 3 yd. 2 ft., 7 yd., 7 yd. 1 ft., 10 yd., 60 in., 2 rd., 4 rd., 2 rd. 3 ft., 2 rd. 2 yd.

6. Express as pounds or fractions of a pound: 32 oz., 8 oz., 4 oz., 1 T., 2 T., $3\frac{1}{2}$ T., 4 cwt., 1 T. 2 cwt.

7. Express as quarts, liquid or dry as the case may be: $20\frac{1}{2}$ gal., 2 gal., 2 bu., 3 pk., 2 pt., 5 gal. 2 qt., 2 pk. 1 qt.

WRITTEN EXERCISE

1. Write 3456 as dozens; as gross; as great gross.

2. Write as inches: 17 ft., 21 ft. 6 in., 3 yd., 4 rd.

3. Write as feet: 36 in., 144 in., 1728 in., 54 yd., $\frac{3}{8}$ rd.

4. Write as pounds: $2\frac{1}{2}$ T., 3.25 T., 144 oz., 3 T. 1520 lb., 4 cwt. 27 lb., 1584 oz.

5. Write as quarts: 53 gal., 26 bu., 17 gal. 3 qt., 98 pt., 31 bu. 2 pk., $31\frac{1}{2}$ gal., $75\frac{3}{4}$ gal., $17\frac{1}{2}$ bu.

6. Express as units: 6 gross 3 doz., $7\frac{1}{2}$ gross, $15\frac{1}{3}$ doz., 6 great gross, $15\frac{3}{4}$ doz., $\frac{1}{6}$ doz., $27\frac{1}{2}$ gross.

7. Explain what is meant by a uniform scale; a varying scale. Give an illustration of each.

8. Tell why a varying scale is not, in general, so good as a uniform scale. Illustrate by examples.

ADDITION REVIEWED

8. The principle of addition. The principle of addition is the same whether we add integers, fractions, denominate numbers, or expressions containing letters.

(1)	(2)	(3)
6 8	6 . 8	$6\frac{8}{11}$
9 7	9 . 7	$9\frac{7}{11}$
(15)(15) =	(15)·(15) =	$15\frac{15}{11}$ =
165, because 15 units = 1 ten + 5 units.	16.5, because 15 tenths = 1 + 5 tenths.	$16\frac{4}{11}$, because $\frac{15}{11} = 1\frac{4}{11}$.

In each case the number of units is the same. In each the addition 7 + 8 requires us to add 1 to the next higher order.

In oral addition, as of 68 and 97, it is usually better to begin at the left. In this case say: "97, 157, 165"; that is, 97 + 60 = 157, 157 + 8 = 165.

ORAL EXERCISE

Add, stating only the answers:

1. 23 49	2. 61 28	3. 37 48	4. 29 63	5. 34 48
6. 34 85	7. 26 96	8. 82 75	9. 68 68	10. 47 53
11. 121 69	12. 132 48	13. 126 57	14. 235 75	15. 128 89

16. Add 2 in., 3 in., 3 in., 7 in., 11 in. Express the result also as feet and a fraction; as feet and inches.

17. What do we mean by addition? by addends? by the sum? How do we check (or prove) our additions? Give examples.

WRITTEN EXERCISE

Add, checking (proving) the work by adding in reverse order. Time yourself.

1.	472	2.	47 ft.	2 in.	3.	4 mi.	7 yd.	2 ft.	4.	47 bu.	2 pk.
	691		69	1		6	9	1		69	1
	580		58			5	8			58	
	432		43	2		4	3	2		43	2
	841		84	1		8	4	1		84	1
	570		57			5	7			57	

5. Reduce $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{6}$ to twelfths and add. Also reduce to twenty-fourths and add.

6. Reduce $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$ to eighths and add. Also reduce to decimal fractions and add, checking by reducing the result to an integer plus a common fraction.

7. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$, by reducing to sixths; to twelfths; to eighteenths. Show that the results are equal.

8. Add $\frac{1}{8}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{2}{3}$, $\frac{7}{12}$. (Which is the better plan in this case, to reduce to the denominator 120, or to decimal fractions?)

9. Add $\frac{3}{5}$, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{7}{10}$. (To what common denominator may these fractions be reduced? Is it better to reduce to decimal fractions? Why?)

10. Add the following:

$5x + 1y$	501	5 ft.	1 in.	5 lb.	1 oz.
$6x + 5y$	605	6	5	6	5
$4x + 3y$	403	4	3	4	3
$7x + 6y$	706	7	6	7	6
$9x + 8y$	908	9	8	9	8

Notice that the first of these results reduces to the second if $x = 100$ and $y = 1$; to the third if $x = 1$ ft., $y = \frac{1}{12}$ ft., or 1 in.; to the fourth if $x = 1$ lb., $y = \frac{1}{16}$ lb., or 1 oz.

Add, checking the work by adding in reverse order. Time yourself.

11.	12.	13.	14.
$31.50	$62.35	$20.00	$35.75
26.75	49.82	35.40	87.66
15.42	81.46	20.76	92.33
51.13	73.98	83.92	83.47
73.06	20.05	99.81	62.54
52.70	30.00	78.56	81.35

15.	16.	17.	18.
$312.75	$402.75	$209.81	$175.54
63.40	75.75	801.19	285.46
281.00	163.87	634.77	87.92
127.05	91.83	75.23	93.41
31.63	182.48	62.97	62.78
428.70	328.62	47.03	88.39

19.	20.	21.	22.
$486.83	$128.92	$634.81	$189.98
642.75	871.08	528.92	235.42
208.05	346.52	365.19	628.75
100.00	853.50	471.08	832.63
378.92	671.18	275.50	798.49
604.83	832.00	325.50	629.36

23.	24.	25.	26.
$1276.35	$3246.81	$8234.61	$2983.41
397.63	2981.26	8327.92	3872.62
428.42	583.42	2842.68	2987.48
608.70	629.87	178.53	293.29
2304.69	3473.92	296.48	981.50
572.96	682.58	378.75	6287.49
3270.05	2987.62	2876.92	832.08

27. The following are some recent statistics of the great industries of the United States.

INDUSTRY	Number of Establishments	Money Invested	Number of Wage Earners	Annual Wages	Cost of Materials	Value of Products
Wool Manuf. .	2,636	$415,075,713	264,021	$92,499,262	$250,805,214	$427,905,020
Cotton Manuf. .	1,051	467,240,157	302,861	86,689,752	176,551,527	339,198,619
Iron and Steel .	661	573,119,275	222,264	120,723,092	522,071,772	803,344,591
Meat Industry .	921	189,198,264	68,534	33,457,013	683,583,577	786,603,670
Lumber . . .	33,035	611,611,524	283,260	104,640,591	317,923,548	566,832,984
Flour	25,258	218,714,104	37,073	17,703,418	475,826,345	560,719,063
Boots and Shoes	1,600	101,795,233	142,922	59,175,883	169,604,054	261,028,580
Publishing . .	15,305	192,443,708	94,604	50,214,051	50,214,904	222,983,569

Find the sums of the various columns.

Pupils should be timed in all such work, and accuracy should be insisted upon by requiring checks.

28. The following are some of the wealthiest countries in the world according to recent statistics, with the approximate money which each has.

COUNTRY	Population	Stock of Gold	Stock of Silver	Stock of Paper Money	Total Stock of Money
United States .	85,000,000	$1,174,600,000	$660,000,000	$437,800,000	
France	40,000,000	903,500,000	419,800,000	134,500,000	
Great Britain .	42,000,000	528,000,000	116,800,000	116,200,000	
Germany . . .	57,000,000	762,800,000	207,500,000	153,400,000	
Russia	131,000,000	714,600,000	103,200,000	——	
Austria-Hungary	47,100,000	257,000,000	80,000,000	39,900,000	
Australasia . .	5,500,000	128,600,000	6,100,000	——	

Add the columns. Fill the last column by adding the three preceding columns crossways to the right. How will you check the work?

SUBTRACTION REVIEWED

9. The principle of subtraction. The principle of subtraction is the same whether we subtract integers, fractions, denominate numbers, or expressions containing letters. The general principle is seen in the following:

(1)	(2)	(3)	(4)
61	6.1	$6\frac{1}{4}$	6 ft. 1 in.
23	2.3	$2\frac{3}{4}$	2 3
38	3.8	$3\frac{2}{4} = 3\frac{1}{2}$	3 ft. 10 in.

In each case the number of units is the same. In each case the subtraction $1 - 3$ requires us to increase the 1 by a unit of the next higher order before subtracting.

In oral subtraction, as of 23 from 61, it is usually better to begin at the left. In this case say: "61, 41, 38"; that is, $61 - 20 = 41$, $41 - 3 = 38$.

ORAL EXERCISE

Subtract, stating only the answers:

1. 82	2. 63	3. 72	4. 81	5. 73
27	19	35	24	49
6. 67	7. 42	8. 95	9. 90	10. 86
38	18	58	32	39
11. 102	12. 105	13. 204	14. 125	15. 143
43	36	95	67	85

16. What is meant by saying that "only units of the same kind can be added"? Can you not add $2 and 3 ct.?

17. What is meant by saying that "only units of the same kind can be subtracted"? Can you not subtract 3 ct. from $2?

WRITTEN EXERCISE

Subtract, checking each result by adding the subtrahend and remainder. Time yourself.

1. $287.50 132.95	2. $351.75 268.19	3. $429.35 228.92	4. $526.41 229.37
5. $391.11 287.52	6. $629.48 297.63	7. $802.05 120.70	8. $329.90 100.95
9. $1235.00 926.75	10. $2634.05 827.50	11. $1963.00 298.50	12. $5283.75 1296.55

13. $285.42 196.55	14. 285 mi. 42 ft. 196 55	15. 285 T. 42 lb. 196 55

Notice the similarity of the figures used in Exs. 13–15.

16. $5432 2354	17. 54 mi. 32 ft. 23 54	18. 54 T. 32 lb. 23 54

Notice the similarity of the figures used in Exs. 16–18.

19. $1,725,342,628 – $632,487,129.

20. $27,142,628,341 – $16,923,437,286.

21. $2,421,002,625.25 – $1,278,101,123.50.

22. 372 153	23. 3.72 1.53	24. 37 ft. 2 in. 15 3	25. 37 lb. 2 oz. 15 3

26. 2 yd. 1 ft. 1 2	27. 2 T. 1 lb. 1 2	28. 2 yr. 1 mo. 1 2

29. 1915 yr. 2 mo. 3 da. 1906 8 15	30. 1920 yr. 0 mo. 6 da. 1909 2 17

31. Write out in your own way a statement of your method of adding numbers.

32. Write out in the same way a statement of your method of subtracting numbers.

33. $2738\frac{3}{7} - 1299\frac{6}{7}$.

34. $8721\frac{1}{7} - 1796\frac{3}{5}$.

35. $6592\frac{1}{3} - 4289\frac{2}{7}$.

36. $2872\frac{4}{5} - 1628\frac{1}{3}$.

37. $7291\frac{1}{2} - 2986\frac{3}{5}$.

38. $9289\frac{3}{8} - 2691\frac{5}{12}$.

39. 40 ft. $4\frac{1}{2}$ in. − 27 ft. $9\frac{1}{3}$ in.

40. 172 lb. $5\frac{1}{2}$ oz. − 69 lb. $10\frac{7}{8}$ oz.

41. 426 gal. $1\frac{3}{8}$ qt. − 197 gal. $3\frac{1}{3}$ qt.

42. $1345.25 + 175\frac{2}{3} + 375 - (\frac{3}{5} + \frac{3}{5} + \frac{3}{5})$.

43. 234 sq. ft. $9\frac{1}{2}$ sq. in. − 175 sq. ft. $98\frac{2}{3}$ sq. in.

44. 148 cu. ft. $521\frac{1}{8}$ cu. in. − 69 cu. ft. $827\frac{3}{5}$ cu. in.

45. 321 da. 6 hr. 2 min. 15 sec. − 125 da. 7 hr. 21.75 sec.

46. 273 da. 5 hr. 3 min. 17 sec. − 169 da. 23 hr. 43.25 sec.

47. The savings bank deposits in this country for the years 1900–1904 are indicated in this table. Find the increase for each year. Also find the increase of 1904 over 1900.

1900 . .	2,449,547,885
1901 . .	2,597,094,580
1902 . .	2,750,177,290
1903 . .	2,935,204,845
1904 . .	3,060,178,611

48. The five states having the largest population are here given, with the population according to a recent census. The population of New York was how much more than that of each other state?

New York . .	7,268,894
Pennsylvania	6,302,115
Illinois . . .	4,821,550
Ohio	4,157,545
Missouri . .	3,106,665

49. By the table of Ex. 48, the population of Pennsylvania was how much more than that of Illinois? of Ohio? of Missouri?

50. By the same table, the population of Illinois was how much more than that of Ohio? of Missouri?

MULTIPLICATION REVIEWED

10. The principle of multiplication. The principle of multiplication is the same for all numbers.

$7.50		
9		
4.50	= 9 times	$0.50
63	= " "	$7
$67.50	= " "	$7.50

7 ft. 5 in.		
9		
45 in.	= 9 times	5 in.
63 ft.	= " "	7 ft.
66 ft. 9 in.	= " "	7 ft. 5 in.

In each case the number of units is the same. The results differ because the first number is on the scale of 10, the second on the scale of 12.

The principle is the same:

Multiply the denominations or orders separately. Then add the partial products, simplifying when possible.

In oral multiplication, as of $7.50 by 9, it is usually better to begin at the left. In this case we say: "$63, $4.50, $67.50."

ORAL EXERCISE

Multiply, stating only the answers:

1. 26 2	2. 37 3	3. 33 4	4. 43 5	5. 53 6
6. 25 7	7. 32 8	8. 54 9	9. 66 11	10. 70 12
11. $2.50 5	12. $7.50 6	13. $8.20 7	14. $3.30 8	15. $5.50 9

16. What is meant by saying that the multiplier must always be an abstract number? Illustrate.

17. What is meant by saying that the product must always be of the same denomination as the multiplicand?

WRITTEN EXERCISE

Multiply as indicated, noticing the similarity in the figures of the multiplicands:

1. By 7: 175, 17 ft. 5 in., 1 mi. 75 ft.
2. By 37: 427, 42 lb. 7 oz., 4 mi. 27 rd.
3. By 13: 936, $9.36, 93 ft. 6 in., 9036.
4. By 25: 428, 4 sq. ft. 28 sq. in., 4028.
5. By 1325: $63.25, 63 mi. 25 rd., 63,025.
6. By 59: $19.20, 19 gal. 2 qt., 19 yd. 2 ft.
7. By 2007: $1271.50, 127 da. 1 hr. 50 min.
8. By 625: 1728, 17 sq. ft. 28 sq. in., 17,028.

Multiply, writing the number of minutes required for Exs. 9–33:

9. 2765	10. 3402	11. 4246	12. 1563
127	348	293	892
13. 4762	14. 2983	15. 5786	16. 5999
987	896	469	987
17. $175.27	18. $287.62	19. $482.92	20. $576.75
325	625	328	476
21. $247.60	22. $409.75	23. $507.63	24. $707.07
908	607	806	999

25. 27 ft. 8 in.	26. 29 ft. 9 in.	27. 36 ft. 4 in.
17	32	43
28. 82 ft. 7 in.	29. 12 lb. 8 oz.	30. 16 lb. 9 oz.
68	2	23
31. 17 lb. 15 oz.	32. 23 lb. 4 oz.	33. 24 ft. 9 in.
27	24	48

11. Multiplying by fractions. In multiplying by fractions we may use the following forms to show the similarity, although we have shorter ones for actual work.

69	69	6.9	$\frac{4}{5}$
$\frac{2}{3}$	$.2$	$.2$	$\frac{2}{3}$
$23 = \frac{1}{3}$ of 69	$6.9 = .1$ of 69	$.69 = .1$ of 6.9	$\frac{4}{15} = \frac{1}{3}$ of $\frac{4}{5}$
2	2	2	2
$46 = \frac{2}{3}$ of 69	$13.8 = .2$ of 69	$1.38 = .2$ of 6.9	$\frac{8}{15} = \frac{2}{3}$ of $\frac{4}{5}$

In multiplying 69 by $\frac{2}{3}$ we simply *think* that $\frac{1}{3}$ of 69 is 23, and we multiply (mentally) 23 by 2.

In multiplying 6.9 by .2 we simply *think* that tenths multiplied by tenths is hundredths, and we multiply 69 by 2 and express the result as hundredths.

To multiply $3\frac{1}{3}$ by $2\frac{1}{2}$ we may say:

$$2\frac{1}{2} = \frac{5}{2},\quad 3\frac{1}{3} = \frac{10}{3},\quad \frac{5}{\cancel{2}} \times \frac{\overset{5}{\cancel{10}}}{3} = \frac{25}{3} = 8\frac{1}{3}.$$

ORAL EXERCISE

1. $\frac{3}{4}$ of $\frac{7}{8}$.	2. $\frac{2}{3}$ of $\frac{4}{7}$.	3. $\frac{2}{3}$ of $\frac{3}{4}$.
4. $\frac{5}{6}$ of $\frac{3}{4}$.	5. $\frac{1}{2}$ of $\frac{4}{5}$.	6. $\frac{1}{3}$ of $\frac{6}{7}$.
7. $\frac{1}{4}$ of $\frac{8}{9}$.	8. $\frac{4}{5}$ of $\frac{5}{8}$.	9. $\frac{7}{8}$ of $\frac{3}{10}$.
10. $\frac{3}{5}$ of $\frac{5}{7}$.	11. $\frac{1}{5}$ of $\frac{10}{11}$.	12. $\frac{5}{7}$ of $\frac{14}{15}$.
13. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$.	14. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.	15. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$.
16. $\frac{3}{5}$ of $\frac{5}{8}$ of 8.	17. $\frac{2}{3}$ of $\frac{3}{4}$ of 10.	18. $\frac{1}{3}$ of $\frac{3}{7}$ of 7.
19. $\frac{5}{6}$ of $\frac{1}{2}$ of $\frac{12}{13}$.	20. $\frac{2}{7}$ of $\frac{1}{2}$ of 14.	21. $\frac{1}{4}$ of $\frac{4}{5}$ of 50.
22. $\frac{1}{3}$ of $2\frac{1}{2}$.	23. $\frac{1}{4}$ of $1\frac{1}{3}$.	24. $\frac{1}{2}$ of $3\frac{1}{2}$.
25. $\frac{3}{4}$ of 44.	26. $\frac{2}{3}$ of 33.	27. $\frac{2}{3}$ of $1\frac{1}{2}$.
28. $\frac{5}{7}$ of 49.	29. $\frac{5}{7}$ of 77.	30. $\frac{4}{5}$ of 25.

31. How much will $\frac{5}{8}$ yd. of silk cost at 64¢ a yard?

32. How much will $\frac{7}{8}$ yd. of velvet cost at \$1.60 a yard?

WRITTEN EXERCISE

1. 1.5×324.
2. $3\frac{4}{5} \times 65$.
3. $9\frac{2}{3} \times 72$.
4. $\frac{1}{2}$ of $73\frac{1}{2}$.
5. $\frac{2}{3}$ of $16\frac{3}{4}$.
6. $\frac{4}{5}$ of $19\frac{1}{2}$.
7. $2\frac{1}{2} \times 3\frac{2}{3}$.
8. $4\frac{2}{5} \times 6\frac{7}{8}$.
9. $5\frac{3}{8} \times 6\frac{1}{5}$.
10. State a brief rule for multiplying by a decimal fraction. Illustrate by 0.3 of 0.22.
11. State a brief rule for multiplying one common fraction by another. Illustrate by $\frac{2}{3}$ of $\frac{5}{7}$.

Multiply as indicated, timing yourself:

12.	23.4 4.5	13.	17.6 8.4	14.	35.7 6.9	15.	82.7 9.8
16.	99.8 7.9	17.	2.75 .35	18.	7.48 1.25	19.	8.09 8.09
20.	.632 8.25	21.	.473 2.48	22.	6.42 3.27	23.	89.2 62.8
24.	34.9 30.5	25.	1.472 3.25	26.	6.043 1.37	27.	2.481 6.25
28.	0.735 6.25	29.	1.326 4.32	30.	2.307 53.6	31.	298.7 .634
32.	812.8 .437	33.	296.4 .0124	34.	82.32 .063	35.	487.6 2.083
36.	$14.92 2.35	37.	$15.75 3.42	38.	$14.87 2.91	39.	$65.04 9.09
40.	1484 $3.2\frac{1}{2}$	41.	2636 $4.1\frac{1}{4}$	42.	3792 $6.23\frac{1}{3}$	43.	178.32 $1.24\frac{2}{3}$

44. 49.74 .0426	45. 82.49 .0487	46. 693.4 83.7

47. $\frac{11}{36}$ of $\frac{18}{55}$. 48. $\frac{23}{24}$ of $\frac{12}{115}$. 49. $\frac{17}{32}$ of $\frac{64}{85}$.

50. $\frac{21}{25}$ of $\frac{75}{84}$. 51. $\frac{25}{32}$ of $\frac{16}{75}$. 52. $\frac{11}{12}$ of $\frac{13}{16}$.

53. \$327.76 $13\frac{1}{2}$	54. \$139.83 $37\frac{2}{3}$	55. \$164.24 $34\frac{3}{8}$
56. \$325.84 $39\frac{3}{4}$	57. \$841.40 $63\frac{5}{7}$	58. \$935.44 $87\frac{3}{4}$
59. 21 ft. 8 in. $15\frac{1}{2}$	60. 38 lb. 2 oz. $23\frac{3}{4}$	61. 62 yd. 27 in. $25\frac{2}{3}$

62. $127\frac{2}{3} \times 135\frac{1}{4}$. 63. $229\frac{5}{8} \times 213\frac{2}{3}$. 64. $723\frac{3}{5} \times 734.8$.

65. $209\frac{5}{8} \times 235.5$. 66. $327\frac{2}{3} \times 333\frac{1}{3}$. 67. $187\frac{3}{16} \times 175\frac{1}{8}$.

68. At \$1.56 a yard, what will $14\frac{3}{4}$ yd. of silk cost?

69. At \$1.28 a yard, what will $37\frac{3}{8}$ yd. of carpet cost?

70. At \$5.60 a ton, what will 17 T. 250 lb. of coal cost?

71. What is the cost of $37\frac{1}{2}$ rd. of fence at \$$1.12\frac{1}{2}$ a rod?

72. When hay is selling at \$$9\frac{3}{4}$ a ton, what will $23\frac{1}{8}$ T. cost?

73. What is the number resulting from taking 32 ft. 9 in. as an addend 37 times?

74. At $78\frac{3}{4}$ lb. to the yard, what is the weight of a mile of double rails for a track?

75. A hexagon of equal sides is measured, and each side is found to be $19\frac{7}{32}$ in. What is the perimeter?

76. What is the number resulting from taking $17\frac{3}{7}$ as an addend 23 times, and then adding $\frac{1}{3}$ of $17\frac{3}{7}$ to the sum?

DIVISION REVIEWED

12. Nature of division. We have seen that division is the inverse of multiplication. That is,

because	2.5 times $\$40 = \$100$,
therefore	$\$100 \div \$40 = 2.5$,
and	$\$100 \div 2.5 = \$40$.

13. Two cases. There are, therefore, two cases of division:

1. *If the dividend and divisor are like numbers, the quotient is abstract.*

2. *If the dividend is concrete and the divisor abstract, the quotient is like the dividend.*

14. Similar classes in the first case. The similarity for various classes of numbers appears from the following:

572 ft. $\div$ 24 in. may be reduced to 572 ft. $\div$ 2 ft. $= 286$, or it may be reduced to 6864 in. $\div$ 24 in. $= 286$;

$55.2 \div 0.24$ may be reduced to $55.20 \div 0.24$, and treated as if it were $5520 \div 24 = 230$;

$\frac{2}{3} \div \frac{4}{5}$ may be reduced to $\frac{10}{15} \div \frac{12}{15}$, and treated as if it were $10 \div 12 = \frac{5}{6}$. (See § 20 for the practical work.)

That is, in this case, *dividend and divisor are reduced to like numbers, or denominations, before dividing.*

15. Similar classes in the second case. In the second of these cases the similarity for various classes of numbers appears from the following examples:

$\$10 \div .8 = \$100 \div 8$, by multiplying each by 10;
$\$10 \div \frac{4}{5} = \$50 \div 4$, " " " " 5,

although we have a simpler method for dividing common fractions, as explained in § 20, where it is shown that $10 \div \frac{4}{5} = 10 \times \frac{5}{4} = 12\frac{1}{2}$.

ORAL EXERCISE

Divide as indicated in Exs. 1–30:

1. 355 ÷ 5.	2. 424 ÷ 4.	3. 126 ÷ 3.
4. 275 ÷ 5.	5. 536 ÷ 4.	6. 315 ÷ 3.
7. 636 ÷ 6.	8. 637 ÷ 7.	9. 728 ÷ 8.
10. 189 ÷ 9.	11. 217 ÷ 7.	12. 640 ÷ 8.
13. 819 ÷ 9.	14. 504 ÷ 7.	15. 656 ÷ 8.
16. 121 ÷ 11.	17. 144 ÷ 12.	18. 156 ÷ 12.
19. 143 ÷ 13.	20. 154 ÷ 14.	21. 315 ÷ 15.
22. 320 ÷ 16.	23. 510 ÷ 17.	24. 360 ÷ 18.
25. 620 ÷ 20.	26. 420 ÷ 21.	27. 660 ÷ 22.
28. 253 ÷ 23.	29. 144 ÷ 24.	30. 625 ÷ 25.

Divide as indicated in Exs. 31–45, stating the remainders:

That is, 186 ÷ 17 = 10, and 16 remainder.

31. $145 ÷ 12.	32. $136 ÷ 13.	33. $150 ÷ 14.
34. $157 ÷ 15.	35. $330 ÷ 16.	36. $181 ÷ 17.
37. $193 ÷ $18.	38. $200 ÷ $19.	39. $219 ÷ $20.
40. $430 ÷ $21.	41. $444 ÷ $22.	42. $691 ÷ $23.
43. 723 ft. ÷ 24.	44. 630 yd. ÷ 25.	45. 631 in. ÷ 30.

Divide as indicated in Exs. 46–63, including fractions in the quotients:

That is, $186 \div 17 = 10\frac{16}{17}$.

46. 1526 ÷ 5.	47. 2751 ÷ 5.	48. 6017 ÷ 5.
49. 2419 ÷ 2.	50. 3221 ÷ 2.	51. 1683 ÷ 2.
52. 3631 ÷ 3.	53. 1234 ÷ 3.	54. 6094 ÷ 3.
55. 4805 ÷ 4.	56. 8005 ÷ 4.	57. 6013 ÷ 6.
58. 7146 ÷ 7.	59. 8009 ÷ 8.	60. 8183 ÷ 9.
61. 6477 ÷ 10.	62. 5239 ÷ 10.	63. 1211 ÷ 11.

16. General principle of division. We have learned how to divide by *long division* and by *short division*. The general principle is, however, the same for both. Consider the case of $625 \div 25$.

$$
\begin{array}{rl}
 & 25 \\
25 & \overline{)625} \\
 & \underline{500} = 20 \text{ times } 25 \\
 & 125 \\
 & \underline{125} = 5 \quad \text{``} \quad 25
\end{array}
\qquad\qquad
\begin{array}{l}
25\underline{)625} \\
25 \\
\text{is the same as} \\
25\underline{)500 + 125} \\
20 + 5 = 25
\end{array}
$$

In each case we see that $600 \div 25 =$ no hundreds; $620 \div 25 = 2$ tens, for 2 tens $\times 25 = 500$, and there is 125 left to be divided. Also, $125 \div 25 = 5$. Therefore the result is 25.

17. Division by a decimal. *In dividing by a decimal fraction always multiply both dividend and divisor by such a number as shall make the divisor a whole number.*

Thus, in the case of $62.5 \div 0.25$, multiply both by 100 and divide 6250 by 25.

WRITTEN EXERCISE

1. Divide 987 by 35 and write out the explanation.

2. Divide 575 by 25 in the three ways shown above.

3. Divide 506 by 22 in the three ways shown above.

4. At $26\frac{1}{2}$ ct. a yard, how many yards of cloth can be bought for \$36.57? If 138 yd. cost \$36.57, what will 1 yd. cost?

5. Derby hats cost from \$24 to \$42 a dozen. Make up two problems, similar to those in Ex. 4, state them, and solve.

6. Name and define the four terms used in division.

7. Write out a rule for dividing one number by another.

Divide as indicated, seeing how many problems you can solve in five minutes; in ten minutes:

8. 1554 ÷ 37.	9. 1924 ÷ 52.	10. 2793 ÷ 57.
11. 3844 ÷ 62.	12. 5593 ÷ 49.	13. 2701 ÷ 73.
14. 5402 ÷ 37.	15. 3402 ÷ 81.	16. 1225 ÷ 18.
17. 1089 ÷ 33.	18. 6822 ÷ 36.	19. 8888 ÷ 808.
20. 5203 ÷ 121.	21. 20,273 ÷ 97.	22. 17,841 ÷ 57.
23. 2541 ÷ 121.	24. 20,698 ÷ 98.	25. 22,248 ÷ 72.
26. 10,449 ÷ 81.	27. 13,024 ÷ 407.	28. 14,838 ÷ 209.
29. 30,092 ÷ 121.	30. 27,968 ÷ 304.	31. 17,094 ÷ 777.
32. 80,091 ÷ 809.	33. 19,908 ÷ 237.	34. 14,985 ÷ 666.
35. 4291)55,783.	36. 2075)47,725.	37. 6772)277,652.
38. 3227)203,301.	39. 7407)674,037.	40. 2469)224,679.

41. \$569.25 ÷ \$24.75.	42. \$570.24 ÷ \$17.82.
43. \$1309.00 ÷ \$29.75.	44. \$2521.48 ÷ \$48.49.

45. If 35 floor tiles together weigh 113.75 lb., what is the average weight of each?

46. If $1548 is divided equally among a dozen persons, how much is the share of each?

47. If $2193.75 is divided equally among 15 persons, how much is the share of each?

48. If a piece of land containing 12.75 acres is cut into 17 equal building lots, what is the area of each?

49. If the dividend is 420, the quotient 32, and the remainder 4, what is the divisor?

50. If the dividend is 73 times the remainder, and the quotient is 24, and the divisor is 12, what is the remainder?

18. Division of denominate numbers by abstract numbers. Divide 33 ft. 9 in. by 15.

We divide 33 ft. by 15 and have 2 ft., with a remainder of 3 ft., which, reduced to inches (36 in.) and added to 9 in., equals 45 in., which is still to be divided. Again, 45 in. ÷ 15 = 3 in.

$$
\begin{array}{r|rr}
 & \text{2 ft.} & \text{3 in.} \\
\hline
15) & \text{33 ft.} & \text{9 in.} \\
 & \text{30 ''} & \\
\hline
 & \text{3 ft.} = & \text{36 in.} \\
\hline
 & & \text{45 in.} \\
 & & \text{45 ''} \\
\hline
\end{array}
$$

19. Division of denominate numbers by denominate numbers. Divide 23 ft. 4 in. by 3 ft. 4 in. Here either

1. 23 ft. 4 in. ÷ 3 ft. 4 in. = $23\frac{1}{3}$ ft. ÷ $3\frac{1}{3}$ ft. = $\frac{70}{3}$ ft. ÷ $\frac{10}{3}$ ft. = 7; or
2. 23 ft. 4 in. ÷ 3 ft. 4 in. = 280 in. ÷ 40 in. = 7.

In the first case we reduce to feet; in the second, to inches.

WRITTEN EXERCISE

1. 2043 gal. 3 qt. ÷ 25.
2. 430 yd. 10 in. ÷ 11.
3. 213 ft. 9 in. ÷ 23 ft. 9 in.
4. 652 ft. ÷ 81 ft. 6 in.
5. 518 yd. ÷ 42 yd. 3 ft. 6 in.
6. 1587 lb. ÷ 132 lb. 4 oz.
7. 355 hr. 33 min. 45 sec. ÷ 15.
8. 4868 cu. ft. 1200 cu. in. ÷ 23.
9. 22 hr. 40 sec. ÷ 5 hr. 30 min. 10 sec.
10. How much is an eighth of 116 lb. 8 oz.?
11. 4557 sq. ft. 54 sq. in. ÷ 147 sq. ft. 90 sq. in.
12. How much is half of 2 hr. 53 min. 18 sec.?
13. How much is a quarter of 11 yd. 2 ft. 8 in.?
14. If a cubic foot of gold weighs 1187 lb. 8 oz., and if gold is 19 times as heavy as water, what does a cubic foot of water weigh?
15. If it takes a man 8 hr. 31 min. 24 sec. to walk a certain distance, how long will it take an automobile to travel the same distance, if it goes 12 times as fast?

ORAL EXERCISE

1. Tell why $\frac{2}{3} \div 2 = \frac{1}{3}$; also why $\frac{1}{3} \div 2 = \frac{1}{6}$.

2. Tell why $1 \div \frac{1}{3} = 3$; also why $2 \div \frac{1}{3} = 6$, and $5 \div \frac{1}{3} = 15$.

3. Because $5 \div \frac{1}{3} = 15$, tell why $5 \div \frac{2}{3} = \frac{1}{2}$ of 15, or $\frac{15}{2}$.

4. Because $5 \div \frac{2}{3} = \frac{15}{2}$, tell why $\frac{5}{7} \div \frac{2}{3} = \frac{1}{7}$ of $\frac{15}{2}$, or $\frac{15}{14}$.

5. Because $\frac{5}{7} \div \frac{2}{3} = \frac{15}{14}$, and because $\frac{3}{2}$ of $\frac{5}{7} = \frac{15}{14}$, what operation may be performed in place of dividing one fraction by another?

20. Dividing fractions. In dividing $\frac{5}{7}$ by $\frac{2}{3}$, we may

1. *Reduce to a common denominator and divide the new numerators.* $\frac{15}{21} \div \frac{14}{21} = \frac{15}{14}$, just as $\$15 \div \$14 = \frac{15}{14}$, or 15 ft. $\div$ 14 ft. $= \frac{15}{14}$. Or we may

2. *Invert the divisor and multiply.* That is, $\frac{5}{7} \div \frac{2}{3} = \frac{5}{7} \times \frac{3}{2} = \frac{15}{14}$. We use this operation rather than the other, because it is easier and gives the same result. Always indicate the multiplication first and cancel if possible.

WRITTEN EXERCISE

1. $\frac{7}{8} \div \frac{3}{4}$.	2. $\frac{3}{5} \div \frac{2}{15}$.	3. $\frac{5}{12} \div \frac{7}{8}$.
4. $\frac{9}{16} \div \frac{3}{8}$.	5. $\frac{15}{16} \div \frac{5}{4}$.	6. $\frac{2}{15} \div \frac{7}{8}$.
7. $\frac{3}{11} \div \frac{5}{13}$.	8. $\frac{5}{11} \div \frac{7}{12}$.	9. $\frac{5}{16} \div \frac{3}{32}$.
10. $\frac{14}{15} \div \frac{11}{12}$.	11. $\frac{7}{18} \div \frac{21}{22}$.	12. $\frac{19}{27} \div \frac{38}{81}$.
13. $\frac{15}{39} \div \frac{10}{13}$.	14. $\frac{17}{52} \div \frac{51}{52}$.	15. $\frac{9}{16} \div \frac{5}{32}$.
16. $\frac{15}{7} \div \frac{5}{14}$.	17. $\frac{7}{144} \div \frac{23}{60}$.	18. $\frac{209}{221} \div \frac{17}{19}$.
19. $\frac{137}{138} \div \frac{32}{33}$.	20. $\frac{16}{41} \div \frac{8}{123}$.	21. $\frac{143}{323} \div \frac{17}{13}$.
22. $\frac{123}{322} \div \frac{63}{144}$.	23. $\frac{303}{122} \div \frac{101}{244}$.	24. $\frac{117}{211} \div \frac{99}{422}$.

21. Dividing mixed numbers. In dividing $15\frac{1}{2}$ by $7\frac{3}{4}$, we might

1. *Reduce to a common denominator, and divide the new numerators.* $\frac{62}{4} \div \frac{31}{4} = 62 \div 31 = 2$. Or

2. *Multiply both dividend and divisor by a common denominator, and then divide.* That is, $15\frac{1}{2} \div 7\frac{3}{4} = 62 \div 31$, by multiplying by 4.

But practically we

3. *Reduce to fractional forms and multiply by the inverted divisor.* That is, $15\frac{1}{2} \div 7\frac{3}{4} = \frac{31}{2} \div \frac{31}{4} = \frac{31}{2} \times \frac{4}{31} = 2$.

WRITTEN EXERCISE

1. $21 \div 3\frac{1}{4}$.	2. $83 \div 9\frac{8}{9}$.	3. $62 \div 5\frac{3}{4}$.
4. $38 \div 5\frac{5}{9}$.	5. $32 \div 4\frac{2}{3}$.	6. $17 \div 2\frac{7}{8}$.
7. $59 \div 8\frac{6}{7}$.	8. $41 \div 8\frac{7}{8}$.	9. $18\frac{5}{8} \div 4\frac{7}{8}$.
10. $33 \div 7\frac{4}{5}$.	11. $15\frac{2}{3} \div 6\frac{1}{3}$.	12. $17\frac{1}{8} \div 2\frac{1}{2}$.
13. $12\frac{1}{2} \div 3\frac{1}{2}$.	14. $15\frac{1}{2} \div 3\frac{1}{3}$.	15. $78 \div 9\frac{5}{32}$.
16. $16\frac{5}{9} \div 4\frac{1}{9}$.	17. $61\frac{4}{5} \div 8\frac{1}{7}$.	18. $324 \div 7\frac{1}{5}$.
19. $15\frac{7}{8} \div 5\frac{1}{3}$.	20. $82 \div 8\frac{3}{16}$.	21. $61\frac{7}{8} \div 5\frac{3}{16}$.
22. $75 \div 6\frac{5}{12}$.	23. $27\frac{1}{2} \div 33\frac{1}{3}$.	24. $32\frac{5}{12} \div 6\frac{1}{3}$.
25. $51\frac{1}{16} \div 2\frac{1}{8}$.	26. $29\frac{1}{5} \div 59\frac{3}{5}$.	27. $62\frac{7}{8} \div 15\frac{1}{3}$.
28. $31\frac{5}{8} \div 15\frac{13}{16}$.	29. $13\frac{9}{10} \div 4\frac{2}{5}$.	30. $258 \div 15\frac{3}{17}$.
31. $776.5 \div 129\frac{5}{12}$.	32. $120\frac{1}{16} \div 14\frac{7}{16}$.	33. $52\frac{1}{11} \div 17\frac{4}{11}$.

34. How many strips of cloth, each $5\frac{3}{8}$ yd. long, can be cut from a strip $37\frac{5}{8}$ yd. long?

35. How many city lots, each $31\frac{3}{4}$ ft. front, can be cut from a piece having $222\frac{1}{4}$ ft. front?

36. How many yards of cloth at $31\frac{1}{3}$¢ a yard can be bought for a dollar and a half?

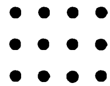
GENERAL PRINCIPLES OF THE OPERATIONS

ORAL EXERCISE

1. If I put 4 ct. with 3 ct., do I have the same result as if I put 3 ct. with 4 ct.? What does this tell about the order of adding numbers?

2. If I wish to add 4, 3, and 5, shall I get the same result by first adding $4 + 3$ and then 5, as by first adding $3 + 5$ and then 4? What does this tell about grouping numbers?

3. How does 3×4 compare with 4×3? What does this tell about the order of multiplying numbers?



22. Law of order in addition. *The sum is the same whatever the order of the addends.*

That is, $2 + 3 = 3 + 2$.

23. Law of grouping in addition. *The sum is the same however the addends are grouped.*

That is, $(2 + 3) + 5 = 2 + (3 + 5)$.

24. Law of order in multiplication. *The product is the same whatever the order of the factors.*

That is, $2 \times 3 = 3 \times 2$.

WRITTEN EXERCISE

1. Make a rectangle $\frac{5}{8}$ in. by $\frac{3}{8}$ in., and separate it into $\frac{1}{8}$-in. squares, and show that $5 \times 3 = 3 \times 5$.

2. Show that $(2 \times 3) \times 4 = 2 \times (3 \times 4)$, and state the law of grouping in multiplication.

3. We have found that $2 + 3 = 3 + 2$, and $2 \times 3 = 3 \times 2$. Does $2 \div 3 = 3 \div 2$? Write out a statement about it.

THE PRACTICAL SHORT METHODS

25. But few practical short methods. There are many short methods of operation, and of these a few are really practical.

People who have a great amount of computing now use machines, just as those who have much writing to do now use a typewriter. Here is a picture of an adding machine such as is used by most of the large banks. There are also machines that multiply, divide, and perform other operations.

26. Short methods in addition. 1. Learn to read the columns like a word, without pronouncing, even to yourself, the separate numbers any more than necessary.

2. Accustom yourself to "catch the tens" as much as possible in adding. Thus, in the annexed column, the eye should at once catch the two tens, and see that the result is 22. There is danger of mistakes in skipping numbers to catch the tens. A good computer, however, will know and detect the other combinations as well as he does the tens.

$$\begin{array}{r} 4 \\ 3 \\ 3 \\ 5 \\ 2 \\ 3 \\ \underline{2} \\ 22 \end{array}$$

3. The shortest way of checking the work is to add in the opposite direction.

4. In adding several long columns, keep the sums of each order separate, on a slip of paper, and add these partial sums. At least keep a note of the figures "carried." This enables you to check more readily.

ORAL EXERCISE

Look at the example and state only the sum:

1. 17	2. 23	3. 42	4. 61	5. 15
32	27	69	71	38

6. 15	7. 21	8. 32	9. 18	10. 21
13	12	14	6	10
22	31	42	61	73

11. 26	12. 33	13. 42	14. 81	15. 16
25	12	63	48	17
32	26	48	53	82

16. 21	17. 32	18. 41	19. 19	20. 28
35	76	88	73	29
42	63	82	35	46

21. $75	22. $31	23. 48 ft.	24. 29 in.	25. 32 yd.
32	92	26	36	48
61	86	35	27	62

26. $32	27. $21	28. 36 yd.	29. 63 ft.	30. 81 lb.
41	23	28	21	28
62	49	42	42	96

31. $12	32. $26	33. 26 lb.	34. 82 yd.	35. 29 ft.
42	47	81	63	27
37	35	29	48	36

WRITTEN EXERCISE

See how long it takes to copy these numbers, and to add and check each result. Write the time with the results.

1. $147.50	2. $748.06	3. $375.42	4. $298.04
35.70	126.53	104.41	127.63
61.80	39.40	630.67	701.96
130.50	86.51	49.60	872.37
16.70	28.42	131.49	286.32

5. $359.68	6. $496.48	7. $396.04	8. $982.69
298.76	127.92	72.63	604.05
400.92	600.00	129.87	81.97
640.32	503.52	871.64	631.83
701.24	872.08	280.93	400.00
248.72	172.96	371.40	180.02

9. $1482.63	10. $4761.42	11. $3147.29	12. $4268.75
689.74	3917.63	2873.41	3942.35
2908.37	5238.58	4192.64	2065.08
8517.37	6082.37	128.46	5095.02
9310.26	4098.07	3029.00	2711.40
7091.63	5901.93	7081.00	1029.60
328.04	670.29	283.96	409.62

13.	127 ft.	9 in.	14.	12 lb.	8 oz.	15.	3 yr.	8 mo.	6 da.
	37	6		9	15		5	2	27
	41	8		17	12		3	10	14
	32	5		15	11		1	0	19
	15	8		21	7		3	7	0

16. A merchant has the following sums owing him: \$147.60, \$21.30, \$29.47, \$63.82, \$173.46, \$391.14, \$2.94, \$6.37, \$98.71. What is the total amount?

17. What was the total number of dwellings in the following states in a certain year? Illinois, 845,836; Indiana, 552,495; Michigan, 521,648; Missouri, 593,528; New York, 1,035,180; Ohio, 857,636; Pennsylvania, 1,236,238; Texas, 575,734.

18. What was then the total population of the following cities? New York, 3,437,202; Chicago, 1,698,575; Philadelphia, 1,293,697; St. Louis, 575,238; Boston, 560,892; Baltimore, 508,957; Cleveland, 381,768; Buffalo, 352,387; San Francisco, 342,782; Cincinnati, 325,902; Pittsburg, 321,616.

19. What was then the total number of school children in the following states? Alabama, 733,222; Arkansas, 529,375; Georgia, 885,725; Illinois, 1,589,915; Indiana, 843,885; Iowa, 767,870; Kansas, 527,560; Kentucky, 798,027; Louisiana, 538,267; Massachusetts, 778,110; Michigan, 790,275; Minnesota, 612,990; Mississippi, 633,026; Missouri, 1,105,258; New Jersey, 572,923; New York, 2,146,764; North Carolina, 753,826; Ohio, 1,338,345; Pennsylvania, 2,031,171; South Carolina, 560,773; Tennessee, 780,421; Texas, 1,215,634; Wisconsin, 730,685.

20. The cost of some of our largest war ships is as follows: *Georgia*, \$3,590,000; *Indiana*, \$3,063,000; *Iowa*, \$3,010,000; *Massachusetts*, \$3,063,000; *Nebraska*, \$3,733,600; *New Jersey*, \$3,405,000; *Oregon*, \$3,222,810; *Rhode Island*, \$3,405,000; *Virginia*, \$3,590,000; *California*, \$3,800,000; *Colorado*, \$3,780,000; *Maryland*, \$3,775,000; *Pennsylvania*, \$3,890,000; *South Dakota*, \$3,750,000; *West Virginia*, \$3,885,000. What is their total cost?

27. Short methods in subtraction. The shortest plan of subtraction is the "making change" method. It is better, however, to follow the method with which the class is familiar. All work should be checked by adding the remainder and the subtrahend, the sum being the minuend.

ORAL EXERCISE

Imagine yourself buying goods at a store, and handing the clerk the first amount given, the goods costing the second amount. State the amount of change due you.

1. $1, 72¢.	2. $2, $1.37.	3. $2, $1.67.
4. $3, $2.17.	5. $4, $3.41.	6. $5, $1.27.
7. $5, $2.40.	8. $5, $3.05.	9. $5, $1.70.
10. $10, $6.55.	11. $10, $7.05.	12. $10, $6.05.
13. $10, $5.70.	14. $10, $3.70.	15. $10, $8.67.
16. $15, $12.75.	17. $15, $13.60.	18. $15, $11.42.
19. $20, $16.92.	20. $20, $17.30.	21. $20, $16.05.
22. $25, $19.60.	23. $35, $32.70.	24. $40, $37.06.

25. Subtract 78 from the sum of 45 and 57.

26. Subtract 49 from the difference between 37 and 88.

27. Subtract the difference between 69 and 72 from their sum.

28. The difference is 48 and the minuend is 63. What is the subtrahend?

29. How much less is the difference between 68 and 81 than that between 49 and 68?

30. How much greater is the difference between 37 and 64 than that between 37 and 19?

WRITTEN EXERCISE

See how long it takes to copy these numbers, and to subtract and check. Write the time with the last result.

1. $175.62	2. $129.87	3. $208.09	4. $400.12
81.98	67.98	98.78	69.84
5. $170.08	6. $370.42	7. $209.18	8. $302.78
69.99	91.64	87.69	62.95
9. $802.65	10. $200.00	11. $608.75	12. $672.93
69.81	93.74	35.82	98.94
13. $409.98	14. $682.70	15. $200.00	16. $325.34
23.49	93.81	187.60	128.92
17. $429.81	18. $308.72	19. $298.05	20. $408.00
149.92	129.85	109.72	129.60
21. $1087.50	22. $2023.40	23. $3029.62	24. $3278.48
963.90	1285.42	2172.58	1284.69

25. 37 ft. 8 in.	26. 41 lb. 8 oz.	27. 39 yd. 9 in.
29 9	29 12	27 16
28. 9 mo. 8 da.	29. 12 yr. 4 da.	30. 6° 9′ 21″
6 15	8 92	1 27 42
31. 15 mi. 125 ft.	32. 134 ft. 3 in.	33. 42° 6′ 23″
8 650	69 9	29 18 50
34. 67 gal. 1 qt.	35. 5 yr. 2 mo.	36. 75° 18′ 48″
19 3	2 11	29 37 59

37. A man deposited $42.96 and $125 in a bank, and drew out $103.75. How much was left?

38. A man had $287.50 in a bank, and he drew out $12.75 and $37.63. How much was left?

39. A man deposited $15 a week in a bank, for 13 weeks. During this time he drew out $7.29, $6.35, $14.20, and $5.75. How much had he left at the end of the time?

40. In a year when this country produced 522,229,505 bu. of wheat, and 800,125,989 bu. of oats, what was the difference in amount?

41. There were 841,201,546 acres of farm land in this country in a certain year, 414,793,191 being improved. How many acres were unimproved?

42. Of the total number of farms in this country, 5,739,-657, in a certain year, there were 5,537,731 with buildings on them. How many had no buildings?

43. The value of this farm property was then $20,514,-001,838, of which $13,114,492,056 was the value of the land. What was the value of the buildings, implements, and stock?

44. By how much did the population of London, 4,536,-063, in a certain year, exceed that of New York, 3,437,202? of Chicago, 1,698,575? of Boston, 560,892?

45. When the irrigation systems of California had cost $19,181,610, and those of Colorado $11,758,703, how much more had California spent on irrigation than Colorado?

46. The states having the greatest railroad mileage in a certain year were: Illinois, 11,116.18 mi.; Pennsylvania, 10,480.35 mi.; and Texas, 10,189.04 mi. By how much did that of Illinois exceed each of the other two?

28. Short methods in multiplication. There are a few multipliers which are used so frequently in business that it is desirable to know the shortest methods of multiplying by them. These include the numbers 5, 10, 25, 125 (with 12.5 or $12\frac{1}{2}$), $33\frac{1}{3}$, and their various per cents, together with 9 and 11.

ORAL EXERCISE

1. Multiply by 10: 43, 6.7, 0.42, 300, $260.10.
2. State the short method of multiplying by 10.
3. Multiply by 100: 75, 8.2, 5.2, $600, $425.75.
4. State the short method of multiplying by 100.
5. Since $5 = \frac{10}{2}$, to multiply by 5 why may we annex a 0, or move the decimal point one place to the right, and then divide by 2? Illustrate by an example.
6. Multiply by 5: 40, 66, 84, 98, 104, 124, 666, 42.2.
7. Since $25 = \frac{100}{4}$, what is the short method of multiplying by 25? Illustrate by an example.
8. Multiply by 25: 48, 64, 82, 34, 81, 13, 448, 204, 216.
9. Since $33\frac{1}{3} = \frac{100}{3}$, what is the short method of multiplying by $33\frac{1}{3}$? Illustrate by an example.
10. Multiply by $33\frac{1}{3}$: 60, 36, 42, 51, 66, 123, 312, 111, 66.3, 42.6, 15.63, 3.3333, 12,000, 93,300, 690,000.
11. Since $125 = \frac{1000}{8}$, what is the short method of multiplying by 125? Illustrate by an example.
12. Multiply by 125: 48, 56, 12, 36, 44, 8, 8.8, 4.8, 4.88, 1.688, 800, 8800, 1600, 16,000, 32,000.

13. 50×44.	14. 50×68.	15. 50×124.
16. 50×240.	17. 250×12.	18. 250×16.
19. 250×24.	20. 250×40.	21. 125×16.
22. 1250×16.	23. 1250×24.	24. 1250×32.

25. Since $5\% = \frac{5}{100} = \frac{1}{20}$, to multiply by 5% is to divide by 10 and by what other number? Illustrate.

26. How much is 5% of 40? of 70? of 90? of \$230? of \$350? of 400 ft.? of \$2.60? of 4220? of 84,000?

27. Since $25\% = \frac{1}{4}$, what is the short method of finding 25% of a number? Illustrate.

28. How much is 25% of 620? of 460? of \$30? of \$924? of \$424? of \$4.32? of 1600? of 32,000?

29. Since $33\frac{1}{3}\% = \frac{1}{3}$, what is the short method of finding $33\frac{1}{3}\%$ of a number? Illustrate.

30. How much is $33\frac{1}{3}\%$ of 48? of \$63? of \$720? of \$1.23? of 456 ft.? of 1230 mi.? of 72,000? of 15.63?

31. Since $12\frac{1}{2}\% = \frac{1}{8}$, what is the short method of finding $12\frac{1}{2}\%$ of a number? Illustrate.

32. How much is $12\frac{1}{2}\%$ of 48? of 56? of \$720? of \$6.40? of 800 ft.? of \$1600? of 72,000? of 16.08?

33. Increase \$48 by 25% of itself.

34. Decrease \$800 by 5% of itself.

35. Increase \$160 by $12\frac{1}{2}\%$ of itself.

36. Decrease \$720 by $33\frac{1}{3}\%$ of itself.

37. What will 125 erasers cost at 5¢ each?

38. What will 72 doz. eggs cost at $12\frac{1}{2}$¢ a dozen?

39. What will 24 yd. of silk cost at \$1.25 a yard?

40. What will 36 yd. of cloth cost at 25¢ a yard?

41. What will 16 yd. of cloth cost at $12\frac{1}{2}$¢ a yard?

42. What will \$8.40 worth of goods cost at 25% off?

43. What will \$3.60 worth of goods cost at $33\frac{1}{3}\%$ off?

44. Find 25% of $33\frac{1}{3}\%$ of 12; of 600; of 1200; of 1500.

45. Find $12\frac{1}{2}\%$ of 50% of 16; of 160; of 1600; of 4800.

46. Find 5% of 10% of $12\frac{1}{2}\%$ of 80; of 1600; of 3200.

29. Summary of short methods of multiplying.

1. *To multiply by* 10, *move the decimal point one place to the right, annexing zero if necessary.*

2. *To multiply by* 100, *move the decimal point two places to the right, annexing zeros if necessary.*

3. *To multiply by* 5, *multiply by* 10 *and divide by* 2.

4. *To multiply by* 25, *multiply by* 100 *and divide by* 4.

5. *To multiply by* 125, *multiply by* 1000 *and divide by* 8.

6. *To multiply by* $33\frac{1}{3}$, *multiply by* 100 *and divide by* 3.

7. *To find* 5%, *divide by* 10 *and by* 2.

8. *To find* 25%, *divide by* 4; 50%, *by* 2; 20%, *by* 5.

9. *To find* $33\frac{1}{3}$%, *take* $\frac{1}{3}$; $66\frac{2}{3}$%, *take* $\frac{2}{3}$.

10. *To find* $12\frac{1}{2}$%, *take* $\frac{1}{8}$; $16\frac{2}{3}$%, *take* $\frac{1}{6}$.

11. *To multiply by* 9, *which is* $10 - 1$, *multiply by* 10 *and then subtract the multiplicand.*

Thus, 9 times $476 = 4760 - 476 = 4284$.

12. *To multiply by* 11, *which is* $10 + 1$, *multiply by* 10 *and then add the multiplicand.*

WRITTEN EXERCISE

1. Multiply by 125: 4832, 5275, 6892, $49,365.
2. Multiply by $33\frac{1}{3}$: 1011, 7227, 3102, $43,392.
3. Find $12\frac{1}{2}$% of $6848, $3272, $42.48, $35,284.
4. Find $16\frac{2}{3}$% of 600, $33.42, $14,286.66, $32,331.36.
5. Find $66\frac{2}{3}$% of $41.10, $33.66, $14,283.12, $68,391.21.
6. Find 25% of $33\frac{1}{3}$% of 12, $298.32, $3802.80, $296.64.
7. Multiply by 9: 329, 746, $981, 628 ft., 1476, $23,481.
8. Multiply by 11: 6123, 3762, $4837, 3972 ft., $31,287.

ORAL EXERCISE

Find the cost of the following:

Do not analyze aloud, at least until after the answer is given. Use short processes (page 37).

1. $3\frac{3}{4}$ yd. of felt @ $1.60.
2. 16 yd. of denim @ 15¢.
3. $7\frac{1}{2}$ yd. of dimity @ 16¢.
4. $3\frac{1}{4}$ lb. of feathers @ 48¢.
5. 32 yd. of cambric @ 11¢.
6. 9 yd. of cashmere @ 75¢.
7. 16 yd. of gingham @ 11¢.
8. 22 yd. of buckram @ 15¢.
9. 17 yd. of crinoline @ 11¢.
10. 15 yd. of percaline @ 23¢.
11. 7 pieces of beading @ 23¢.
12. 8 yd. of eider down @ 65¢.
13. 12 yd. of chambray @ 25¢.
14. $22\frac{1}{2}$ yd. of nainsook @ 20¢.
15. 16 yd. of linen lawn @ 75¢.
16. A dozen whalebones @ 24¢.
17. 35 yd. of cheese cloth @ 5¢.
18. 12 yd. of jute burlap @ 35¢.
19. $3\frac{1}{2}$ doz. Clark's cotton @ 42¢.
20. $7\frac{3}{4}$ yd. of torchon lace @ 12¢.
21. $4\frac{1}{2}$ doz. basting cotton @ 24¢.
22. 15 yd. of cotton scrim @ 22¢.
23. $5\frac{1}{2}$ doz. darning cotton @ 24¢.
24. 7 gross of bone buttons @ 22¢.

25. 16 gross of shoe buttons @ 7¢.
26. $8\frac{1}{2}$ yd. of farmer's satin @ 44¢.
27. $3\frac{1}{2}$ yd. of damask linen @ 98¢.
28. $5\frac{3}{4}$ yd. of cotton flannel @ 16¢.
29. 8 yd. of cotton burlap @ $1.12.
30. $7\frac{1}{2}$ lb. of hammock cord @ 22¢.
31. $22\frac{1}{2}$ yd. of Persian lawn @ 30¢.
32. 11 yd. of striped flannel @ 65¢.
33. 14 yd. of plain gingham @ 25¢.
34. 15 pieces of mohair braid @ 9¢.
35. 6 balls of crochet cotton @ 35¢.
36. 8 pieces of seam binding @ 12¢.
37. 19 skeins of Prisma cotton @ 5¢.
38. $17\frac{1}{2}$ yd. of butcher's linen @ 50¢.
39. 15 yd. of huckaback linen @ 45¢.
40. 17 yd. of unbleached muslin @ 7¢.
41. 17 skeins of Poseidon cotton @ 3¢.
42. 7 yd. of arras-cloth burlap @ $1.25.
43. 16 yd. of Alpine rose muslin @ $9\frac{1}{4}$¢.
44. 15 yd. of Java cream canvas @ 70¢.
45. 25 yd. of Java colored canvas @ 45¢.
46. 15 yd. of single-thread canvas @ 60¢.
47. 11 yd. of white narrow canvas @ 38¢.
48. 9 yd. of basket-weave burlap @ $1.15.
49. $\frac{1}{2}$ doz. boxes for materials @ 35¢ each.
50. A gross of pearl buttons @ 13¢ a dozen.
51. 7 pieces of feather-stitched braid @ 24¢.
52. 16 balls of knitting cotton @ 4 for a quarter.

53. If 6 lb. of coffee cost \$1.86, what will 5 lb. cost?

54. If 12 lb. of butter cost \$3, what will 18 lb. cost?

55. If $\frac{4}{5}$ of a building lot is worth \$2200, what is the lot worth?

56. At \$24 a month, what is the rent of a house for $3\frac{1}{2}$ mo.?

57. If 3% of a number is 36, what is 5 times the number?

58. If 7% of a number is 63, what is 20 times the number?

59. If the divisor is 13 and the quotient is 12, what is the dividend?

60. If a man buys a horse for \$150 and sells it at an advance of 30%, what is the selling price?

61. A grocer bought coffee at 25¢ a pound and sold it at a profit of 20%. What was the selling price?

62. If a man can build a fence in 32 days, in how many days should he build $12\frac{1}{2}$% of it?

63. The product of three numbers is 242. Two of the numbers are 11 and 2. What is the third number?

64. If a man owned $\frac{3}{4}$ of the stock of a certain company and sold $\frac{8}{9}$ of his share, how much did he have left?

65. A man buys \$11.88 worth of goods and sells $\frac{1}{3}$ of them at the same rate. How much does he receive?

66. How many square rods are there in a field 20 rd. square? What is the land worth at \$$\frac{3}{8}$ a square rod?

State the profit in buying the following goods at the price first given, and selling at the second price:

67. 12 lb. mixed candy, $1.92; 25¢ a pound.

68. 25 bars peanut brittle, $1.56; 10¢ a bar.

69. 216 chocolate bars, 45¢; 3 bars for a cent.

70. 1 doz. boxes chocolate chips, $1.92; 25¢ a box.

71. 6 lb. fine chocolate creams, $2.40; 60¢ a pound.

72. 75 packages peanut brittle, $4.68; 10¢ a package.

73. 40 packages chocolate drops, $1.25; 5¢ a package.

74. 12 packages chocolate creams, 72¢; 10¢ a package.

75. 2 boxes chocolate cream bars, 48 to the box, 76¢; 1¢ a bar.

76. 4 boxes gum drops, 24 packages to the box, $3; 5¢ a package.

77. 3 boxes caramels, 40 packages to the box, $3.75; 5¢ a package.

78. 5 boxes licorice drops, 48 packages to the box, $7.20; 5¢ a package.

79. 4 boxes chocolate almonds, 40 packages to the box, $5; 5¢ a package.

80. 9 cases lemon soda, 2 doz. bottles to the case, $4.50; 5¢ a bottle.

81. 6 boxes salted peanuts, 24 packages to the box, 75¢ a box; 5¢ a package.

82. 4 boxes pepsin chewing gum, 20 packages to the box, $2.20; 5¢ a package.

83. 3 boxes vanilla chocolates, 2 doz. packages to the box, $2.25; 5¢ a package.

30. Short methods in division. There are a few divisors which are used so frequently in business that it is desirable to know the shortest methods of dividing by them. These include the numbers 5, 10, 25, 100, 125 (with 12.5 or $12\frac{1}{2}$), and $33\frac{1}{3}$.

ORAL EXERCISE

1. Divide by 10 : \$420, \$5700, 330, \$425, 375 lb., 6 mi.

2. State the short method of dividing by 10.

3. Since $\frac{1}{5} = \frac{2}{10}$, we may take $\frac{1}{5}$ of a dollar by taking how many tenths of it? State a short method of dividing by 5, or of taking $\frac{1}{5}$ of a number.

4. Divide by 5 : \$1110, \$3240, 5280 ft., 4230 mi., 3221.

5. Divide by 100 : \$600, 7000, 4800, \$560, \$275, \$48.

6. State the short method of dividing by 100.

7. State the short method of finding $\frac{1}{25}$ (or $\frac{4}{100}$) of a dollar; of dividing a number by 25.

8. Divide by 25 : \$2200, \$1200, \$3100, 4100, \$32, \$11.

9. State the short method of dividing by 125. ($\frac{1}{125} = \frac{8}{1000}$.)

10. Divide by 125 : \$11,000, 2000, \$4000, \$70, \$600.

11. State the short method of dividing by $33\frac{1}{3}$. (Take $\frac{3}{100}$.)

12. Divide by $33\frac{1}{3}$: \$200, 500, \$7, \$12, \$30,000, \$11,000.

13. In a certain year Delaware had 346.2 mi. of railroad, and the District of Columbia had $\frac{1}{10}$ as many. How much had the latter?

14. France then had 26,600 mi. of railroad, and Korea had 0.001 as many. How many had Korea?

15. The United States then had 193,300 mi. of railroad, and Ecuador had 0.001 as many. How many had Ecuador?

31. Summary of short methods of division.

1. *To divide by* 10, *move the decimal point one place to the left.*

2. *To divide by* 100, *move the decimal point two places to the left, and so on for other powers of* 10.

3. *To divide by* 5, *multiply by* 2 *and divide by* 10.

4. *To divide by* 25, *multiply by* 4 *and divide by* 100.

5. *To divide by* 125, *multiply by* 8 *and divide by* 1000.

6. *To divide by* $33\frac{1}{3}$, *multiply by* 3 *and divide by* 100.

WRITTEN EXERCISE

1. State a short method of dividing by $66\frac{2}{3}$, or $\frac{200}{3}$.

2. State a short method of dividing by $16\frac{2}{3}$, or $\frac{100}{6}$.

3. State a short method of dividing by $12\frac{1}{2}$, or $\frac{100}{8}$.

4. $6484 \div 66\frac{2}{3}$.	5. $7280 \div 66\frac{2}{3}$.	6. $9276 \div 66\frac{2}{3}$.
7. $2937 \div 16\frac{2}{3}$.	8. $4831 \div 16\frac{2}{3}$.	9. $9134 \div 16\frac{2}{3}$.
10. $9237 \div 12\frac{1}{2}$.	11. $7396 \div 12\frac{1}{2}$.	12. $8187 \div 12\frac{1}{2}$.
13. $6889 \div 33\frac{1}{3}$.	14. $4959 \div 33\frac{1}{3}$.	15. $8892 \div 33\frac{1}{3}$.
16. $4936 \div 125$.	17. $67{,}400 \div 125$.	18. $39{,}888 \div 125$.

19. Divide by 25: 13,428, $37,642, $135,621, $427,693.

20. Divide by 125: 17,200, $68,300, $276,400, $317,500.

21. Divide by $33\frac{1}{3}$: 69,720, $32,960, $48,750, $276,320.

22. At 25¢ a yard, how many yards of cloth can be bought for $37.25?

23. At $33\frac{1}{3}$¢ a yard, how many yards of cloth can be bought for $98?

24. Divide 3,742,697 by 25, by long division, and see how many seconds it takes. Do the same by the short method stated above. Write the time with the answer.

ORAL EXERCISE

1. To multiply by $\frac{1}{2}$ is the same as to divide by what number? To divide by $\frac{1}{2}$ is the same as to multiply by what number? Illustrate on the blackboard.

2. Multiply by $\frac{1}{2}$: 48, 250, 380, 460, 666, 1110, 1234.

3. Divide by $\frac{1}{2}$: 70, 35, 60, 300, 450, 700, 1450, 8112.

4. To multiply by $\frac{1}{3}$ is the same as to divide by what number? To divide by $\frac{1}{3}$ is the same as to multiply by what number? Illustrate on the blackboard.

5. Multiply by $\frac{1}{3}$: 36, 90, 63, 45, 75, 300, 123, 321.

6. Divide by $\frac{1}{3}$: 30, 70, 50, 25, 80, 400, 2000, 4211.

7. Multiply by $\frac{2}{3}$: 30, 60, 90, 33, 63, 300, 600, 1230.

8. Divide by $\frac{2}{3}$: 20, 12, 16, 10, 60, 100, 200, 500, 1000.

9. Multiply by $\frac{1}{4}$: 8, 16, 32, 48, 64, 84, 100, 200, 1000.

10. Divide by $\frac{1}{4}$: 9, 11, 21, 25, 60, 70, 91, 125, 250, 800.

11. Multiply by $\frac{3}{4}$: 8, 16, 20, 32, 40, 44, 48, 60, 80, 444.

12. Divide by $\frac{3}{4}$: 9, 12, 15, 21, 30, 60, 66, 90, 120, 300.

13. Since $\frac{1}{5} = \frac{2}{10}$, what is the short method of multiplying by $\frac{1}{5}$? of dividing by $\frac{1}{5}$?

14. Multiply by $\frac{1}{5}$: 21, 31, 34, 123, 112, 321, 132, 125.

15. Divide by $\frac{1}{5}$: 22, 300, 34, 160, 444, 664, 486, 1010.

16. Since $\frac{2}{5} = .4$, state a short method of multiplying and of dividing by $\frac{2}{5}$. Do the same for $\frac{3}{5}$ and $\frac{4}{5}$.

17. Multiply by $\frac{5}{6}$: 30, 60, 66, 120, 600, 720, 840, 1200.

18. Because $.12\frac{1}{2} = \frac{1}{8}$, what is the short method of multiplying by $.12\frac{1}{2}$?

19. At $12\frac{1}{2}$¢ a yard, what will 16 yd. cost? 72 yd.?

20. At 48¢ a dozen, what will $\frac{1}{2}$ doz. eggs cost?

21. At \$1.28 a yard, what will $\frac{5}{8}$ yd. of silk cost?

22. At \$1.20 a yard, what will $\frac{3}{4}$ yd. of silk cost?

23. At \$2.40 a yard, what will $\frac{3}{8}$ yd. of lace cost?

24. At \$1.68 a yard, what will $\frac{3}{4}$ yd. of lace cost?

25. At 78¢ a yard, what will $\frac{1}{3}$ yd. of velvet cost?

26. At \$18.60 a dozen, what will $\frac{2}{3}$ doz. spoons cost?

27. At $12\frac{1}{2}$¢ a yard, how much cloth will \$1.50 buy?

28. How much will $12\frac{1}{2}$ doz. forks cost at \$16 a dozen?

29. How much will $6\frac{1}{4}$ doz. eggs cost at 16¢ a dozen?

30. At 16¢ a pound, how much will $3\frac{1}{4}$ lb. of meat cost?

31. At 24¢ a pound, how much will $2\frac{3}{4}$ lb. of meat cost?

32. At 64¢ a pound, what will $\frac{3}{4}$ lb. shelled almonds cost? What will 2 lb. 4 oz. cost?

33. At 36¢ a dozen, how much must a dealer pay for $2\frac{3}{4}$ doz. small bottles of mucilage?

34. At \$19.20 a dozen, how much must a dealer pay for $1\frac{1}{2}$ doz. fountain pens? for 2 doz.?

35. At 15 mi. an hour, how far will an automobile travel in 20 min.? in 40 min.? in 30 min.?

36. At $12\frac{1}{2}$¢ a dozen, how many dozen eggs will \$9.25 buy? How many dozen will \$12.50 buy?

37. At 35 mi. an hour, how far will a train travel in 12 min.? in 24 min.? in 36 min.? in 48 min.?

38. At 47 mi. an hour, how far will a train travel in 12 min.? (12 min. $= \frac{1}{5}$ hr.) How far in 15 min.?

WRITTEN EXERCISE

1. $\frac{1}{2}$ of \$73.10.	2. $\frac{1}{2}$ of $3\frac{2}{3}$.	3. $\frac{1}{2}$ of 375 ft.
4. $\frac{1}{4}$ of \$61.20.	5. $\frac{1}{4}$ of $5\frac{1}{8}$.	6. $\frac{1}{4}$ of 239 ft.
7. $\frac{3}{4}$ of \$27.40.	8. $\frac{3}{4}$ of $6\frac{3}{5}$.	9. $\frac{3}{4}$ of 297 lb.
10. $\frac{1}{3}$ of \$71.40.	11. $\frac{1}{3}$ of $4\frac{7}{8}$.	12. $\frac{1}{3}$ of 237 yd.
13. $\frac{2}{3}$ of \$62.10.	14. $\frac{2}{3}$ of $2\frac{3}{8}$.	15. $\frac{2}{3}$ of 128 mi.
16. $\frac{1}{5}$ of \$67.30.	17. $\frac{2}{5}$ of $4\frac{2}{3}$.	18. $\frac{3}{5}$ of 293 yd.
19. $12\frac{3}{4} \times 16\frac{4}{5}$.	20. $32\frac{1}{2} \times 63\frac{1}{3}$.	21. $41\frac{2}{3} \times 33\frac{1}{5}$.
22. $12\frac{1}{2}\%$ of \$72.40.	23. $12\frac{1}{2}\%$ of $6\frac{1}{2}$.	24. $12\frac{1}{2}\%$ of 375 ft.

25. 275 in. $\div \frac{1}{3}$ in.	26. 268 ft. $\div \frac{1}{3}$ ft.
27. 826 yd. $\div \frac{1}{3}$ yd.	28. 2372 ft. $\div \frac{1}{5}$ ft.
29. 1635 yd. $\div \frac{1}{5}$ yd.	30. \$12.60 $\div$ \$0.12$\frac{1}{2}$.
31. \$32.40 $\div$ \$0.12$\frac{1}{2}$.	32. 268 yd. $\div 12\frac{1}{2}$ yd.

33. At $12\frac{1}{2}$¢ a yard, how much cloth will \$876 buy?

34. At $12\frac{1}{2}$¢ a gross, how many gross of paper fasteners will \$17.75 buy?

35. At $33\frac{1}{3}$¢ a hundred, how many photograph cards can be bought for \$1.83?

36. At $66\frac{2}{3}$¢ a gross, how many gross of pens can be bought for \$24?

37. At $\frac{3}{8}$ yd. each, how many strips of ribbon can be cut from a piece 27 yd. long?

38. When a train is traveling at the rate of 32 mi. in 40 min., what is its rate per hour?

39. When a horse is trotting at the rate of $\frac{1}{2}$ mi. in 3 min., what is his rate per hour?

MEASURES

32. Tables for reference. Although these tables have been learned, they are here inserted for reference and review.

Table of Length

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet = 1 rod (rd.).
320 rods or 5280 feet = 1 mile (mi.).

The *hand* (4 in.) is used in measuring the height of horses at the shoulder. Sailors use the *fathom* (6 ft.) and *cable length* (120 fathoms) for measuring depths, and the *knot* (nautical mile, 1.15 common or statute miles, or 6080.27 ft.) for distances. A *furlong* is $\frac{1}{8}$ mi., a *league* is 3 mi., and a *cubit* is 18 in.

Table of Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.).
160 square rods = 1 acre (A.).
640 acres = 1 square mile (sq. mi.).

Carpenters, architects, and mechanics often write 8″ for 8 in., and 5′ for 5 ft. In this book both of these forms are used.

Table of Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).
128 cubic feet = 1 cord (cd.).

A perch of stone or masonry is usually considered as 1 rd. long, 1 ft. high, and $1\frac{1}{2}$ ft. thick, and it contains $24\frac{3}{4}$ cu. ft. It varies, however. A cubic yard of earth is considered a load.

Table of Weight

16 ounces (oz.) = 1 pound (lb.).
2000 pounds = 1 ton (T.).

The ton of 2000 lb. is sometimes called the *short ton*, there being a *long ton* of 2240 lb. which is used in some wholesale transactions in mining products. A hundredweight (cwt.) is 100 lb.

Goldsmiths still use an old table of *Troy weight*, and it is here inserted for reference. In this table

24 grains (gr.) = 1 pennyweight (pwt. or dwt.).
20 pennyweights = 1 ounce (oz.).
12 oz. = 1 pound (lb.).

The avoirdupois pound contains 7000 gr., the Troy pound 5760 gr. Therefore 1 lb. of iron is heavier than 1 lb. of gold.

A *carat* weight, used in weighing diamonds, varies, but is commonly taken in the United States as 3.2 Troy grains. The word *carat* is also used in speaking of the purity of gold, meaning $\frac{1}{24}$, "16 carats fine" meaning $\frac{16}{24}$ pure gold.

There is also a table of *Apothecaries' weight*, used by physicians and druggists in prescriptions. It is here inserted for reference.

20 grains (gr.) = 1 scruple (sc. or ℈).
3 scruples = 1 dram (dr. or ʒ).
8 drams = 1 ounce (oz. or ℥).
12 ounces = 1 pound (lb.).

Table of Liquid Measure

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).

A gallon contains 231 cu. in. Barrels (bbl.) vary in size, although in estimating the capacity of tanks and cisterns 31.5 gal. are considered a barrel, and 2 bbl. a hogshead.

There is also a table of Apothecaries' liquid measure, in which 16 fluid ounces make 1 pint.

Table of Dry Measure

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

A bushel contains 2150.42 cu. in. The dry quart contains 67.2 cu. in., the liquid quart only 57.75 cu. in.

Table of Time

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
12 months (mo.) = 1 year (yr.).

"Thirty days hath September,
April, June, and November."

The other months have 31 days, except February, which has 28 days in common years and 29 days in leap years. The common year has 365 days, or 52 weeks and 1 day; the leap year 366 days. A century is 100 years.

Table of Value

10 mills (m.) = 1 cent (ct. or ¢).
10 cents = 1 dime (d.).
10 dimes = 1 dollar ($).

The term *eagle* (for $10) is no longer used. The mill is not coined.

Arc and Angle Measure

60 seconds (60″) = 1 minute (1′).
60 minutes = 1 degree (1°).
360 degrees = 1 circumference.

In measuring angles 360° = 4 right angles.

ORAL EXERCISE

1. Express 640 rd. as miles.
2. Express 64 oz. as pounds.
3. Express 2 A. as square rods.
4. Express 33 ft. as rods; as yards.
5. Express 54 cu. ft. as cubic yards.
6. Express 9 ft. as inches; as yards.
7. Express 11 da. as hours; as weeks.
8. Express 17 qt. as pints; as gallons.
9. Express 9 pk. as quarts; as bushels.
10. How many cubic inches in 10 cu. ft.?
11. How many rods in 3 mi.? in 10 mi.?
12. How many yards in 8 rd.? in 10 rd.?
13. How many cubic feet in 2 cd. of wood?
14. Express 300 min. as hours; as seconds.
15. How many quarts in a barrel of $31\frac{1}{2}$ gal.?
16. How many pints in a 60-gallon hogshead?
17. Express 80 sq. rd. as a fraction of an acre.
18. How many right angles in 270°? in 180°?
19. How many days from May 23 to June 25?
20. How many days from June 11 to July 17?
21. What part of a 16-carat ring is pure gold?
22. Express 11 yd. as rods; as feet; as inches.
23. Express 30 pt. as quarts; as gills; as gallons.
24. How many feet in 2 rd. 8 ft.? in 2 rd. 10 ft.?
25. How many feet in 10 rd.? in 5 rd.? in 2 rd.?
26. How many degrees in 540′? in 420′? in 660′?

27. How many mills in $2.75?

28. How many days from August 15 to October 15?

29. How many acres in a tract of land 10 mi. square?

30. How many days from January 17 to February 17?

31. How many square feet in 10 sq. yd.? in 5 sq. yd.?

32. At 6 qt. a day, how long will 3 bu. of oats last a horse?

33. What is the perimeter of a square that is 4 ft. 9 in. on a side?

34. What is the side of a square whose perimeter is 14 ft. 4 in.?

35. Since a gallon contains 231 cu. in., how many gallons in 693 cu. in.?

36. How many square feet in a surface having an area of 720 sq. in.?

37. Express a long ton as a short ton and hundredths of a short ton.

38. What is the perimeter of an equilateral triangle that is 3 ft. 8 in. on a side?

39. If school closes June 21, how many days from that day to the fourth of July?

40. How many quarts will a 10-bushel bin hold? a 5-bushel bin? How many pecks in each?

41. A man buys a building lot 2 rd. front. How many square feet of sidewalk 5 ft. wide must he have?

42. A cellar $13\frac{1}{2}$ ft. by 20 ft. is to be excavated to a depth of 6 ft. How many loads (cubic yards) of earth must be removed?

43. If a man buys a load of coal weighing 1 T. 600 lb., how many pounds does he buy? Express this in tons and tenths of a ton.

33. Illustrative problems. 1. Express $35\frac{1}{2}$ yd. as feet.

Since 1 yd. = 3 ft.,

$35\frac{1}{2}$ yd. = $35\frac{1}{2}$ times 3 ft., or $106\frac{1}{2}$ ft.

2. Express 81 oz. as pounds and ounces.

Since 1 oz. = $\frac{1}{16}$ lb.,

81 oz. = 81 times $\frac{1}{16}$ lb. = $5\frac{1}{16}$ lb., or 5 lb. 1 oz.

3. Express 2 ft. 8 in. as inches.

Since 1 ft. = 12 in.,

2 ft. = 2 times 12 in. = 24 in.

2 ft. 8 in. = 24 in. + 8 in. = 32 in.

WRITTEN EXERCISE

1. Express 92 rd. 3 ft. as feet; as inches.
2. Express 32 yd. 2 ft. as feet; as inches.
3. Express $3\frac{1}{2}$ A. as acres and square rods.
4. Express 21 sq. ft. 82 sq. in. as square inches.
5. Express 95 cu. ft. as cubic yards; as cubic inches.
6. Express 27 sq. rd. 4 sq. yd. as square yards; as square feet.
7. Express 8 sq. yd. 7 sq. ft. as square feet; as square inches.
8. Express 35 cu. yd. 15 cu. ft. as cubic yards; as cubic feet; as cubic inches.
9. Express 4 T. 1735 lb. as tons and the decimal part of a ton; as pounds; as ounces.
10. Express 39,426 lb. as tons and the decimal part of a ton; as tons and pounds; as ounces.
11. Express 27 qt. 1 pt. as gallons and the decimal part of a gallon; as quarts and the decimal part of a quart; as pints.

34. Surveyors' table of length:

7.92 inches = 1 link (li.).
100 links = 4 rods = 1 chain (ch.).
80 chains = 5280 ft. = 1 mile.

City property is usually measured by feet and decimal fractions of a foot; farm property, by rods or chains.

WRITTEN EXERCISE

1. Express 1 mi. as chains; as rods; as links.

2. Express 142 ch. as miles and chains; as rods.

3. Express 17 rd. as chains; as links; as inches.

4. How many feet in 60 li.? in 10 ch.? in 125 li.?

5. Express 17 mi. 32 ch. as miles and the decimal part of a mile; as chains; as rods.

6. It is 30 ch. 44 li. around a square field. How long is each side? Suppose it were 97 ch. 64 li. around?

7. An equilateral triangle is 4.72 ch. on a side. How many links in the perimeter? How many chains?

8. By using the surveyors' table, find how many inches there are in a mile. Check by using the common table.

Add:

9.	4 ch. 3 rd. 9 2	10.	38 ch. 93 li. 19 78	11.	9 mi. 275 rd. 8 192

Subtract:

12.	26 ch. 1 rd. 17 3	13.	19 ch. 15 li. 13 87	14.	16 mi. 12 rd. 7 234

Multiply:

15.	4 ch. 2 rd. 27	16.	29 ch. 78 li. 16	17.	8 mi. 192 rd. 37

35. Surveyors' table of square measure:

16 sq. rd. = 1 square chain (sq. ch.).
10 sq. ch. = 1 acre (A.).

The square rod is sometimes called a *perch*. The word *rood* is sometimes met in reading, meaning 40 sq. rd. or ¼ acre. 640 acres = 1 sq. mi., and 36 sq. mi. = 1 township in the government surveys.

ORAL EXERCISE

1. Add 6 ch. 2 rd. and 3 ch. 3 rd.
2. Express 3 A. as square chains.
3. From 7 ch. subtract 4 ch. 2 rd.
4. From 5 ch. subtract 1 rd. 40 li.
5. Add 7 ch. 90 li. and 6 ch. 70 li.
6. Express 2 mi. as rods; as chains.
7. Multiply 1 ch. 2 rd. by 2; by 3; by 4.
8. Express as square rods: 3 sq. ch., 5 sq. ch.
9. How many acres in 30 sq. ch.? 50 sq. ch.?
10. How many square chains in 7 A.? in 17 A.?
11. How many feet in 10 mi.? How many chains?
12. Express as rods: 3 ch., 5 ch., 7 ch., 16 ch., 30 ch.
13. How many square chains in a field 6 ch. by 9 ch.? How many acres?
14. How many square chains in a field 15 ch. by 11 ch.? How many acres?
15. How many square chains in a field 5 ch. by 4 ch.? How many acres?
16. Divide 15 ch. by 2, expressing the result as chains; as rods; as chains and rods.

36. Illustrative problem. How many acres in a field 32 ch. long and 6 ch. 30 li. wide?

6 ch. 30 li. = 6.30 ch.

32 times 6.30 times 1 sq. ch. = 201.6 sq. ch.

Since 1 sq. ch. = $\frac{1}{10}$ A.,

201.6 sq. ch. = 201.6 times $\frac{1}{10}$ A. = 20.16 A.

Actual measurements of this kind in connection with the school ground and other pieces of land are valuable.

WRITTEN EXERCISE

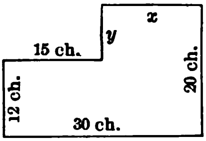
Find the areas in acres:

1. 16 ch. by 14 ch.
2. 8 ch. by 23 ch.
3. 23 ch. by 29 ch.
4. 32 ch. by 34 ch.
5. 5 ch. 2 rd. by 11 ch.
6. 1 ch. 1 rd. by 2 rd.
7. 5 ch. 30 li. by 17 ch.
8. 6 ch. 48 li. by 9 ch.
9. 6 ch. 80 li. by 13 ch.
10. 5 ch. 18 li. by $3\frac{1}{2}$ ch.
11. 8 ch. 10 li. by $2\frac{1}{2}$ ch.
12. 8 ch. 42 li. by 12 ch.
13. 11 ch. 75 li. by 6 rd.
14. 8 ch. 30 li. by 10 ch. 40 li.
15. 12 ch. 45 li. by 15 ch. 75 li.
16. 13 ch. 80 li. by 14 ch. 65 li.
17. 32 ch. 75 li. by 27 ch. 50 li.
18. 27 ch. 42 li. by 31 ch. 60 li.
19. 34 ch. 27 li. by 42 ch. 83 li.
20. 14 ch. 15 li. by 13 ch. 17 li.
21. Express 1 A. as square yards; as square feet.
22. Express 1 sq. rd. as square feet; as square inches.
23. Express 1 sq. mi. as square rods; as square yards; as square feet.
24. Express 3 sq. mi. 32 A. as square miles and the decimal part of a square mile; as acres.

25. Express 7 A. 8 sq. ch. as acres and the decimal part of an acre; as square chains; as square rods.

26. How many square feet in a city lot 31.8 ft. wide by 107.6 ft. long?

27. What is the difference in area between a piece of ground 20 rd. square and one containing 20 sq. rd.?



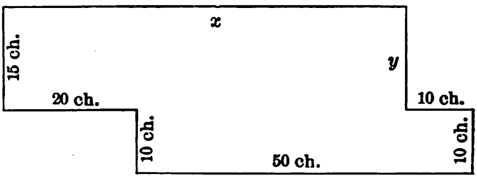
28. In the field here shown find the length of x and y; the perimeter; the number of acres.

29. How many square feet in a school playground $8\frac{1}{2}$ rd. wide and 160 ft. long?

30. A piece of land 34 ch. by 15 ch. is sold for $75 an acre. What is the price?

31. Which costs the more, a piece of land 42 ch. by 27 ch. at $65 an acre, or a piece 37 ch. by 29 ch. at $70 an acre? How much more?

32. In the field here shown find the length of x and y; the perimeter; the number of acres.



33. Two fields have equal perimeters, 80 ch. One is a square and the other a rectangle 30 ch. long. Having equal perimeters, have they equal areas? What is the area of each?

Find the value of the following pieces of land:

34. 24 ch. by 30 ch. at $75 an acre.

35. 42 ch. by 65 ch. 50 li. at $85 an acre.

36. 37 ch. 50 li. by 42 ch. 75 li. at $60 an acre.

ORAL EXERCISE

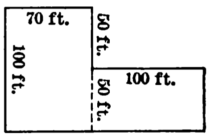
1. If a rectangle has an area of 50 sq. in., and is 10 in. long, how wide is it? What is its perimeter?

State only the answers on this page. Reserve explanations for written work.

State the lengths of rectangles whose areas and widths are as follows:

2. 72 sq. ft., 8 ft.
3. $\frac{15}{32}$ sq. in., $\frac{3}{4}$ in.
4. $\frac{5}{12}$ sq. in., $\frac{2}{3}$ in.
5. 45 sq. rd., 5 rd.
6. 56 sq. ch., 7 ch.
7. 54 sq. ch., 6 ch.
8. 132 sq. ft., 11 ft.
9. 143 sq. in., 11 in.
10. 108 sq. ch., 9 ch.
11. 750 sq. in., 20 in.

12. How many square feet in the area of this city lot? (Take the parts on opposite sides of the dotted line, find the area of each separately, and add.)



13. Find the areas of squares whose sides are 6 in., 12 in., 4 in., 11 in., 20 in., 40 in., 100 in.

14. A square has an area of 25 sq. in. What is the length of each side?

State the lengths of the sides of squares whose areas are as follows:

15. 49 sq. ch.
16. 81 sq. ft.
17. 64 sq. in.
18. 100 sq. mi.
19. 144 sq. in.
20. 121 sq. ft.
21. 36 sq. mi.
22. 400 sq. mi.
23. 900 sq. ft.
24. 1600 sq. ft.
25. 2500 sq. ft.
26. 3600 sq. ft.

37. Illustrative problems. 1. If a rectangle has an area of 15 sq. in. and is 5 in. long, how wide is it?

1. If a rectangle is 5 in. long and 1 in. wide, area = 5 sq. in.
2. Therefore the number of inches in width is 15 sq. in. $\div$ 5 sq. in. = 3. Therefore it is 3 in. wide.

Pupils should construct the figure by paper folding or cutting, or should draw it, if necessary. Another form of solution is given in Ex. 2.

2. A rectangle $32\frac{1}{2}$ in. long has an area of $227\frac{1}{2}$ sq. in. What is the width?

Work:

$$32\tfrac{1}{2}\overline{)227\tfrac{1}{2}}$$

$$\begin{array}{r} 7 \\ 65\overline{)455} \\ \underline{455} \end{array}$$

1. If w = the number of inches in width, then w times $32\frac{1}{2}$ times 1 sq. in. = $227\frac{1}{2}$ sq. in.
2. Therefore $w = 227\frac{1}{2}$ sq. in. $\div$ $32\frac{1}{2}$ sq. in.
3. Therefore $w = 7$, the number of inches in width.

38. We therefore see that

The number of units of area of a rectangle, divided by the number of units of length, equals the number of units of width.

Or, briefly, *Area divided by length equals width.*

WRITTEN EXERCISE

1. What is the area of a rectangle $5' \times 4'$? Draw the figure and explain all of your work.

2. How wide is a 12.95-acre farm 17 ch. long? Consider such farms as rectangular.

3. How long is a 272-acre farm 40 ch. wide?

4. A room having a floor area of $205\frac{5}{9}$ sq. ft. is 16 ft. 8 in. long. How wide is it?

5. A school playground 63 ft. wide has an area of 4473 sq. ft. How long is it?

6. How wide is a 295.2-acre farm 72 ch. long? Draw to scale a plan of the farm.

7. What is the area of a rectangle 3.25 in. by 5.75 in.?

8. A rectangle has an area of 294 sq. ft. It is 17 ft. 6 in. long. How wide is it?

9. A floor has an area of $358\frac{7}{8}$ sq. ft. The room is 21 ft. 9 in. long. What is the width?

10. A floor has an area of 560 sq. ft. The width of the room is $17\frac{1}{2}$ ft. What is the length?

11. Find the cost of a farm 120 rd. by 242 rd., at $75 an acre. Draw to scale a plan of the farm.

12. A floor has an area of 288 sq. ft. 80 sq. in. The room is 17 ft. 8 in. long. What is the width?

13. A floor has an area of 788 sq. ft. 18 sq. in. The room is 24 ft. 3 in. wide. What is the length?

14. A rectangular table top is $42\frac{3}{4}$ in. long, and it has an area of $1296\frac{3}{4}$ sq. in. What is the width?

15. A page of a book is $7\frac{1}{8}$ in. by $4\frac{5}{8}$ in., and the book has 400 pages. How many square feet of page surface?

16. A rectangular playground has an area of 188 sq. yd. 540 sq. in. It is 14 yd. 9 in. long. What is the width?

17. How many square yards of oilcloth are needed for a kitchen 15 ft. by 21 ft., allowing 5% more than the area for matching?

18. Two fields have each a perimeter of 376 rd. One is 71 rd. wide and the other is 93 rd. wide. Find the length and area of each, and draw a plan.

19. Two fields have each an area of 4750 sq. rd. One is 40 rd. wide and the other 50 rd. What is the length of each? the perimeter? Draw each to a scale.

20. A city building lot, rectangular in shape, is advertised as having an area of 2107 sq. ft. The lot has a frontage of 21 ft. 6 in. What is the depth of the lot?

FARM MEASURES

WRITTEN EXERCISE

1. A farmer buys three tracts of land of the following dimensions: 51 ch. 12 li. × 20 ch., 51 ch. 13 li. × 40 ch., 52 ch. × 21 ch. How many acres in each? in all?

51 ch. 12 li. × 20 ch. means 51 ch. 12 li. by 20 ch.

2. He paid $48.50 an acre for the land. What was the total cost?

3. A piece of the property 7 ch. long and 2 ch. 28 li. wide was not tillable. What was the area of this part? of the tillable part?

4. The farmer grew wheat on a piece $66\frac{2}{3}$ ch. long and 15 ch. wide, and the yield was 13 bu. to the acre. He sold the wheat at 85¢ a bushel. How much did he receive for it?

5. He grew oats on a piece 15 ch. long and 10 ch. wide, and the yield was 42 bu. to the acre. He sold the oats at 59¢ a bushel. How much did he receive for it?

6. He grew corn on a piece 20 ch. long and 15 ch. wide, and the yield was 34 bu. to the acre. He sold the corn at 56¢ a bushel. How much did he receive for it?

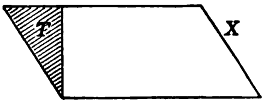
7. He used an acre for his house and lawn, this piece having a frontage of 120 ft. What was the depth of the lot? What was the length of fence necessary to inclose it?

8. The house is 50 ft. long and 30 ft. wide, and is rectangular in ground plan. How many square feet are left from the acre?

9. The farmer took for a vegetable garden a rectangular piece of land 60 rd. in perimeter, the length being twice the width. What was the area of the piece?

ORAL EXERCISE

1. If we cut triangle T from this parallelogram and place it where X is, what kind of a figure have we? Try it. What does this tell us about the area of a parallelogram?



State the areas of the parallelograms whose bases and altitudes are given in Exs. 2–9:

2. 160 ft., 25 ft.
3. 66 ft., $16\frac{2}{3}$ ft.
4. 64 in., $12\frac{1}{2}$ in.
5. 150 ft., $33\frac{1}{3}$ ft.
6. 444 in., 25 in.
7. 303 ft., $33\frac{1}{3}$ ft.
8. 800 in., 125 in.
9. 300 ft., $66\frac{2}{3}$ ft.
10. State a rule for finding the area of a parallelogram.

39. Parallelogram. Illustrative problem. What is the area of a parallelogram of base 10 ft. and altitude 6 ft.?

1. A rectangle 10 ft. by 6 ft. has an area 10 times 6 sq. ft.
2. Therefore the area of the parallelogram is also 60 sq. ft.

WRITTEN EXERCISE

1. What is the area of a parallelogram of base 16.9 ft. and altitude 7.2 ft.?

2. Also of base 17 ft. 8 in. and altitude 27 in.?

3. Also of base 2 yd. 17 in. and altitude 2 ft. 3 in.?

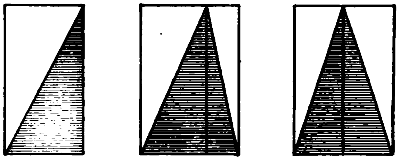
4. What is the altitude of a parallelogram whose base is 129.9 ft. and area 303.1 sq. yd.?

5. Also of one with base 26.46 in. and area 5.88 sq. ft.?

6. What is the area of a parallelogram with a base 42.5 ft. and altitude 27.8 ft.? with a base 6.7 in. and altitude 3.9 in.? with a base 345.25 ft. and altitude 127.37 ft.?

ORAL EXERCISE

1. How do the areas of these triangles compare with the areas of the rectangles? Then a triangle is what part of a rectangle of the same base and same height?



State the areas of the triangles whose bases and altitudes are given in Exs. 2–7:

2. 80 in., 75 in.
3. 80 in., 25 in.
4. 16 in., 125 in.
5. 44 in., 50 in.
6. 60 in., $33\frac{1}{3}$ in.
7. 120 in., $16\frac{2}{3}$ in.
8. State a rule for finding the area of a triangle.

40. Triangle. Illustrative problem. What is the area of a triangle of base 18 in. and altitude 7 in.?

$\frac{1}{2}$ of 18 times 7 sq. in. = 63 sq. in.

WRITTEN EXERCISE

Find the areas of the triangles whose bases and altitudes are given in Exs. 1–8:

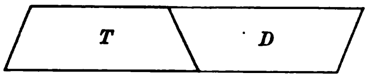
1. 43 ft., 32 ft.
2. 9.3 ft., 4.9 ft.
3. 7.9 in., 8.7 in.
4. $37\frac{1}{4}$ ft., $42\frac{1}{2}$ ft.
5. 8.7 in., 6.3 in.
6. 2 yd. 32 in., 27 in.
7. 324 in., 47 in.
8. 3 ft. 4 in., 2 ft. 8 in.

9. A right-angled triangle has its three sides 30 rd., 40 rd., 50 rd. What is its area in square rods? in acres?

10. What is the base of a triangle whose altitude is 6.4 rd. and area $\frac{1}{2}$ acre? altitude 7.2 in. and area $\frac{1}{2}$ sq. ft.?

41. Trapezoid. A four-sided figure with two parallel sides is called a *trapezoid.*

42. Area of a trapezoid. If a trapezoid T have a duplicate cut from paper and turned over and fitted to it, like D, the two together make a parallelogram. (Illustrate by paper cutting.) Therefore



The area of a trapezoid equals half that of a rectangle with the same altitude and with a base equal to the sum of the two parallel sides.

43. Illustrative problem. What is the area of a trapezoid of altitude 4 in. and parallel sides 8 in. and 10 in.?

As above explained, the area is $\frac{1}{2}$ of 4 times $(8 + 10)$ sq. in., or 36 sq. in.

ORAL EXERCISE

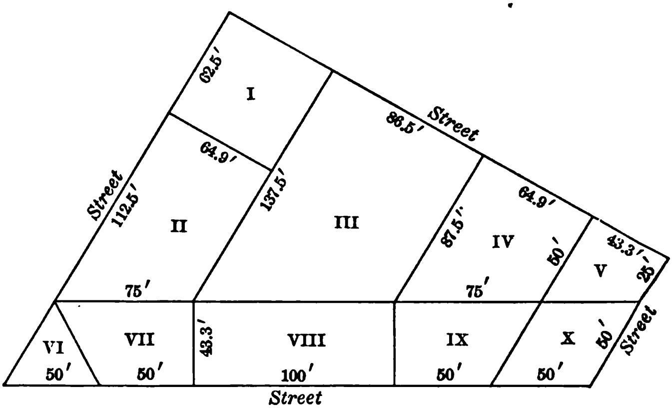
State the areas of the trapezoids whose altitudes are first given in Exs. 1–10, followed by the two parallel sides:

1. 6 in., 7 in. and 13 in.
2. 8 in., 3 in. and 12 in.
3. 10 in., 12 in. and 13 in.
4. 12 in., 7 in. and 8 in.
5. 14 in., 14 in. and 16 in.
6. 13 in., 6 in. and 14 in.
7. 22 in., 21 in. and 29 in.
8. 15 in., $7\frac{1}{2}$ in. and $12\frac{1}{2}$ in.
9. 17 in., $8\frac{1}{3}$ in. and $11\frac{2}{3}$ in.
10. 11 in., $11\frac{1}{4}$ in. and $18\frac{3}{4}$ in.
11. If the area of a trapezoid is 48 sq. in., and the parallel sides are 5 in. and 7 in., what is the altitude?
12. If the area of a trapezoid is 48 sq. in. and the altitude is 6 in., what is the sum of the parallel sides? If one of these sides is 9 in., what is the other?

WRITTEN EXERCISE

Find the areas of the trapezoids whose altitudes are first given in Exs. 1–7, followed by the two parallel sides:

1. 5 rd., 6 rd. 7 ft. and 9 rd.
2. 322 ft., 427 ft. and 534 ft.
3. 127 ft., 129 ft. and 148 ft.
4. 236 in., 208 in. and 235 in.
5. 34 ft., 27 ft. 8 in. and 33 ft.
6. 62 ft. 3 in., 59 ft. and 78 ft.
7. 4 yd., 2 yd. 27 in. and 6 yd.
8. If the area of a trapezoid is 21 sq. ft. 16 sq. in., the altitude 5 ft. 4 in., and one of the parallel sides 4 ft. 9 in., how long is the other parallel side?

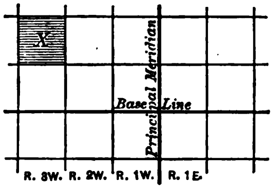


An irregular city block is divided into lots as shown. Find the area, in square feet, of lots numbered as follows:

9. I.	10. II.	11. III.	12. IV.	13. V.
14. VI.	15. VII.	16. VIII.	17. IX.	18. X.

44. Laying out public lands. In the more recently settled parts of the country land is laid out as here described.

45. Principal meridian. Through a given tract a meridian is chosen as the *principal meridian.*



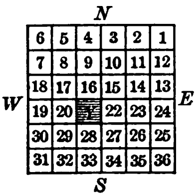
46. Base line. An east and west line is chosen as the *base line.*

The principal meridian and base line are here shown.

47. Townships. Lines are run parallel to the principal meridian and base line, at intervals of 6 mi. This divides the land into *townships.*

48. Ranges. The north and south rows of townships are called *ranges.*

X on the first map is numbered T. 2 N., R. 3 W.; that is, the second township north of the base line, in the third range west of the meridian.

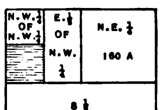


49. Sections. A township is divided into *sections,* each 1 mi. square.

This map shows the method of numbering these sections.

Each section is then divided as shown in the third map.

If this map represents the shaded part *Y* of the second map, and that represents the shaded part *X* of the first map, the shaded part here shown would be thus described: S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 21, T. 2 N., R. 3 W. This means the southwest quarter of the northwest quarter of section 21, second township north, third range west.



In sections of the country where land is not laid out in this way little attention should be given to this subject.

WRITTEN EXERCISE

Write the description, plot, and find the area:

1. S.E. $\frac{1}{4}$, Sec. 5, T. 3 S., R. 3 W.
2. N.E. $\frac{1}{4}$, Sec. 8, T. 2 N., R. 2 W.
3. E. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 2, T. 2 N., R. 3 E.
4. S. $\frac{1}{2}$ of S.E. $\frac{1}{4}$, Sec. 20, T. 2 S., R. 3 W.
5. N. $\frac{1}{2}$ of S.W. $\frac{1}{4}$, Sec. 10, T. 3 S., R. 2 E.
6. E. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 30, T. 2 S., R. 3 E.
7. N.E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$, Sec. 5, T. 1 N., R. 1 W.
8. S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 32, T. 1 S., R. 3 E.
9. N.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 6, T. 3 N., R. 3 W.
10. How much is this farm worth at $65 an acre: W. $\frac{1}{2}$ of S. $\frac{1}{2}$, Sec. 3, T. 2 N., R. 2 W.?
11. How much is this farm worth at $75 an acre: N.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$, Sec. 5, T. 2 S., R. 2 E.?
12. Find the area of this farm: S.W. $\frac{1}{4}$, Sec. 10, T. 2 S., R. 2 E. Draw a plan of a township and locate the farm.
13. Find the area of this farm: N. $\frac{1}{2}$, Sec. 6, T. 1 N., R. 1 E. Draw a plan of a township and locate the farm.
14. Mr. Simmons owns the S. $\frac{1}{2}$, N.E. $\frac{1}{4}$, Sec. 3, T. 2 N., R. 3 E. How many rods of fence are needed to inclose it?
15. How far is it from Mr. Taylor's farm, N.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 16, T. 1 N., R. 3 W., to Mr. Hunt's farm, S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 28, T. 1 N., R. 3 W.? Draw the map.
16. A road running straight through a farm is $\frac{1}{2}$ mi. long and 3 rd. wide. How many acres in the road? If hay can be cut from the sides, averaging $\frac{1}{4}$ of the area, and the amount of hay averages 2 tons to the acre and is worth $9.50 a ton, what is gained by attending to this crop?

II. MEASURES. PERCENTAGES. PROPORTION

VOLUMES

ORAL EXERCISE

1. Give the table of dry measure.

2. Give the table of cubic measure.

3. Give the table of liquid measure.

4. A cellar is $21' \times 18'$, and $9'$ deep. Express the dimensions in yards; the volume in cubic yards.

5. At 50 ct. a cubic yard, how much will it cost to excavate a cellar containing 126 cu. yd.? 175 cu. yd.?

$21' \times 18'$ means 21 ft. by 18 ft. $6'' \times 5''$ means 6 in. by 5 in.

6. What is the volume of a box $7'' \times 3'' \times 8''$?

7. What is the volume of a box $6'' \times 5'' \times 10''$?

8. How many cubic feet in a room $10' \times 12' \times 9'$?

9. How many cubic feet in a room $12' \times 12' \times 10'$?

Find the volumes of solids of the following dimensions:

10. $3'' \times 4'' \times 7''$.
11. $2'' \times 8'' \times 9''$.
12. $6' \times 8' \times 10'$.
13. $5' \times 9' \times 11'$.
14. $20' \times 30' \times 5'$.
15. $3' \times 8' \times 11'$.
16. $11'' \times 11'' \times 10''$.
17. 6 ft. $\times$ 8 ft. $\times$ 11 ft.
18. 6 yd. $\times$ 8 yd. $\times$ 2 yd.
19. 4 ft. $\times$ 9 ft. $\times$ 11 ft.
20. 21 in. $\times$ 20 in. $\times$ 10 in.
21. 5 yd. $\times$ 2 yd. $\times$ 7 yd.
22. 11 in. $\times$ 12 in. $\times$ 10 in.
23. 2 yd. $\times$ 7 yd. $\times$ 11 yd.
24. 12 in. $\times$ 12 in. $\times$ 10 in.
25. 3 yd. $\times$ 9 yd. $\times$ 11 yd.
26. 23 in. $\times$ 20 in. $\times$ 10 in.
27. 5 yd. $\times$ 8 yd. $\times$ 12 yd.
28. 42 ft. $\times$ 11 ft. $\times$ 10 ft.
29. 5 rd. $\times$ 16 rd. $\times$ 20 rd.

50. Illustrative problem. What is the volume of a box 3 ft. wide, $6\frac{1}{3}$ ft. long, 2 ft. high?

If it were 1 ft. each way, the volume would be 1 cu. ft.
But it is 3 times as wide, $6\frac{1}{3}$ times as long, and 2 times as high.
Therefore the volume is

3 times $6\frac{1}{3}$ times 2 times 1 cu. ft., or 38 cu. ft.

If there is any difficulty in understanding this fact, in this review, inch cubes may be used as in the earlier classes.

WRITTEN EXERCISE

1. How many cords in a pile of 4-ft. wood, 5 ft. high and 56 ft. long? 6 ft. high and 128 ft. long?

2. How many loads (cubic yards) of earth must be taken out in excavating a cellar $24' \times 18'$, and $7'$ deep?

3. How much will it cost to excavate a ditch $150' \times 2' \times 6'$, at 50¢ per cubic yard? at $62\frac{1}{2}$¢ per cubic yard?

4. A schoolroom is 32 ft. 8 in. long, $18\frac{1}{3}$ ft. wide, and $12\frac{1}{2}$ ft. high. What is the number of cubic feet in the room?

5. A tunnel 720 ft. long has a cross-section area of 180 sq. ft. How much earth and rock were excavated? (Imagine the cross section $10' \times 18'$.)

6. In preparing a flower bed in a park it was necessary to fill in a space $60' \times 100'$ to an average depth of $1\frac{1}{2}'$. The earth cost $33\frac{1}{3}$¢ a load. What was the total cost of the earth used?

7. A freight car is $8' \times 34'$, and the interior is $7'$ high. How many cubic feet does it contain? If it is filled with grain to a height of $5'$, what is the weight of the grain at 60 lb. to the bushel, allowing $1\frac{1}{4}$ cu. ft. to the bushel?

8. A fish pond has been excavated to a depth of 5 ft. The dimensions of the bottom are 40 ft. and 30 ft. How much earth was excavated?

9. The size of an ordinary Nebraska farm-wagon box is 10 ft. by 3 ft., and 24 in. to 26 in. deep. Find the contents in cubic feet for each of these depths; also in cubic inches.

10. Such a wagon box 24 in. deep contains 50 bu. of shelled corn. How many bushels of shelled corn will such a box 26 in. deep contain?

11. If a wagon box contains 50 bu. of wheat worth 89¢ a bushel, how many loads must be drawn to carry $311.50 worth? to carry $133.50 worth?

12. Prairie hay is heavier than eastern meadow hay, a cube of the former 7 ft. on an edge weighing a ton (the measurement taken 30 days after the hay is stacked). If it takes $33\frac{1}{3}\%$ more in bulk of a certain eastern meadow hay to make a ton, how many cubic feet to a ton of the latter?

13. What is the weight of prairie hay that will fill a space 30 ft. long, 17 ft. 6 in. wide, and 11 ft. deep, taking the number of cubic feet to the ton suggested in Ex. 12?

14. What is the weight of the eastern meadow hay mentioned in Ex. 12 required to fill the space mentioned in Ex. 13?

15. A haystack is estimated to contain 2600 cu. ft. If prairie hay, what does it weigh, to the nearest half ton? If eastern meadow hay (Ex. 12)?

16. A schoolroom is 30 ft. long, 20 ft. wide, and 15 ft. high. If it is occupied by 29 children and the teacher, how many cubic feet of air are allowed to each? If 2400 cu. ft. of fresh air per hour should be allowed to each person, how many times an hour should the air be changed?

Problems in Excavation

WRITTEN EXERCISE

1. How many cubic yards of earth must be removed in digging a tunnel 492 ft. long, 39 ft. wide, and 19 ft. 6 in. high?

2. If a dirt car is 27 ft. long, 6 ft. wide, and $3\frac{1}{4}$ ft. deep, how many cubic yards will it carry? How many will 12 cars carry?

3. If one laborer can shovel a cubic yard of earth into a car in 30 min., how long will it take him to load a car? How long will it take 15 men? (See Ex. 2.)

4. How many cars of the above dimensions will it take to remove the earth from the tunnel mentioned in Ex. 1? How many trains of 9 cars each? Allowing 30 ft. for the total length of a car, how long a train of cars would be needed?

5. If one laborer can remove the earth from a space 20 ft. long, 6 ft. wide, and 6 ft. deep in a day, how long will it take 100 men to remove the earth from the tunnel above mentioned?

6. If a laborer is paid at the rate of 10¢ a load (cubic yard) of earth removed, what will he earn in a day, removing the amount specified in Ex. 5?

7. In the first great subway in New York City 9733 cu. yd. of brick were used. Taking the size of the brick as $8\frac{1}{4}'' \times 4'' \times 2''$, how many bricks were used?

8. What will be the earnings of a laborer who can excavate a space a yard wide, a yard deep, and 12 ft. long, in solid rock, in one day, at 75¢ per cubic yard? How much at $87\frac{1}{2}$¢ per cubic yard?

ORAL EXERCISE

In Exs. 1–12, x *represents a missing number. What is the number?*

1. $2 \times 6 \times x = 24$.
2. $2 \times 3 \times x = 42$.
3. $3 \times 4 \times x = 84$.
4. $5 \times 6 \times x = 90$.
5. $2 \times 6 \times x = 60$.
6. $8 \times 8 \times x = 64$.
7. $5 \times 6 \times x = 210$.
8. $6 \times 9 \times x = 108$.
9. $5 \times 5 \times x = 125$.
10. $2 \times 6 \times x = 144$.
11. x times 13 cu. ft. = 130 cu. ft.
12. x times 16 cu. ft. = 320 cu. ft.
13. If the product of three numbers is 400, and two of the numbers are 5 and 10, what is the third one?
14. If the product of three numbers is 540, and the product of two of them is 90, what is the third number?
15. If a box is 9 in. by 10 in. by 11 in., what is its cubic contents? If the cubic contents of a box 9 in. wide and 11 in. long is 99 cu. in., how deep is the box?
16. If a box contains 54 cu. in., and is 9 in. long and 3 in. wide, how deep is it?
17. If a box contains 24 cu. in., and the area of the base is 6 sq. in., how deep is the box?

Given the following volumes of boxes, and the length and breadth, find the depth:

18. 60 cu. in., 5 in., 4 in.
19. 64 cu. in., 8 in., 4 in.
20. 18 cu. in., 3 in., 2 in.
21. 108 cu. in., 9 in., 6 in.
22. 63 cu. in., 7 in., 3 in.
23. 160 cu. in., 8 in., 4 in.
24. 240 cu. in., 10 in., 6 in.
25. 350 cu. in., 10 in., 7 in.

51. Illustrative problems. 1. How thick is a block 5 ft. long and 4 ft. wide, containing 60 cu. ft.?

Analysis:

If it is x ft. thick, x times 5 times 4 cu. ft. $= 60$ cu. ft. and $x = \dfrac{60 \text{ cu. ft.}}{5 \text{ times } 4 \text{ cu. ft.}} = 3$. Therefore it is 3 ft. thick.

Work:

$$\frac{60}{5 \times 4} = 3$$

We may, if we prefer, give this analysis:

Since it is 5 ft. long and 4 ft. wide, the area of the base is 20 sq. ft.

If it were 1 ft. high, the volume would be 20 cu. ft.

Therefore it is as many times 1 ft. high as 60 cu. ft. ÷ 20 cu. ft., or 3.

52. Hence we see that

The number of cubic units of volume, divided by the product of the numbers of units of two dimensions, equals the number of units of the third dimension.

This is sometimes less accurately expressed: *The volume divided by two dimensions equals the third dimension.*

2. What is the area of the base of a block $2\frac{1}{2}$ in. high, containing 70 cu. in.?

If it were 1 in. high, the volume would be 70 cu. in. ÷ $2\frac{1}{2} = 28$ cu. in. But the base of such a block is 28 sq. in.

$$70 \div \frac{5}{2} = \frac{2 \times 70}{5} = 28$$

This might be briefly stated as in § 52: *The volume divided by the height equals the area of the base.* This has a meaning only as we think of the measures as abstract numbers.

3. What is the thickness of a block containing 100 cu. in., the base containing 30 sq. in.?

If it were 1 in. thick, it would contain 30 cu. in. Therefore it is as many times 1 in. thick as 100 cu. in. ÷ 30 cu. in. $= 3\frac{1}{3}$. Therefore it is $3\frac{1}{3}$ in. thick.

$$\frac{100}{30} = 3\frac{1}{3}$$

WRITTEN EXERCISE

1. How many cubic feet in a hall $80' \times 60' \times 23'\ 4''$?

2. A tank $9' \times 16\frac{2}{3}'$ contains 900 cu. ft. How deep is it?

3. A box $13'' \times 20''$ contains 3900 cu. in. How deep is it?

4. A tank 4′ deep contains 450 cu. ft. What is the area of the base?

5. A hall 22′ high and 30′ wide contains 28,600 cu. ft. Find the length.

6. A block containing 700 cu. ft. has a base area of 56 sq. ft. How thick is it?

7. A hall contains 41,341.3 cu. ft. It is 41.3 ft. long and 36.4 ft. wide. How high is it?

8. A box contains 42.159 cu. in. The area of the bottom is 10.81 sq. in. How deep is the box?

9. A block of granite is 14 in. long and 8.3 in. wide. It contains 836.64 cu. in. How thick is it?

10. The floor of a hall contains 1386 sq. ft. The hall contains 15,246 cu. ft. Required the height.

11. A storeroom is 14.3 ft. long, 13 ft. 6 in. wide, and has a capacity of 2316.6 cu. ft. How high is the room?

12. The floor of a room is square and contains 144 sq. ft. The room contains 1368 cu. ft. Required the three dimensions of the room.

13. A room 24 ft. long and 9 ft. high, containing 3240 cu. ft., is carpeted with plain ingrain carpet 1 yd. wide. What is the cost of the carpet at 75¢ a yard?

14. A room contains 2110.68 cu. ft. The room is 16.4 ft. long, and the floor area is 234.52 sq. ft. What are the three dimensions of the room?

15. Express in bushels the volume of a bin 6 ft. by 4 ft. by 3 ft. $6\frac{1}{2}$ in., allowing 2150 cu. in. to the bushel.

16. How many bars of soap, each 4 in. long, $2\frac{3}{4}$ in. wide, and $1\frac{1}{2}$ in. thick, can be packed in a box 16 in. long, $13\frac{3}{4}$ in. wide, and 1 ft. deep?

17. What is the value of the bricks in a kiln 60 ft. long, 25 ft. wide, and 9 ft. high, each brick being 8 in. by 4 in. by 2 in., at $9 a thousand?

18. The length of a tank is 100% greater than its width, and its width is 200% of its depth. If the width is 2 yd., how many gallons does it hold?

19. A cord of stone having the same volume as a cord of wood, what is a pile of stones 27 ft. long, 5 ft. wide, and 6 ft. high worth at $4.20 a cord?

20. If it takes 550 cu. ft. of clover hay to make a ton, how many tons will fill a mow 32 ft. long, the width being double the depth and half the length?

21. Allowing 231 cu. in. to the gallon, how many gallons in a watering trough that is 6 ft. long and 16 in. wide, the ratio of its depth to its width being 3 : 4?

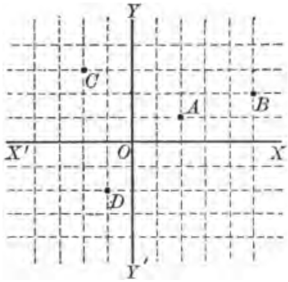
22. A bin 21 ft. 6 in. long and 10 ft. wide is filled with wheat to the depth of 5 ft. Allowing 2150 cu. in. to the bushel, what is the wheat worth at 80¢ a bushel?

23. Find the cost of the carpet needed for a room 11 ft. 3 in. wide, 10 ft. high, containing 2278 cu. ft. 216 cu. in., the carpet being 27 in. wide and costing $1.35 a yard, allowing 9 in. for matching on each strip except the first.

24. A cellar is 32 ft. long, 17 ft. wide, and 8 ft. 6 in. deep. How much will it cost to build the wall, 1 ft. thick, at 35¢ per cubic foot, no allowance being made for openings, and the length of the wall being determined by the outside measure (thus doubling the corners)?

LONGITUDE AND TIME

53. Axes. A point on any surface may be located by *two* measures taken from two intersecting lines XX' (read "XX prime") and YY'. These lines are called *axes.*



In this figure the point A is 2, 1, and the point B is 5, 2.

Distances to the left of YY', or below XX', may be marked $-$. Thus, C is $-2, 3$, and D is $-1, -2$.

54. Prime meridian. An arc on the earth's surface, from the north pole to the south pole, is called a *meridian.* The one through the Royal Observatory at Greenwich, England, is taken by most nations as the *prime meridian.*

55. Points on a map. On a map the lines taken for locating points are the *equator* and the *prime meridian.*

56. Latitude and longitude. Instead of giving the distances from these lines in miles, they are given in degrees. Thus, St. Louis is located when we say that it is $90° 12' 17''$ W., $38° 38' 3.6''$ N. The distance in degrees east and west from the prime meridian is called *longitude;* north and south from the equator, *latitude.*

WRITTEN EXERCISE

1. Draw two axes and indicate the points 2, 4; 7, 6.
2. In the same way, indicate $-2, 4$; $-6, 1$; $-5, 5$.
3. In the same way, indicate the points $-2, -3$; $-1, -4$.
4. In the same way, indicate the points $3, -2$; $2, -1$
5. Draw a rough map, indicating a place 40° N., 75° W.

57. Correspondence of longitude to time. Since the earth turns about on its axis once every 24 hours, the place in which we live will pass through 360° between now and this time to-morrow. That is, to the time 24 hours will correspond the longitude 360°. Therefore

360° of long. correspond to 24 hr.						
1° " "	corresponds "	$\frac{1}{360}$ of 24 hr.,	or	$\frac{1}{15}$ hr.,	or	4 min.
1′ " "	" "	$\frac{1}{60}$ " 4 min.,	"	$\frac{1}{15}$ min.,	"	4 sec.
1″ " "	" "	$\frac{1}{60}$ " 4 sec.,	"	$\frac{1}{15}$ sec.		

58. Apparent motion of the sun. Because the earth *really* turns from west to east, the sun *appears* to pass from east to west, as trees *seem* to move when we are on the cars.

59. Earlier west, later east. Because the earth revolves 360° in 24 hours, the sun appears to pass through 15° an hour. Therefore when it is noon here it is an hour later, or 1 P.M., 15° east of here; an hour earlier, or 11 A.M., 15° west of here; and 6 hours earlier, or 6 A.M., 90° west of here.

This work belonging partly to geography, no illustrations are here introduced. The teacher should use the globe as needed.

ORAL EXERCISE

1. When it is 9 A.M. here, what time is it 15° east of here? 30° west of here? directly south of here?

2. When it is noon here, what time is it 30° east of here? 90° west of here? 150° east of here? directly north of here?

3. What difference of time corresponds to a difference of longitude of $7\frac{1}{2}$°? 45°? 60°? 105°? 120°? 165°? 180°?

4. Since 1′ of longitude corresponds to 4 sec. of time, to what does 7′ correspond? 11′? 15′? 30′? 45′? 60′? 120′?

5. Since 1″ of longitude corresponds to $\frac{1}{15}$ sec. of time, to what does 30″ correspond? 60″? 45″? 5″? 1″? 150″?

60. Illustrative problem. Two ships at sea are 65° 7′ 30″ of longitude apart. What is their difference in time?

1. Since 1° corresponds to $\frac{1}{15}$ hr., 65° correspond to 65 times $\frac{1}{15}$ hr. $= 4\frac{1}{3}$ hr. $=$ 4 hr. 20 min.

2. Since 1′ corresponds to $\frac{1}{15}$ min., 7′ correspond to 7 times $\frac{1}{15}$ min. $= \frac{7}{15}$ min. $=$ 28 sec.

3. Since 1″ corresponds to $\frac{1}{15}$ sec., 30″ correspond to 30 times $\frac{1}{15}$ sec. $=$ 2 sec.

4. Therefore the difference in time is 4 hr. 20 min. 30 sec.

It is apparent that the mere figures of the answer could be obtained by this division, although the reason would not be so clear.

$$15^\circ\overline{)65^\circ\ \ 7'\ 30''}$$
$$4\ \ 20\ \ 30$$

WRITTEN EXERCISE

1. 27° 4′ 15″ of longitude corresponds to what difference in time?

2. Two ships are 75° 30′ 30″ of longitude apart. What is their difference in time?

3. A ship at 62° 3′ 40″ W. receives a wireless telegram from one at 60° 1′ 10″ W. at 11 A.M. When was it sent?

4. For time purposes the longitude of Berlin is 15° E., and that of Chicago 90° W. What is the difference in longitude? Illustrate by a rough map. When it is 2 P.M. in Chicago, what time is it in Berlin?

5. For time purposes the longitude of San Francisco is taken as 120° W. When it is noon in London (on the prime meridian), what time is it in San Francisco? When it is noon in San Francisco, what time is it in London?

6. A steamer 68° 10′ 30″ W. sends a wireless telegram to the Nantucket light-ship, 70° W., at 8 : 30 A.M. At what time is it received? If transmitted to New York, 75° W., without loss of time, when is it received there?

61. Correspondence of time to longitude. Since 24 hours correspond to 360° of longitude, we have the following:

24 hr. correspond to 360°.
1 hr. corresponds to $\frac{1}{24}$ of 360°, or 15°.
1 min. " " $\frac{1}{60}$ " 15°, " $\frac{1}{4}$°, or 15′.
1 sec. " " $\frac{1}{60}$ " 15′, " $\frac{1}{4}$′, " 15″.

62. Illustrative problem. The difference in time between two ships is 3 hr. 7 min. 3 sec. What is the difference in longitude?

1. Since 1 hr. corresponds to 15°, 3 hr. correspond to 3 times 15° = 45°.

2. Since 1 min. corresponds to 15′, 7 min. correspond to 7 times 15′ = 105′ = 1° 45′.

3. Since 1 sec. corresponds to 15″, 3 sec. correspond to 3 times 51″ = 45″.

4. Therefore the difference in longitude is 46° 45′ 45″.

Evidently the mere figures of the answer could be obtained by calling 3 hr. 7 min. 3 sec. 3° 7′ 3″ and multiplying by 15, although the reason would not be so clear.

$$\begin{array}{r} 3^\circ\ \ 7'\ \ 3'' \\ 15 \\ \hline 46^\circ\ 45'\ 45'' \end{array}$$

WRITTEN EXERCISE

1. The difference of time between two ships is 2 hr. 3 min. 10 sec. What is the difference in longitude?

2. When it is noon at Denver, it is 7 P.M. at Greenwich. What longitude does Denver use for its time purposes?

3. The difference in time between the Harvard and Columbia observatories is 8 min. 22.7 sec. What is the difference in longitude?

4. At 10:4 A.M. a steamer in longitude 26° 30′ W. sends a wireless message to another steamer. It is received at 10:19 A.M. What is the longitude of the second steamer?

5. When it is noon in Greenwich it is 9 P.M. in Japan. What longitude does Japan use for its time purposes?

6. When it is noon in Melbourne it is 2 A.M. in Greenwich. What longitude does Melbourne use for its time purposes?

7. A traveler sails from New York with his watch set by the time of 75° W. If the watch indicates 2:30 P.M. when it is 5 P.M. by the ship's time, in what longitude is he?

8. A traveler sails from England with his watch set by Greenwich time. When he reaches 52° 30′ 45″ W., is his watch faster or slower than the time of that place, and how much?

9. Suppose a telegram sent from an observatory 121° 32′ 48″ W. at 8:40 A.M. to another observatory 73° 8′ 16″ W., and to require 16 min. for transmission. At what time will it be received?

10. Suppose a telegram sent at 4 P.M. from a place in Germany 15° E. to a place in California 120° W. At what time will it reach its destination, allowing 30 min. for repeating at various terminal offices?

11. A ship's captain observing the sun finds that when the ship is on the meridian (that is, at noon) the time is 3 hr. 7 min. 35 sec. P.M. by a chronometer set by Greenwich time. What is the ship's longitude?

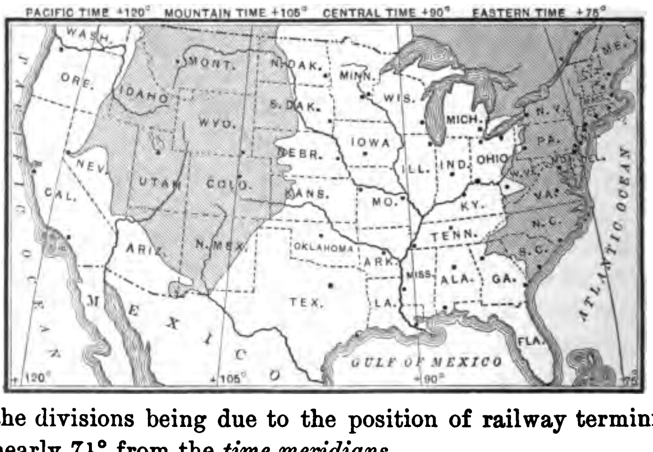
12. Two observatories A and B are connected by a telegraph line. A telegram sent from A at 2 hr. 4 min. 6 sec. P.M. reaches B at 4 hr. 8 min. P.M. The longitude of B is 79° 12′ 42.2″ W. What is the longitude of A?

13. The officers of a steamer in the Mediterranean Sea find that when the sun is on the meridian the time is 10 o'clock 50 min. 15 sec. A.M. by a chronometer set by Greenwich time. What is the ship's longitude?

63. Standard time. It is so much trouble to think of differences of time like 1 hr. 2 min. 17.5 sec., that most civilized countries have adopted a system of standard time. They have considered all places in one section as having the same longitude, some multiple of 15°, so that the differences in time from Greenwich (the prime meridian) shall be exact hours, and therefore all differences in time shall also be exact hours.

For this reason New York (73° 58′ 25.5″ W.) is considered to have the longitude 75° W., and Chicago (87° 36′ 42″ W.) the longitude 90° W. Then when it is noon in England (which is all considered of longitude 0°) it is 5 hours earlier in New York, or 7 A.M., and it is 6 A.M. in Chicago.

64. Standard time map. This map shows the standard time sections in the United States, the irregularities of



the divisions being due to the position of railway termini nearly 7½° from the *time meridians.*

When it is noon in New York it is 11 A.M. in the Central Time section, 10 A.M. in the Mountain Time section, and 9 A.M. in the Pacific Time section.

65. Standard time problems. Since for practical purposes standard time is generally used, the exercises refer to that time unless the contrary is stated. The cities selected are near the time meridians. Cities on the line dividing two time sections usually adopt one or the other.

Questions concerning the date line and the change of day do not involve arithmetic, but belong rather to geography.

ORAL EXERCISE

1. When it is 10 A.M. in Iowa, what time is it in New York? in Wisconsin? in Louisiana? in Ohio?

2. What time is it now in your school? in Boston? in Florida? in Arkansas? in Colorado? in California?

3. When it is 10 A.M. in San Francisco, what time is it in Denver? in St. Louis? in Cleveland? in Albany?

4. When it is 9 A.M. in Denver, what time is it in Los Angeles? in New Orleans? in Cincinnati? in Boston?

5. When it is noon in California, what time is it in Illinois? in Massachusetts? (Consult the map, page 80.)

6. When it is 3 P.M. in Milwaukee, what time is it in Oregon? in Louisville? in New York? in Philadelphia?

7. When it is 1 P.M. in New York, what time is it in Richmond? in Philadelphia? in Chicago? in California?

8. When it is 9:30 A.M. in New Orleans, what time is it in Alabama? in Maine? in Wyoming? in San Francisco?

9. Suppose a telegram sent from New York at 1 P.M. to San Francisco. If it goes without loss of time, what time will it be when it crosses the Mississippi River? when it crosses Colorado? when it reaches San Francisco?

WRITTEN EXERCISE

1. France uses the time of Paris (2° 20′ 15″ E.). When it is noon, standard time, in Iowa, what is the time in France?

2. Japan uses the time of 135° E. When it is noon at Greenwich, what is the time in Tokyo? in St. Louis? in Portland, Oregon?

3. All of Great Britain uses Greenwich time. If it is 5 : 30 P.M. at Liverpool, what is the local time of a ship in 17° 16′ 30″ W. longitude?

4. The longitude of Chicago is 87° 36′ 42″ W. What is the difference between the local (that is, the real meridian) and standard time?

5. Italy uses the Central European time (15° E.). When it is 7:30 A.M. in Springfield, Illinois, what time is it in Rome (7 hr. later)? in London?

6. When it is 1 P.M. in London, what is the time in Paris? (See Exs. 1, 3.)

7. The longitude of San Francisco is 122° 25′ 40.8″ W. Which is the later, local or standard time, and how much?

8. Berlin uses the standard time of 15° E. The longitude of Berlin is 13° 23′ 43.5″. What is the difference between the local and standard time?

9. What is the difference in standard and in local time between Berlin and San Francisco? (See Exs. 7, 8.)

10. At midnight, December 31, 75° time, the United States Naval Observatory sends an electric time signal to various parts of the world, indicating that a new year is beginning at Washington. When does the signal reach California? Colorado? Alabama? England? Germany?

66. Table of longitudes, for reference:

Albany	73° 44′ 48″ W.	New York	73° 58′ 25½″ W.
Berlin	13° 23′ 43½″ E.	Paris	2° 20′ 15″ E.
Chicago	87° 36′ 42″ W.	San Francisco	122° 25′ 40⅘″ W.

WRITTEN EXERCISE

1. What is the difference between the local and standard time of New York? of Albany?

2. What is the difference in local time between Berlin and Paris? Berlin and New York?

3. What is the difference in local time between Albany and San Francisco? Albany and Paris?

4. What is the difference in local time between New York and Chicago? Chicago and San Francisco?

5. When it is 1 P.M., standard time, at Chicago, what is the standard time at San Francisco? the local time?

6. When it is noon, local time, at Albany, what is the standard time there? What is then the standard time at Chicago?

What differences of local time correspond to these differences of longitude?

7. 8° 8′ 8″.	8. 15° 27′ 30″.	9. 7° 23′ 48″.
10. 24° 10′ 3″.	11. 17° 42′ 15″.	12. 8° 19′ 32″.
13. 25° 16′ 9″.	14. 32° 48′ 19″.	15. 6° 17′ 19″.
16. 48° 16′ 17″.	17. 63° 19′ 30″.	18. 9° 13′ 45″.

What differences of longitude correspond to these differences of local time?

19. 3 hr. 4 min. 8 sec.	20. 10 hr. 4 min. 4 sec.
21. 5 hr. 17 min. 5 sec.	22. 7 hr. 9 min. 18 sec.
23. 8 hr. 19 min. 7 sec.	24. 5 hr. 50 min. 55 sec.

PERCENTAGE REVIEWED

ORAL EXERCISE

1. F is what part as large as E? what per cent?
2. F is what part as large as D? what per cent?
3. F is what part as large as C? what per cent?
4. F is what part as large as B? what per cent?
5. F is what part as large as A? what per cent?

Tell what per cent

6. E is of C.	7. E is of D.	8. D is of C.
9. E is of A.	10. E is of B.	11. C is of B.
12. E is of F.	13. D is of F.	14. C is of F.

E is twice F, or $\frac{200}{100}$ of F, or 200% of F.

15. A is of D.	16. C is of E.	17. D is of E.

18. Find 10%, or $\frac{1}{10}$, of 50, 175, 212, \$327, \$475.50.
19. Find 50%, or $\frac{1}{2}$, of 86.4, \$16.44, \$244.50, \$862.20.
20. Find $12\frac{1}{2}$%, or $\frac{1}{8}$, of 16, 80, 96, 728, 488, 648, \$840.
21. Find $16\frac{2}{3}$%, or $\frac{1}{6}$, of 36, 66, 90, 720, 366, 540, \$636.
22. Find 20%, or $\frac{1}{5}$, of 75, 90, 250, 305, 555, 750, \$3.50.
23. Find 25%, or $\frac{1}{4}$, of 64, 96, 324, 4.40, 2.40, 320, \$840.
24. Find $33\frac{1}{3}$%, or $\frac{1}{3}$, of 63, 96, 3.33, 1.50, 630, 900, 324.

Sam is 6 years old, Nora is 9, and Tom is 12. Tell the following per cents:

25. Sam's age is what per cent of Tom's?

26. Sam's age is what per cent of Nora's?

27. Nora is what per cent older than Sam?

28. Sam is what per cent younger than Nora?

29. Nora's age is what per cent of Tom's? of Sam's?

30. Tom is what per cent older than Sam? than Nora?

31. Nora's age is what per cent of the sum of Sam's and Tom's?

32. Nora's age is what per cent of the product of Sam's and Tom's?

33. Sam's age is what per cent of the difference between Nora's and Tom's?

It is 3000 miles from London to New York, 1000 miles from New York to Chicago, and 2000 miles from Chicago to San Francisco. Tell the following per cents:

34. The distance from Chicago to San Francisco is what per cent of that from London to Chicago?

35. The distance from New York to Chicago is what per cent of that from London to San Francisco?

36. The distance from New York to Chicago has what ratio to that from Chicago to San Francisco?

37. The distance from New York to San Francisco has what ratio to that from London to San Francisco?

38. The distance from London to New York is what per cent greater than that from New York to Chicago?

39. The distance from London to New York is what per cent less than that from London to San Francisco?

67. Terms used. When we say that 5% of \$400 is \$20, we call 5% the *rate*, \$400 the *base*, and \$20 the *percentage*, terms already defined in the earlier treatment of the subject.

68. *To find any per cent of a number, multiply the number by the rate.*

That is, *the percentage is the product of the base and rate.*

69. The most important per cents. The per cents studied on page 84 are among the most important of all, and their common-fraction values must be thoroughly known before taking up the business arithmetic on page 138.

WRITTEN EXERCISE

1. How much is \$448.64 plus 25% of itself?
2. How much is \$369.36 less $33\frac{1}{3}$% of itself?
3. Express as per cents: $\frac{4}{7}$, $\frac{5}{8}$, $\frac{3}{11}$, $\frac{4}{15}$, $\frac{5}{32}$, $\frac{5}{12}$.
4. How much is 125% of \$246.80? of \$1234.40?
5. What is the interest on \$750 for 1 yr. at $3\frac{1}{2}$%?
6. How much is 93% of \$623? of \$275? of \$350?
7. What is the discount on \$950 worth of goods at 15%?
8. If a village formerly of 1460 inhabitants has gained 5% in population, what is its population now?

Express as common fractions in their simplest forms:

9. 48%.	10. 40%.	11. 85%.	12. $28\frac{4}{7}$%.
13. $16\frac{3}{4}$%.	14. $22\frac{1}{2}$%.	15. $55\frac{5}{9}$%.	16. $37\frac{1}{2}$%.
17. $77\frac{7}{9}$%.	18. $27\frac{3}{11}$%.	19. $14\frac{2}{7}$%.	20. $17\frac{4}{5}$%.
21. $32\frac{3}{8}$%.	22. $27\frac{9}{10}$%.	23. $67\frac{4}{7}$%.	24. $53\frac{9}{32}$%.

Using common-fraction equivalents, find:

25. $16\frac{2}{3}$% of 102.	26. $33\frac{1}{3}$% of \$111.21.
27. $66\frac{2}{3}$% of \$729.	28. $12\frac{1}{2}$% of \$336.48.

ORAL EXERCISE

1. Since $125\% = 1\frac{1}{4}$, find 125% of \$400.
2. Find 150% of \$200, \$300, \$500, \$1000.
3. Find $133\frac{1}{3}\%$ of \$300, \$36, \$360, \$450, \$600.
4. Find $116\frac{2}{3}\%$ of \$60, 600 ft., 360 mi., 420 lb., 48 yd.
5. Since $75\% = 1 - \frac{1}{4}$, find 75% of 40, 400, 200, \$360.
6. Since $66\frac{2}{3}\% = 1 - \frac{1}{3}$, find $66\frac{2}{3}\%$ of 30, 66, 45, \$300.
7. Since $87\frac{1}{2}\% = 1 - \frac{1}{8}$, find $87\frac{1}{2}\%$ of 8, 16, 32, 56, 88.
8. Since $80\% = 1 - \frac{1}{5}$, find 80% of 50, 45, 75, \$5000.
9. Since $83\frac{1}{3}\% = 1 - \frac{1}{6}$, find $83\frac{1}{3}\%$ of 600, 60, 66, \$36.

Find 125% *of the following:*

10. 44 ft.	11. 50 ft.	12. \$120.	13. \$8.40.
14. 30 in.	15. 84 yd.	16. \$150.	17. \$1.80.
18. \$300.	19. 500 ft.	20. \$1000.	21. \$3000.

Find 150% *of the following:*

22. 66 ft.	23. 82 in.	24. \$150.	25. \$2.40.
26. 35 in.	27. 45 in.	28. \$250.	29. \$1.10.

Find $133\frac{1}{3}\%$ *of the following:*

30. \$330.	31. \$630.	32. \$120.	33. \$2.40.
34. \$1200.	35. 36 in.	36. 48 ft.	37. 75 rd.

Find $116\frac{2}{3}\%$ *of the following:*

38. \$1200.	39. \$660.	40. 2400.	41. \$3.60.

Find 75% *of the following:*

42. \$440.	43. \$300.	44. \$150.	45. \$220.

Find $66\frac{2}{3}\%$ *of the following:*

46. \$3.30.	47. \$7.50.	48. \$1.20.	49. \$5.40.

WRITTEN EXERCISE

1. If 20% of the days of a common year are stormy, how many days are not stormy?

2. A dealer bought $167.40 worth of clocks and sold them at a profit of 33⅓%. How much did he gain?

3. How much is the interest on $642.25 for 1 yr. at 4%? How much is it for 2 yr.? for ½ yr.? for 2 yr. 6 mo.?

4. A village of 3600 inhabitants increased in population 15%. It then decreased 5%. What was its population then?

5. If in a period of 72 days 33⅓% are cloudy, and if it rains on 16⅔% of the cloudy days, how many of the days are rainy?

6. A dealer bought $1260 worth of goods and marked them 33⅓% above cost. He sold them at 10% less than the marked price, at "bargain sales." What did he make?

7. A dealer bought $644.80 worth of shoes and sold them at a profit of 25%. How much did he gain? If he failed to collect 10% of this gain, how much did he really make?

8. A dry goods dealer bought $2460.30 worth of Persian lawn, and marked it 33⅓% above cost. He sold half of it, and then sold the rest at 10% off the marked price. How much did he gain?

9. The number of pupils in a school three years ago was 235. The next year there were 20% more. Last year there were 33⅓% less than two years ago. This year there are 25% more than last year. How many are there this year?

ORAL EXERCISE

1. $33\frac{1}{3}$ is what per cent of $66\frac{2}{3}$? of 100?

2. 50 is what part of 100? what per cent?

3. $75 is what per cent of $150? of $37.50?

4. 1 ft. equals what per cent of 1 yd.? of 4 ft.?

5. 40 is what per cent of 80? of 100? of 60? of 40?

6. 120 is what per cent of 100? of 60? of 40? of 120?

7. If a quarter of the days of last month were rainy, what per cent were not rainy?

8. 25 is what per cent of 50? of 75? of 100? of 125? of 150? of 200? of 2500? of 5000?

9. $11 is what part of $33? what per cent? what per cent of $22? of $55? of $1100? of $2200?

The first of these numbers is what per cent of the second?

10. 50, 40.	11. 60, 20.	12. 75, 15.	13. 90, 60.
14. 15, 20.	15. 21, 28.	16. 63, 21.	17. 48, 12.
18. 51, 17.	19. 96, 24.	20. 11, 55.	21. 81, 27.
22. 57, 57.	23. 12, 24.	24. 80, 120.	25. 16, 24.
26. 75, $37\frac{1}{2}$.	27. $12\frac{1}{2}$, $6\frac{1}{4}$.	28. $66\frac{2}{3}$, $33\frac{1}{3}$.	29. 90, 150.

The first of these numbers is what per cent greater than the second?

30. 75, 50.	31. 60, 40.	32. 44, 33.	33. 99, 66.
34. 66, 55.	35. 36, 27.	36. 93, 62.	37. 21, 14.

The first of these numbers is what per cent less than the second?

38. 21, 28.	39. 28, 32.	40. 44, 48.
41. 42, 63.	42. 300, 400.	43. 210, 240.
44. 52, 104.	45. 108, 120.	46. 150, 200.

70. Illustrative problem. What per cent of \$35 is \$11.66⅔?

Work in steps:

1. If $x\%$ of \$35 = \$11.66⅔,
2. Then $x\%$ = \$11.66⅔ ÷ \$35
   = 0.33⅓.
3. Therefore \$11.66⅔ is 33⅓% of \$35.

Or we may simply note that \$11⅔ is $\frac{11\frac{2}{3}}{35}$ of \$35, which is 33⅓%.

Actual work:

$$\begin{array}{r} 0.33\frac{1}{3} \\ \$35\overline{)\$11.66\frac{2}{3}} \\ \underline{10\ 5\phantom{0\frac{2}{3}}} \\ 1\ 16\frac{2}{3} \\ \underline{1\ 05\phantom{\frac{2}{3}}} \\ 11\frac{2}{3} \end{array}$$

$$\frac{11\frac{2}{3}}{35} = \frac{35}{3 \times 35} = \frac{1}{3}$$

71. *The percentage (part) divided by the base (whole) equals the rate.*

WRITTEN EXERCISE

1. \$82.67 is what per cent of \$248.01?

2. \$17.42 is what per cent of \$156.78?

3. The 45 pupils in our grade are what per cent of the 225 in school?

4. If a man saves \$187.50 out of his salary of \$1250, what per cent does he save?

5. The 45 minutes devoted to arithmetic to-day are what per cent of the 4½ hours that we spend in school?

6. By how much does the area of Texas, 265,780 sq. mi., exceed 7% of that of the United States, 3,616,484 sq. mi.?

7. The 9,789,012 tons of sugar produced in the world in a certain year is what per cent of the 10,876,680 tons produced the following year?

8. The 6.27 inches of rainfall here last September was what per cent of the total rainfall of 41.8 inches for the year ending with that month?

Dressmaking Problems

WRITTEN EXERCISE

1. A dressmaker bought 16 yd. of velvet at $3 a yard, selling 9 yd. at a profit of $16\frac{2}{3}\%$ and the rest at a rate of profit half as great. What was the rate of gain on the whole?

2. She bought a 25-yd. box of chiffon velvet at $4 a yard, with 10% off for cash, selling it at $4.35 a yard. What was her gain per cent?

3. She bought a 75-yd. piece of silk skirt lining at 65 ct. a yard. She sold 28 yd. at 90 ct., 15 yd. at 95 ct., and the remainder, at the close of the season, at 70 ct. What was her per cent of gain?

4. She bought a 50-yd. piece of silk waist lining at 75 ct. a yard. She sold 12 yd. at $1 and 10 yd. at 95 ct., but the remainder, being kept in stock over the season, had to be sold at 65 ct. What was her per cent of gain or loss?

5. She bought a 20-yd. silk dress pattern at $2.10 a yard, being allowed, as a dressmaker, a discount of 5%, and 6% off for cash. She charged her customer the marked price, $2.10. What was her per cent of profit?

6. She charged her customer $25.50 for 3 yd. of Honiton lace, which had cost her $7 a yard. What was her per cent of profit?

7. She charged her customer $2 for findings for the dress. These consisted of 4 spools of silk at 10 ct. each, 1 spool of thread at 5 ct., 3 yd. of featherbone at 10 ct., a card of hooks and eyes at 8 ct., skirt braid 16 ct., plaiting 30 ct., waist binding 30 ct., and collar 10 ct. What was her gain per cent on the findings?

Grocery Problems

WRITTEN EXERCISE

1. If a grocer pays 15¢ a dozen for eggs, how many must he sell for 25¢ to gain $33\frac{1}{3}\%$ on the cost?

2. A grocer buys eggs at 20¢ a dozen, and sells them at the rate of 10 for 25¢. What is his gain per cent?

3. If a man buys sugar at 5¢ a pound, what must he charge for $3\frac{1}{2}$ lb. in order to gain 20% on the cost?

4. A grocer buys sugar at $12 per barrel of 300 lb., and sells it at the rate of 3 lb. 8 oz. for 17¢. What was his gain per cent?

5. A grocer buys canned fruit at the rate of 17¢ a can and sells it at the rate of $5.10 per case of 24 cans. What is his gain per cent?

6. My butcher's bill was $8 last week, my grocer's bill was 25% more, and my baker's bill was 80% less than my grocer's bill. What was the total of the three bills?

7. My grocer offers me 60 cakes of one kind of soap for $3.24, or 100 cakes of another kind for $4.38. Which is the more expensive per cake, and what per cent more expensive is it?

8. A grocer buys a crate of eggs containing 40 doz., paying 15¢ a dozen. He drops the crate and breaks 5 doz. How many must he sell for 25¢ so as still to gain $16\frac{2}{3}\%$ on the total cost?

9. A boy in a grocery store receives $6 a week. He spends 25% of it for board, 20% of the remainder for clothes, and $1 in other ways. If he saves the rest, how much will he save in a year (52 wk.)?

ORAL EXERCISE

1. 25 is $\frac{1}{2}$ of what number? 50% of what number?

2. 40 is $\frac{1}{3}$ of what number? $33\frac{1}{3}\%$ of what number?

3. Of what number is 50 one half? one third? two thirds? 10%? 25%? 50%? 100%? 125%? 200%?

The following numbers are the given per cents of what other numbers?

Think of the rate as a common fraction. If 7 is 10% of a number, it is $\frac{1}{10}$ of the number. Hence the number is 70. If 20 is $66\frac{2}{3}\%$ of a number, it is $\frac{2}{3}$ of the number, and $\frac{1}{3}$ is 10. Hence the number is 30.

4. 21, 10%.	5. 17, 50%.	6. 32, $33\frac{1}{3}\%$.
7. 49, 50%.	8. 63, $33\frac{1}{3}\%$.	9. 48, $66\frac{2}{3}\%$.
10. 150, 75%.	11. 400, 80%.	12. 12, $16\frac{2}{3}\%$.
13. 100, $8\frac{1}{3}\%$.	14. 21, $12\frac{1}{2}\%$.	15. 55, 125%.
16. 330, 150%.	17. 720, 200%.	18. 350, $166\frac{2}{3}\%$.
19. 99, $112\frac{1}{2}\%$.	20. 81, $112\frac{1}{2}\%$.	21. 180, $112\frac{1}{2}\%$.
22. 500, $166\frac{2}{3}\%$.	23. 250, $166\frac{2}{3}\%$.	24. 276, $110\frac{1}{2}\%$.

25. If \$6 is 3% of a number, what is 1% of the number? What is the number?

26. If \$9.12 is the interest on a certain sum for 1 yr. at 3%, what is the sum?

27. If \$4.50 is 5% of a certain sum, what is the sum? If it is $2\frac{1}{2}\%$, what is the sum?

28. If a piece of cloth shrinks 2 yd., the shrinkage being 5%, what was the original length?

29. A kettle of water loses 25% of its contents in boiling, and then contains 2 qt. less than it did at first. How much did it contain at first?

What is the number which, if decreased by 25% of itself, becomes the following?

If 150 is $\frac{3}{4}$ of a number, what is $\frac{1}{4}$? $\frac{4}{4}$?

30. 150. **31.** 600. **32.** 450. **33.** 750.
34. 3750. **35.** 2250. **36.** 9000. **37.** 6300.

Also, if decreased by $33\frac{1}{3}\%$ of itself, becomes the following?

38. 240. **39.** 400. **40.** 500. **41.** 600.
42. 1000. **43.** 1200. **44.** 1600. **45.** 2000.

Also, if decreased by 20% of itself, becomes the following?

46. 480. **47.** 840. **48.** 640. **49.** 320.
50. $80. **51.** $60. **52.** $24. **53.** $36.

Also, if increased by $16\frac{2}{3}\%$ of itself, becomes the following?

If 140 is $\frac{7}{6}$ of a number, what is $\frac{1}{6}$? $\frac{6}{6}$?

54. 140. **55.** 350. **56.** 280. **57.** 210.
58. $42. **59.** $70. **60.** $63. **61.** $49.

Also, if increased by $12\frac{1}{2}\%$ of itself, becomes the following?

62. $\frac{9}{8}$. **63.** 90. **64.** 63. **65.** 81.
66. $27. **67.** $18. **68.** $45. **69.** $99.

Also, if increased by $66\frac{2}{3}\%$ of itself, becomes the following?

70. $\frac{5}{3}$. **71.** 25. **72.** 45. **73.** 75.
74. $60. **75.** $80. **76.** $100. **77.** $500.

72. Illustrative problem. **$13.80 is 6% of what amount?**

Work in steps:		*Actual work:*
1. Since	6% of x = $13.80,	.06)$13.80
2. Therefore	x = $13.80 ÷ 0.06,	6) $1380
by dividing equals by 6%.		$230
3. That is,	x = $230.	

Or we may think that since $13.80 is 6% of some number, 1% is $\frac{1}{6}$ of $13.80, or $2.30, and 100% is 100 times $2.30, or $230.

73. *The percentage (part) divided by the rate equals the base (whole).*

WRITTEN EXERCISE

1. $0.85 is 2% of what sum? 5% of what sum?

2. $1.43 is 4% of what sum? 5% of what sum?

3. $1.75½ is 6% of what sum? 10% of what sum?

4. $22.68 is 7% of what sum? 20% of what sum?

5. On what sum is $7 the interest for 1 yr. at 4%?

6. If $13.50 is the interest on a certain sum for 2 yr. at 3%, what is the interest for 1 yr.? What is the principal?

7. A class measuring a certain distance on a wall made an error of 0.28 ft., which was 0.07% of the total distance. What was the distance?

8. A certain city has increased 4230 in population in ten years, which is 15% of the population ten years ago. What was the population then?

9. A merchant gained $4375 last year, which was 17½% of his capital. This year he has lost $2750. What per cent of his capital has he lost this year?

10. A man lost $247.50 in business last year, or 15% of his capital at the beginning of the year. How much was his capital at the beginning of the year?

74. Illustrative problems. 1. A man gained $16\frac{2}{3}\%$ on his capital of \$18,000. How much did he then have?

1. His capital + $16\frac{2}{3}\%$ of his capital = $116\frac{2}{3}\%$ of his capital.
2. $116\frac{2}{3}\%$ of \$18,000 = \$21,000, the amount after the increase.

2. A village having a population of 1900 ten years ago has since lost 13%. What is its present population?

1. Population − 13% of the population = 87% of the population.
2. 87% of 1900 = 1653, the present population.

In the first of these examples we might, of course, find $16\frac{2}{3}\%$, or $\frac{1}{6}$, and add, and in the second we might find 13% and subtract.

WRITTEN EXERCISE

1. Increase \$2750 by 15% of itself; by 12% of itself.

2. Decrease \$3500 by 12% of itself; by 8% of itself.

3. What is the amount of \$350 and a year's interest at 6%? at 5%? at $3\frac{1}{2}\%$? at $4\frac{1}{2}\%$?

4. A school had 204 pupils last year and has $16\frac{2}{3}\%$ more this year. How many has it now?

5. What is the amount of \$825 and two years' interest, the annual rate being 4%? 5%? 6%? $4\frac{1}{2}\%$?

6. A merchant starts the year with \$15,500, and loses 15% in his business. What does he then have?

7. Decrease \$750 by 50% of itself; that result by 50% of itself; and that result by 50% of itself. The result is what per cent of the \$750?

8. A man started in business with \$8500. The first year he gained 15%, and the next year he lost 15% of what he then had. How much had he then?

9. Write out a rule for finding the base increased or decreased by a certain per cent of itself, given the base and rate.

THE CATTLE INDUSTRY

WRITTEN EXERCISE

1. A ranchman owned 960 head of cattle in the fall, but the losses by storms and disease brought the number down to 928 in the spring. What was his per cent of loss?

2. In a certain year cattle on the range in the Southwest were worth $8.40 a head; a few years later they were worth $13.44 a head. What was the rate of increase in value?

3. In another year cattle in the same locality were selling for $22.50 a head, an increase of 80% in their value of five years before. What was their value then?

4. On the upland pastures in the Southwest it averages about 20 acres per head to pasture a steer through the winter. How many square miles for 1000 head?

5. A western farmer bought in November of one year 30 steers at $48.14 each, and fed them six months, or exactly 183 days. The corn cost him $9 a day for all the steers, and the hay 10¢ a day for each. He sold them for $121.06 each. How much did he lose on the lot, not considering the interest on his investment?

Oyster Industry

WRITTEN EXERCISE

Small "seed oysters" are taken from natural beds off the coast. They are "planted" in shallow water in "beds" rented from the state, and there are fattened. The large ones are called "box" oysters; the average or small ones, "cullings."

1. A man gathered 90 bu. of box oysters and 150 bu. of cullings. Each was what per cent of the other? of all?

2. There are 250 box oysters to a bushel, and $33\frac{1}{3}\%$ more cullings to a bushel. How many box oysters did the man gather? How many cullings?

3. He sold the box oysters for \$6.75 a thousand, which was $192\frac{6}{7}\%$ as much as he received per thousand for cullings. How much did he receive for all of his cullings?

4. He paid \$66 rent for one oyster bed and 12% less for another. What did he pay for both?

5. The oyster man paid 15¢ a bushel for shifting his oysters to another bed, and $66\frac{2}{3}\%$ as much for freight. What was the cost for both items on 250 bu.? He also paid a man \$2.50 a day for 12 days for dredging (gathering) and culling (sorting). What was the total of these three items of expense?

SIMPLE INTEREST REVIEWED

ORAL EXERCISE

Find the interest on the following sums for the times and at the rates specified:

1. \$2 for 1 yr. at 5%; at 6%.
2. \$25 for 1 yr. at 4%; at 2%.
3. \$300, at 5%, for 1 yr.; for 2 yr.
4. \$400, at 5%, for 1 yr.; for 3 yr.
5. \$500, at 6%, for 1 yr.; for 6 mo.
6. \$700, at 6%, for 1 yr.; for $1\frac{1}{2}$ yr.
7. \$400, at 6%, for 2 yr.; for 2 yr. 6 mo.
8. \$250, at 4%, for 1 yr.; for 1 yr. 6 mo.
9. \$1000, at 5%, for 3 yr.; for 3 yr. 6 mo.
10. \$2500, at 4%, for 1 yr.; for 2 yr.; for $2\frac{1}{4}$ yr.
11. \$3000, at 6%, for 1 yr. 6 mo.; for 1 yr. 4 mo.
12. \$2000, at 5%, for 1 yr. 6 mo.; for 2 yr. 6 mo.
13. \$1500, at 3%, for 1 yr.; for 2 yr.; for 1 yr. 4 mo.
14. \$5000, at 6%, for 1 yr.; for 3 yr.; for 6 mo.; for 3 mo.; for 1 mo.; for 15 da.

Find the amount of principal and interest together, given the following principals, rates, and times:

15. \$100, at 6%, for 1 yr.; for 2 yr.
16. \$300, at 5%, for 1 yr.; for 2 yr.
17. \$250, at 4%, for 1 yr.; for 3 yr.
18. \$500, at 4%, for 1 yr.; for 5 yr.
19. \$1000, at $4\frac{1}{2}$%, for 1 yr.; for 2 yr.
20. \$2000, at $5\frac{1}{2}$%, for 1 yr.; for 2 yr.

75. Illustrative problem. Find the interest on $500, at 6%, for 2 yr. 3 mo. 6 da.

1. The interest for one yr. is 6% of $500, or $30.
2. For 2 yr. it is 2 times $30 = $60.
   " 3 mo., or $\frac{1}{4}$ yr., it is $\frac{1}{4}$ of $30 = 7.50
   " 6 da., or $\frac{1}{5}$ of $\frac{1}{12}$ yr., it is $\frac{1}{5}$ of $\frac{1}{12}$ of $30 = .50
3. Therefore the total interest is $68.00

76. How to find interest. Therefore, to find interest,

Find the interest for one year, and multiply by the number of years or the part of a year.

WRITTEN EXERCISE

1. Find the interest on $275 for 3 yr. 6 mo. at 5%.
2. Find the interest on $1250 for 7 yr. 11 mo. at 6%.
3. Find the interest on $630 for 2 yr. 4 mo. 5 da. at 6%.
4. Find the interest on $345 for 2 yr. 3 mo. 3 da. at 5%.
5. Find the interest on $290 for 1 yr. 6 mo. 5 da. at 5%.
6. Find the interest on $350 for 1 yr. 9 mo. 15 da. at 4%.
7. Find the interest on $650 for 2 yr. 8 mo. 10 da. at 6%.
8. Find the interest on $1575 for 3 yr. 1 mo. 6 da. at $5\frac{1}{2}$%; at $4\frac{1}{2}$%; at 6%.
9. A man borrowed $250 on January 15, at 6%. He paid the principal and interest on October 15. How much did he pay?
10. A man borrowed $350 on January 27, at 6%. He paid the principal and interest on February 27 of the following year. How much did he pay?
11. A man bought 12 head of cattle on February 1, at $45 a head, giving his promissory note at 6%. He paid it on the 16th of the following April. What was the amount of principal and interest?

77. Cancellation used in interest. It is often advisable to shorten the work by cancellation. Find the interest on \$256.75 for 2 yr. 2 mo. 10 da. at $4\frac{1}{2}\%$.

Since $4\frac{1}{2}\% = \frac{9}{2}\% = \frac{9}{200}$, and 2 yr. 2 mo. 10 da. $= (730 + 60 + 10)$ da. $= 800$ da., we have

$$\frac{\overset{4}{\cancel{800}} \text{ times } 9 \text{ times } \$\overset{51.35}{\cancel{256.75}}}{\underset{73}{\cancel{365}} \text{ times } \cancel{200}} = \$25.32.$$

When the time exceeds 1 yr., and cancellation is used, it is better to take 365 da. to the year. Of course this gives exact interest, as described on page 108.

WRITTEN EXERCISE

Find the interest, given the principal, time, and rate:

1. \$172.60, 3 yr. 2 mo. 7 da., 4%.
2. \$391.75, 3 yr. 8 mo. 17 da., 4%.
3. \$235.50, 3 yr. 1 mo. 9 da., 6%.
4. \$175.50, 2 yr. 8 mo. 10 da., 6%.
5. \$142.80, 1 yr. 3 mo. 15 da., 5%.
6. \$273.40, 2 yr. 8 mo. 20 da., 5%.
7. \$172.60, 2 yr. 5 mo. 6 da., $4\frac{1}{2}\%$.
8. \$182.60, 4 yr. 8 mo. 17 da., $5\frac{1}{4}\%$.
9. \$163.40, 3 yr. 9 mo. 15 da., $5\frac{1}{2}\%$.
10. \$625.50, 2 yr. 11 mo. 25 da., $5\frac{3}{4}\%$.
11. \$475.50, 3 yr. 10 mo. 15 da., $4\frac{1}{4}\%$.
12. \$265.50, 2 yr. 10 mo. 20 da., $4\frac{3}{4}\%$.
13. \$3562.50, 3 yr. 2 mo. 5 da., $5\frac{1}{2}\%$.

78. The Six Per Cent Method. The following short method, known as the Six Per Cent Method, is often convenient.

Required the interest on $420 for 5 mo. 10 da. at 6%.

1. Since 2 mo. = $\frac{1}{6}$ yr., the rate for 2 mo. = $\frac{1}{6}$ of 6% = 1%.			
2. Therefore the int. for 2 mo. = 1% of $420			= $4.20
and for	2 mo. more,		4.20
" "	1 mo. "	$\frac{1}{2}$ of $4.20,	2.10
" "	10 da. "	$\frac{1}{3}$ of $2.10,	.70
3. Therefore the total interest is			$11.20

Therefore the interest at 6% for 60 days is 0.01 of the principal, for 6 days 0.001 of the principal, and for other periods the interest can be found from the interest for these periods.

Various other forms of this method are given, but none is so simple as the above. This is sometimes spoken of as the Banker's Method. The following method may be taken, if desired, in which case the above should not be taken.

Since the rate for 1 yr. is $\frac{6}{100}$ of the principal,

and for 1 mo. is $\frac{1}{12}$ as much, or $\frac{\frac{1}{2}}{100}$ of the principal,

and for 1 da. is $\frac{1}{30}$ of this, or $\frac{\frac{1}{6}}{1000}$ of the principal, therefore

79. *Multiply the principal by 6 times the number of years and $\frac{1}{2}$ the number of months as hundredths, adding the product of the principal by $\frac{1}{6}$ the number of days as thousandths.*

Thus, to find the interest, at 6%, on $120, for 2 yr. 3 mo. 9 da.:

First method:

$7.20	int. for	1 yr.
7.20		" "
1.20	(1% of $120)	2 mo.
.60	($\frac{1}{2}$ of $1.20)	1 "
.18	($\frac{3}{10}$ of $0.60)	$\frac{9}{30}$ "
$16.38		

Second method:

$$\frac{2\times 6+\frac{3}{2}}{100}\text{ of }\$120+\frac{\frac{9}{6}}{1000}\text{ of }\$120$$

$$=\left(\frac{27}{200}+\frac{9}{6000}\right)\text{of }\$120=\$16.38.$$

80. Further uses of the Six Per Cent Method. After finding the interest at 6%, the interest for other rates is easily found. Thus, take the interest at 6% and

Subtract $\frac{1}{6}$ of itself	to find the interest at 5%.
" $\frac{1}{3}$ " "	" " " " " 4%.
" $\frac{1}{4}$ " "	" " " " " $4\frac{1}{2}$%.
Divide by 2	" " " " " 3%.
Add $\frac{1}{6}$ of itself	" " " " " 7%.
" $\frac{1}{3}$ " "	" " " " " 8%.

ORAL EXERCISE

Find the interest at 6%:

1. On $750, 60 da.
2. On $275, 60 da.
3. On $350, 60 da.
4. On $425, 60 da.
5. On $300, 30 da.
6. On $250, 30 da.
7. On $460, 30 da.
8. On $840, 30 da.
9. On $200, 90 da.
10. On $400, 90 da.
11. On $500, 12 da.
12. On $400, 12 da.
13. On $400, 15 da.
14. On $600, 15 da.
15. On $600, 10 da.
16. On $720, 10 da.

Find the interest at 5%:

17. On $600, 60 da.
18. On $120, 60 da.
19. On $660, 60 da.
20. On $120, 30 da.
21. On $1200, 30 da.
22. On $720, 90 da.

Find the interest at 4%:

23. On $300, 60 da.
24. On $120, 60 da.
25. On $600, 60 da.
26. On $900, 60 da.
27. On $1200, 30 da.
28. On $900, 90 da.

WRITTEN EXERCISE

Find the interest:

1. $320, 8 mo., 6%.
2. $240, 7 mo., 5%.
3. $175, 9 mo., 6%.
4. $185, 2 yr. 6 mo., 5%.
5. $330, 7 mo., 8%.
6. $240, 3 yr. 8 mo. 2 da., 4%.
7. $1200, 2 yr. 6 mo. 3 da., 3%.
8. $475, 2 yr. 2 mo. 2 da., 4½%.
9. $2435, 2 yr. 3 mo. 9 da., 4%.
10. $1765, 3 yr. 2 mo. 5 da., 4½%.
11. $4200, 1 yr. 3 mo. 15 da., 5%.
12. $381.25, 1 yr. 6 mo. 18 da., 3%.
13. $426.40, 1 yr. 6 mo. 20 da., 5%.
14. $692.50, 2 yr. 8 mo. 15 da., 4½%.

Find the amount of principal and interest:

15. $175, 1 yr. 2 mo. 6 da., 6%.
16. $342, 2 yr. 3 mo. 4 da., 5%.
17. $1725, 2 yr. 8 mo. 6 da., 4½%.
18. $2175, 3 yr. 6 mo. 15 da., 5%.
19. $312.50, 2 yr. 7 mo. 10 da., 4%.
20. $427.25, 1 yr. 8 mo. 3 da., 5½%.
21. $375.50, 2 yr. 6 mo. 4 da., 3½%.
22. $137.50, 1 yr. 7 mo. 2 da., 4½%.
23. $243.75, 3 yr. 2 mo. 8 da., 2½%.

24. A man can buy $1750 worth of goods on 90 days' credit, or he can get them for $1720 in cash, which he can borrow for the 90 days at 6%. Which is the better plan, and how much better is it?

81. Difference in time. In working without tables, 360 days are usually considered a year, and 30 days a month.

Thus, to find the time from July 15 to October 5 it is customary to subtract as here shown, and say that the difference is 2 mo. 20 da., or 80 da. A banker, however, will see by his tables that July 15 is the 196th day of the year and October 5 is the 278th day, and that the difference in time is 82 da.

10 mo.	5 da.
7	15
2 mo.	20 da.

278
196
82

82. Table of time. The following table may be used:

	Jan.	Feb.	Mar.	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January . .	365	31	59	90	120	151	181	212	243	273	304	334
February . .	334	365	28	59	89	120	150	181	212	242	273	303
March . . .	306	337	365	31	61	92	122	153	184	214	245	275
April . . .	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June . . .	214	245	273	304	334	365	30	61	92	122	153	183
July . . .	184	215	243	274	304	335	365	31	62	92	123	153
August . .	153	184	212	243	273	304	334	365	31	61	92	122
September .	122	153	181	212	242	273	303	334	365	30	61	91
October . .	92	123	151	182	212	243	273	304	335	365	31	61
November .	61	92	120	151	181	212	242	273	304	334	365	30
December .	31	62	90	121	151	182	212	243	274	304	335	365

The exact number of days from any day of any month to the corresponding day of any month within a year is found opposite the first month and under the second. For example, from November 5 to May 5 is 181 days; to May 17, 12 days more.

WRITTEN EXERCISE

Find from the table the number of days from:

1. July 7 to November 7.
2. October 8 to March 15.
3. January 20 to May 10.
4. November 25 to July 7.
5. September 7 to January 7.
6. August 5 to December 10.

Using the table on page 105 to find the difference in time, find the interest:

7. On \$275, from May 2 to August 17, at 6%.
8. On \$650, from July 14 to October 29, at 6%.
9. On \$675, from March 25 to October 5, at 5%.
10. On \$675.50, from March 1 to July 25, at 3%.
11. On \$645.50, from June 2 to October 19, at 3%.
12. On \$185, from February 10 to August 2, at 5%.
13. On \$725, from October 3 to December 27, at 6%.
14. On \$420.50, from May 15 to November 2, at 4½%.
15. On \$350.75, from April 13 to September 20, at 4½%.

Find the amount of principal and interest:

16. On \$980, from January 7 to July 2, at 5%.
17. On \$145, from March 3 to January 5, at 6%.
18. On \$75, from May 2 to September 17, at 6%.
19. On \$442.50, from May 9 to January 6, at 4½%.
20. On \$275, from August 15 to February 7, at 6%.
21. On \$875, from February 12 to August 5, at 5%.
22. On \$630, from September 14 to March 9, at 5%.
23. On \$675.90, from June 6 to February 1, at 4½%.
24. On \$825.30, from July 15 to January 20, at 4½%.
25. On \$1027.50, from August 9 to February 2, at 3%.
26. On \$2075.50, from September 3 to January 15, at 3%.
27. What is the interest on a note for \$275, dated June 17, and due the following January 3, at 5%?
28. What is the amount of principal and interest on a note for \$450, dated September 21, and due the following August 1, at 5½%?

83. Interest tables. Bankers generally use interest tables based on 360 days to the year, although some use those based on 365 days, the latter plan being the fairer but yielding less interest. The following shows part of an interest table.

3 MONTHS, 6%

Total Days	1000	2000	3000	4000	5000	6000	7000	8000	9000	Days over 3 Mo.
90	15.00	30.00	45.00	60.00	75.00	90.00	105.00	120.00	135.00	0
91	15.17	30.33	45.50	60.67	75.83	91.00	106.17	121.33	136.50	1
92	15.33	30.67	46.00	61.33	76.67	92.00	107.33	122.67	138.00	2
93	15.50	31.00	46.50	62.00	77.50	93.00	108.50	124.00	139.50	3
......	......	......	......	......	......	......	......	......	......	......

84. Illustrative problem. From the table find the interest on $275 for 3 mo. 3 da. at 6%.

The int. on $200 is 0.1	that for $2000,	or	$3.10
" " " $70 " 0.01	" " $7000,	"	$1.09
" " " $5 " 0.001	" " $5000,	"	$0.08
" " " $275 "			$4.27

WRITTEN EXERCISE

Find the interest at 6%, in Exs. 1–6, using the table:

1. $2500, 3 mo.
2. $3450, 92 da.
3. $250.75, 91 da.
4. $5200, 93 da.
5. $475.50, 3 mo. 3 da.
6. $24,300, 3 mo.

Find the interest at 6%, taking the exact number of days from the table on page 105, and using the above table:

7. $350.75, December 19 to March 21.
8. $1250, November 17 to February 15.
9. $650, March 1 to June 2; May 4 to August 5.
10. $775, February 9 to May 13; to May 10; to May 12.

85. Exact interest. When 365 days are taken as a year, and the exact number of days between the dates is taken, the interest is called *exact interest.*

The government and some banks use exact interest.

86. Illustrative problem. What is the exact interest on \$1750 from July 5 to September 5 at 5%?

1. The exact difference in time is 62 da. (See table, page 105.)
2. The interest for 1 yr. is 5% of \$1750, or \$87.50.
3. The interest for 62 da. is $\frac{62}{365}$ of \$87.50, or \$14.86.

WRITTEN EXERCISE

1. Find the exact interest on \$4800 for 93 da. at 5%.

2. Find the amount of \$525 for 2 yr. 3 mo. 6 da. at 5%, exact interest; also at 6%, $4\frac{1}{2}$%, $3\frac{1}{2}$%.

3. Find the interest on \$1260 for 73 da. at 6%, 360 da. to the year; also the exact interest for 73 da. at 6%.

4. Find the exact interest on \$450 from January 3 to February 9 at 4%; from May 17 to July 20 at 5%.

5. What is the difference between the common interest at 360 days to the year, and the exact interest, on \$5000 for 60 days, at 6%?

Find the exact interest, and also the common interest at 360 days to the year, on each of the following:

6. \$420, 6%, 93 da.
7. \$3250, 2%, 192 da.
8. \$275, 5%, 63 da.
9. \$252.50, 6%, 60 da.
10. \$450, $5\frac{1}{2}$%, 70 da.
11. \$1250, $4\frac{1}{2}$%, 300 da.
12. \$3200, 4%, 3 mo. 9 da.
13. \$455.80, 5%, 275 da.
14. \$6000, 4%, from April 1 to June 1.
15. \$4850, 3%, from February 3 to March 10.

87. To find the rate. Illustrative problem. If the common interest on $250 for 3 mo. 6 da. is $4, what is the rate?

1. The interest for 1 da. is $\frac{\$4}{96}$, and the interest for 360 da. is 360 times $\frac{\$4}{96}$, or $15.

2. $15 is $\frac{15}{250}$ of $250, and $\frac{15}{250} = 0.06$.

3. Or we may say that, since $r\%$ of $\$250 = \$15$, $r\% = \$15 \div \$250 = 0.06$.

That is,

88. *The rate equals the interest for one year divided by the principal.*

WRITTEN EXERCISE

Given the principal, common interest, and time, find the rate:

1. $275, $27.50, 2 yr.
2. $250.50, $30.06, 2 yr.
3. $420, $50.40, 3 yr.
4. $480, $36, 1 yr. 6 mo.
5. $121.25, $9.70, 2 yr.
6. $240, $24, 2 yr. 6 mo.
7. $320.50, $57.69, 3 yr.
8. $325, $45.50, 3 yr. 6 mo.
9. $275.25, $55.05, 5 yr.
10. $240, $10.50, 1 yr. 3 mo.
11. $3500, $87.50, 10 mo.
12. $160, $19.80, 2 yr. 3 mo.
13. $220, $9.35, 1 yr. 5 mo.
14. $480, $26.60, 1 yr. 7 mo.
15. $31, $0.93, 1 yr. 6 mo.
16. $540, $60.75, 2 yr. 6 mo.
17. $720, $58.40, 2 yr. 10 da.
18. $360, $22, 1 yr. 6 mo. 10 da.
19. $360, $29.22, 1 yr. 4 mo. 7 da.
20. $1240, $172.05, 2 yr. 9 mo. 9 da.

89. To find the time. Illustrative problem. How long will it take the interest on \$320 to amount to \$28 at 5%?

Since the interest for 1 yr. is 5% of \$320, or \$16, \$28 is the interest for $\frac{28}{16}$ of a year, or $1\frac{3}{4}$ yr.

Or since the interest for 1 yr. is \$16, for x years it is \$16 x.

Therefore $\$16\,x = \$28.$

$$x = \$28 \div \$16 = 1\tfrac{3}{4}.$$

Therefore the time is $1\frac{3}{4}$ yr., or 1 yr. 9 mo.

90. *The number of years equals the total interest divided by the interest for one year.*

WRITTEN EXERCISE

Find the time it will take to gain the common interest stated in Exs. 1–10:

1. \$240, \$36 int., 6%.
2. \$280, \$28 int., 4%.
3. \$3000, \$155 int., 2%.
4. \$350, \$17.50 int., 3%.
5. \$175, \$12.25 int., 4%.
6. \$525, \$47.25 int., 6%.
7. \$600, \$42.75 int., 5%.
8. \$230, \$20.70 int., $4\frac{1}{2}$%.
9. \$360, \$19.91 int., $5\frac{1}{2}$%.
10. \$2500, \$37.50 int., $4\frac{1}{2}$%.
11. How long will it take a sum to double itself at 6%?
12. How long will it take \$220, together with the interest, to amount to \$253, at 6%?
13. A girl had \$250 invested for her, at 6%, on her birthday. When she became 21 it amounted, with interest, to \$400. How old was she when it was invested?
14. A father gave his son his promissory note for \$225, due when the son became 21 years old. The rate of interest was 5%, and when the note became due the principal and interest together amounted to \$303.75. How old was the son when the note was given?

ORAL EXERCISE

State without explanation the interest on:

1. \$100, 2 yr., 6%.
2. \$200, 1 yr., 6%.
3. \$400, 1 yr., 3%.
4. \$200, 2 yr., 3%.
5. \$500, 2 mo., 6%.
6. \$400, 4 mo., 6%.
7. \$850, 1 mo., 6%.
8. \$750, 6 mo., 6%.
9. \$640, 3 mo., 6%.
10. \$880, 3 mo., 6%.
11. \$660, 2 mo., 5%.
12. \$800, 3 mo., 5%.
13. \$360, 2 mo., 4%.
14. \$900, 2 mo., 4%.
15. \$500, 4 mo., 3%.
16. \$650, 4 mo., 3%.
17. \$800, 2 yr., $2\frac{1}{2}$%.
18. \$700, 2 yr., $2\frac{1}{2}$%.

The rate of $2\frac{1}{2}$% for a year is the same as 5% if the time is 2 yr.

19. \$650, 3 yr., $3\frac{1}{3}$%.
20. \$975, 3 yr., $3\frac{1}{3}$%.

Given the principal, interest, and time, find the rate:

21. \$400, \$20, 1 yr.
22. \$500, \$60, 2 yr.
23. \$600, \$72, 3 yr.
24. \$300, \$36, 4 yr.
25. \$1000, \$25, 6 mo.
26. \$200, \$6, 9 mo.
27. \$750, \$7.50, 2 mo.
28. \$640, \$3.20, 1 mo.

Given the interest, time, and rate, find the principal:

29. \$12, 1 yr., 6%.
30. \$9, 1 yr., 3%.

If \$9 is 3% of the principal, what is 1%? 100%?

31. \$32, 2 yr., 4%.
32. \$30, 2 yr., 3%.
33. \$12, 6 mo., 4%.
34. \$63, 3 yr., 3%.
35. \$4, 1 mo., 4%.
36. \$10, 2 mo., 5%.

Given the principal, interest, and rate, find the time:

37. \$400, \$20, 5%.
38. \$300, \$36, 6%.

WRITTEN EXERCISE

Find the common interest in Exs. 1–4:

1. \$3200, 93 da., 5%.
2. \$2100, $1\frac{1}{2}$ yr., 4%.
3. \$85.50, $6\frac{4}{15}$ mo., 6%.
4. \$175, 2 yr. $4\frac{1}{10}$ mo., 6%.

Find the exact interest in Exs. 5–8:

5. \$3200, 128 da., 5%.
6. \$5000, 193 da., 4%.
7. \$2750, 227 da., 6%.
8. \$32,460, 230 da., 5%.

Given the principal and common interest, find the rate:

9. \$108, \$6, 1 yr. 1 mo. 10 da.
10. \$36, \$3.66, 2 yr. 6 mo. 15 da.
11. \$1440, \$77.70, 1 yr. 6 mo. 15 da.
12. \$2750, \$115.50, 1 yr. 2 mo. 12 da.

Find the time, given the principal, interest, and rate:

13. \$5500, \$230, $3\frac{1}{2}$%.
14. \$375, \$54.25, 6%.
15. \$425, \$63.75, 3%.
16. \$2880, \$155.40, $3\frac{1}{2}$%.

17. How long must a man have \$2750 invested at 5% so that the principal and interest would amount to \$3231.25?

18. A man borrowed \$8000 in Boston at $4\frac{1}{2}$%, and loaned it in Oregon at 8%. How much did he gain in three years by the transaction?

19. A man has \$250 in a savings bank at 3%. He leaves the principal, but draws out the interest as it is due. How much does he draw out in four years?

20. A man having \$13,500 invested at 5% reinvests \$4000 of it at $5\frac{1}{2}$%, \$5000 of it at $3\frac{1}{2}$%, and leaves the rest invested as before. What is the annual difference in income?

21. Which yields the better income, $1675 at 5% or $1375 at 6%? What is the difference for 2 yr. 6 mo.?

22. A man receives $1718.75 interest in 2 yr. 6 mo. on an investment of $17,500. What is the rate of interest?

23. A boy has $300 given to him the day he is 14 years old. His father invests it for him in a 5% promissory note, due on the day he is 21 years old. What is the amount on the day the note is due?

24. A real estate dealer buys 360 acres of farm land at $30 an acre, and after keeping it 7 months sells it at an advance of $3.75 an acre. The money being worth to him 5% a year, how much does he gain by the transaction?

25. A man having $26,750 invested in business has found that his annual profits average 18% a year. He is offered $35,000 for the business, which he can invest at 4¼%. If he sells out and retires, what will be his loss in income?

26. On April 10 a coal dealer borrowed $33,250 at 5%, with which he purchased his summer's supply of coal at $4.75 a ton. He sold the coal at $5.65 a ton, the buyers paying for unloading and delivery, and paid his debt on November 16. How much did he gain?

27. I own a house which I rent at $25 a month. My taxes are $50 a year, my repairs $50, and my insurance $10. Would it be better for me to sell the house for $5000 and invest the money at 4%, if my taxes would then be reduced to $10? What is the difference in income?

28. On October 15 a dealer purchased $1750 worth of goods for the holiday trade on 30 days credit. At the end of that time he gave his note for 2 months at 6% a year. He sold the goods for $2105, and paid the note when due. What per cent did he make on the original purchase price?

Factory Problems

WRITTEN EXERCISE

1. A certain canning factory uses the product of 75 acres of peas during the three weeks' season. In this time it puts up 48,000 cans of peas. At 6 days to the week, how many acres of peas does it use a day? How many cans does it put up every 3 hours, allowing 8 hours to the working day?

2. A $\frac{1}{2}$-lb. stick of solder is used to seal 30 of these cans. How many pounds of solder are required for a week's work? This solder being 20% lead, how many pounds of lead are used in a day?

3. The steam kettle in which the peas are cooked will hold 3 baskets, each containing enough peas for 240 cans. How many cans of peas can be cooked in one forenoon (7.50 A.M. to 12 M.) if 25 min. are allowed to each kettleful?

4. A farmer owns 3 of the 75 acres mentioned in Ex. 1. He therefore furnishes the peas for how many cans? If he takes for his pay 50% of the canned goods for which he furnished the peas, how many cases of 24 cans each should he receive?

5. A workman in the factory puts together 200 of these cases a day. How long will it take him to put together cases enough for the season's output mentioned in Ex. 1? How much will he earn in that time, at 90¢ per hundred cases?

6. The factory runs from 8 A.M. to 5 P.M., with an hour out at noon. One day some machinery broke down, stopping all work from 10 A.M. to 2 : 30 P.M. What was the total output that day?

Problems of a Grocer

WRITTEN EXERCISE

1. Mr. F. T. Barker has $20,000 invested in his building and grocery stock. Last year his profits were $2500. What was his rate of profit?

2. Of the $20,000 he owes $8000, paying 5½% interest. How much did this take from his $2500 profit? What rate of income is he receiving on his own money?

3. Last summer he bought 100 muskmelons for $6.50. He sold half of them at 9¢ each, 33 of them at the rate of 3 for 25¢, and the rest he had to throw away. Did he gain or lose, and what per cent?

4. He bought 300 bananas for $4 and sold them at the rate of 22¢ a dozen. What was his gain per cent? At 25¢ a dozen, what would it have been?

5. He bought 500 lb. of clover honey for $70 and sold all but 50 lb. of it at a profit of 32%. The rest he sold at a loss of 55%. Did he gain or lose, and how much?

6. He employs 4 order clerks, paying them $12 a week and ½% commission on all goods sold. Their average sales being $280, $220, $240, and $270 a week respectively, what is the average weekly income of each?

7. He buys sugar at \$20 per barrel of 344 lb., and sells it at 6¢ a pound. How much does he gain on 135 lb.?

8. If he can sell a barrel of sugar (see Ex. 7) at the rate of 48¢ for 7 lb., how much will he make on a barrel?

9. If he buys a 45-lb. chest of tea for \$16.20, and sells it at 50¢ a pound, what is his rate of profit?

10. If 12 chests of the above tea were damaged by being stored in a damp place, and brought 25¢ a pound, what was the loss? What was the per cent of loss?

11. He bought coffee by the 100-lb. bag at \$20 a bag. At what price per pound must he sell this to make 45%?

12. If he buys flour at \$4.75 a barrel and sells it at \$5.25, what profit does he realize on 51 bbl.? What is the per cent of profit?

13. He pays \$4 a box for laundry soap, 100 cakes to the box, and sells the soap at 6 cakes for a quarter. What is his gain on a dozen boxes? his per cent of gain?

14. He bought 15 boxes of starch at \$1.60 a box. There being 40 lb. to the box, what is his gain if he sells it all at 5¢ a pound? What is the rate of gain?

15. He bought 9 cases of tomatoes, 2 doz. cans to the case, at 92¢ a dozen. He sold them at 3 cans for a quarter. What was his gain or loss? the per cent of gain or loss?

16. A box of condensed milk contains 4 doz. cans. If he buys 18 boxes at \$3.84 each, and sells $\frac{1}{3}$ at 7¢ a can and the rest at a profit of $12\frac{1}{2}\%$, does he gain or lose on the lot, and how much?

17. He buys 100 5-lb. caddies of Hyson tea for \$420. He sells 75 caddies at a profit of 15%, and the rest at a loss of 5%. How much did he gain? At what price apiece did he sell the 25 caddies?

RATIO AND PROPORTION REVIEWED

ORAL EXERCISE

1. What are the two common ways of expressing the ratio of 3 to 5? Write them on the blackboard.

2. What must be the nature of the terms of a ratio, as to similarity? Illustrate.

3. In the ratio 4 ft. : 20 ft., which term is the antecedent? What is the name of the other term?

4. What does the ratio of $4 to $2 equal? of 4 ft. to 2 ft.? What is the nature of the ratios of concrete numbers?

State the simplest values of the following ratios:

5. $\frac{2}{4}$. 6. $\frac{3}{6}$. 7. $\frac{12}{36}$. 8. $\frac{17}{51}$. 9. $\frac{21}{28}$.

10. $\frac{30}{40}$. 11. $\frac{25}{75}$. 12. $\frac{45}{90}$. 13. $\frac{15}{60}$. 14. $\frac{36}{12}$.

15. $5 : $10. 16. 6 in. : 3 in. 17. 10 ft. : 30 ft.

18. $6 : $42. 19. $68 : $17. 20. 13 mi. : 104 mi.

State the value of x *in each of the following:*

21. $\frac{x}{3} = 7$. 22. $\frac{x}{5} = 9$. 23. $\frac{x}{4} = 7$. 24. $\frac{x}{6} = 8$.

25. $\frac{x}{9} = 3$. 26. $\frac{x}{5} = 8$. 27. $\frac{x}{7} = 6$. 28. $\frac{x}{8} = 8$.

29. $x : 3 = 4$. 30. $x : 4 = 9$. 31. $x : 9 = 4$. 32. $x : 8 = 6$.

33. $x : 7 = 7$. 34. $x : 8 = 3$. 35. $x : 3 = 11$. 36. $x : 9 = 9$.

37. $\frac{5}{x} = 5$. 38. $\frac{10}{x} = 2$. 39. $\frac{12}{x} = 4$. 40. $\frac{15}{x} = 3$.

41. $\frac{20}{x} = 10$. 42. $\frac{30}{x} = 3$. 43. $\frac{45}{x} = 15$. 44. $\frac{55}{x} = 5$.

45. $\frac{x}{2} = \frac{3}{6}$. 46. $\frac{x}{4} = \frac{6}{8}$. 47. $\frac{x}{15} = \frac{3}{5}$. 48. $\frac{x}{20} = \frac{3}{10}$.

91. Ratio. The relation of one quantity to another of the same kind, as expressed by division, is called the *ratio* of the first to the second.

The ratio of \$4 to \$6 is written $\frac{\$4}{\$6}$ or \$4 : \$6. It equals $\frac{4}{6}$, or $\frac{2}{3}$.

92. Antecedent. The first term is called the *antecedent*.

93. Consequent. The second term is called the *consequent*.

94. *A ratio is always abstract, and its terms may be written as abstract numbers.*

95. Illustrative problems. 1. If $\frac{x}{3.7} = 2.7$, what does x equal?

Multiplying by 3.7, $\cancel{3.7} \times \frac{x}{\cancel{3.7}} = 3.7 \times 2.7,$

or $x = 9.99.$

2. Find the value of x, given $\frac{5.44}{x} = 1.7.$

Multiplying by x, $5.44 = 1.7\,x,$

Hence $x = \frac{5.44}{1.7} = 3.2.$

WRITTEN EXERCISE

Find the value of x *in each of the following:*

1. $\frac{x}{4.9} = 6.7.$
2. $\frac{x}{71.2} = 6.42.$
3. $\frac{x}{31.2} = 0.61.$
4. $\frac{x}{12.6} = 17.3.$
5. $\frac{2x}{31} = 4.2.$
6. $\frac{3x}{1.7} = 4.2.$
7. $\frac{5x}{7.3} = 2.5.$
8. $\frac{7x}{9.3} = 6.3.$
9. $\frac{2.55}{x} = 5.$
10. $\frac{2.97}{x} = 2.7.$
11. $\frac{9.03}{x} = 4.3.$
12. $\frac{45.6}{x} = 5.7.$
13. $\frac{x}{17} = \frac{2.3}{4}.$
14. $\frac{x}{2.3} = \frac{4.1}{7}.$
15. $\frac{x}{1.6} = \frac{4}{7}.$

96. Illustrative problem. Required to separate $1540 into two parts having the ratio of 5 to 9.

Since one part contains $5 when the other contains $9, the two will then contain $14.

Therefore $\frac{5}{14}$ belongs to the first part and $\frac{9}{14}$ to the second.

$\frac{5}{14}$ of $1540 = $550, and $\frac{9}{14}$ of $1540 = $990.

Check. $550 + $990 = $1540.

97. Partitive proportion. The separation of a number into parts having a given ratio is called *partitive proportion.*

ORAL EXERCISE

Separate into parts having the ratios stated:

1. 20, 3 : 7.	2. 20, 2 : 3.	3. 26, 4 : 9.
4. 32, 5 : 11.	5. 32, 7 : 9.	6. 34, 6 : 11.
7. 18, 5 : 13.	8. 40, 7 : 13.	9. 39, 2 : 11.
10. 90, 13 : 17.	11. 26, 11 : 2.	12. 24, 11 : 13.

13. Two partners are to divide $1500 of profits in the ratio of 1 to 2. What is the share of each?

14. A 20-ft. beam is to be divided in the ratio of 2 to 3. The point of division is how far from each end?

15. The ratio of cloudy to clear days in a certain November was 2 : 3. How many of the days were clear?

16. The ratio of cloudy to clear days in February, 1904, in a certain place, was 12 : 17. How many days were clear?

17. In a class of 30, 4 out of 5 were promoted. What was the ratio of promoted to nonpromoted? How many failed of promotion?

18. Out of 44 boys, some are defending a fort and some are attacking. The ratio of the defenders to the others is 1 to 3. How many are attacking?

WRITTEN EXERCISE

Separate into parts having the ratios stated:

1. 528, 17:31. 2. 702, 13:41. 3. 450, 21:29.
4. 1050, 34:41. 5. 13.23, 31:32. 6. 15.54, 21:53.

7. Separate 165 into 3 parts having the ratio of 3 to 5 to 7.

This means that the first is to the second as 3 to 5, and the second to the third as 5 to 7. That is, $\frac{3}{15}$ of 165 is the first, $\frac{5}{15}$ the second, $\frac{7}{15}$ the third, or $\frac{1}{5}$, $\frac{1}{3}$, $\frac{7}{15}$, making 33 the first part.

Separate into parts having the ratios stated:

8. 273, 5 : 7 : 9. 9. 651, 7 : 11 : 13.
10. 2173, 11 : 13 : 17. 11. 33.67, 9 : 11 : 17.
12. 4.455, 11 : 21 : 23. 13. 40.47, 17 : 19 : 21.

14. Water is composed of two gases, hydrogen and oxygen, combined in the proportion of 2 to 1, respectively. In 390 parts of water, how many parts of each gas?

15. Two partners agree to divide their profits in the ratio of the amounts invested in the business. One invests $4000 and the other $3000, and they make $2800. What is the share of each?

16. Two farmers agree to pay for the rent of a pasture in the ratio of the number of head of cattle pastured. One pastures 23 head and the other 32, and they pay $68.75. What is the share of each?

17. Two men receive $25.90 for picking apples, to be divided in the ratio of the number of bushels picked by each. For every 66 bu. picked by the first the second picked $2\frac{1}{2}$ bu. less. How was the money divided?

The common expression "to divide in the proportion of 2 to 3" means the same as "to divide in the ratio of 2 to 3."

98. Proportion. An expression of the equality of two ratios is called a *proportion.*

99. Illustrative problems. 1. In the proportion $x : 17.2 = 4.3 : 8$, find the value of x.

$$\frac{x}{17.2} = \frac{4.3}{8}. \quad \text{Multiplying by 17.2, as in § 95,} \quad x = \frac{\overset{4.3}{\cancel{17.2}} \times 4.3}{\underset{2}{\cancel{8}}} = 9.245.$$

2. If $21.3 : 17.1 = 3 : x$, find the value of x.

1. Because $21.3 : 17.1 = 3 : x$, we must have $17.1 : 21.3 = x : 3$.
2. That is, $\frac{x}{3} = \frac{17.1}{21.3}$, and $x = \frac{\cancel{3} \times 17.1}{\underset{7.1}{\cancel{21.3}}} = 2\frac{29}{71}$.

WRITTEN EXERCISE

Find the value of x:

1. $x : 18.2 = 9.1 : 7.3$.
2. $x : 4.1 = 82.7 : 82$.
3. $x : 21.7 = 2.4 : 3.3$.
4. $x : 1.37 = 2.4 : 13.7$.
5. $x : 3.21 = 4.12 : 6.6$.
6. $x : 0.32 = 7.21 : 1.8$.
7. $3.17 : x = 4.28 : 3.2$.
8. $2.4 : x = 3.8 : 123$.
9. $8.21 : 3 = 4.105 : x$.
10. $3.9 : 4 = x : 3200$.

11. A certain circle has a circumference of 182.73 in. What is the circumference of a circle whose radius is to that of the first as $1.7 : 2.91$?

12. A certain water pipe has a diameter of 1.91 inches. What is the diameter of a pipe whose circumference is to that of the first as $2.6 : 3.42$?

13. A certain fountain pen will write on an average 4210 words with one filling. The amount of ink it holds is to that held by a similar pen as $10 : 13$. How many words will the second pen average at one filling?

100. Extremes and means. The first and last terms of a proportion are called the *extremes;* the second and third, the *means.*

101. It is an interesting property of proportion, as seen in a case like

$$\frac{3}{7} = \frac{15}{35},$$

where $3 \times 35 = 7 \times 15$, that

The product of the means equals the product of the extremes.

102. We also see that if $x : 21 = 35 : 105$,

$$x = \frac{21 \times 35}{105}.$$

Therefore, in any proportion,

The product of the means divided by the given extreme equals the required extreme.

103. Illustrative problem. If $x : 7 = 13 : 20$, what is the value of x?

By § 102, $$x = \frac{7 \times 13}{20} = 4\tfrac{11}{20}.$$

WRITTEN EXERCISE

1. A miller uses 18 bu. of wheat for 4 bbl. of flour. How many barrels can he make from 207 bu.?

2. He sells 25 bbl. of this flour for $112.50. At the same rate, what will he receive for 476 bbl.?

3. The Bennington Battle monument casts a shadow 125 ft. 10 in. long at the same time that a 3-ft. post casts a 15-in. shadow. What is the height of the monument?

4. To find the height of a church spire a boy measured its shadow, 152 ft. 3 in., at the same time that a post 3 ft. 6 in. high was casting a shadow 5 ft. 3 in. long. How high was the spire?

5. How long will it take a train to go 148 mi., at the rate of $55\frac{1}{2}$ mi. in 1 hr. 30 min.?

6. If 9 men can complete some work in 20 hr., how many less men can complete it in 16 hr. more time?

7. If a man can skate 75 yd. in $8\frac{1}{5}$ sec., how long would it take him to skate 200 yd. if he could maintain this rate?

8. If a man can skate 220 yd. in 19 sec., how long would it take him to skate 5 mi. if he could maintain this rate?

9. If 750 lb. can be transported 100 mi. for a certain sum, how far should $\frac{1}{2}$ T. be transported at the same rate?

10. If 6 men can complete some work in 15 hr., how many men must be added to complete it in 5 hr. less time?

11. If 10 men can complete some work in 30 hr., how many men must be added to complete it in 10 hr. less time?

12. If a man is rowing at the average rate of 1 mi. in 6 min. $23\frac{41}{105}$ sec., how long will it take him to row 1 mi. 550 yd.?

13. If a man can swim 45 yd. in 23 sec., how long would it take him to swim 2 mi. if he could maintain this rate?

14. When a sum of money is divided among 7 persons each receives $16.80. How much would each receive if the same sum were divided among 8 persons?

15. One man can do a piece of work in 10 da., and another in 9 da. If the wages of the first are $3.60 a day, what should be the wages of the second?

104. Compound proportion. An expression of the equality of the products of ratios is called a *compound proportion.*

For example, $\frac{2}{3}$ of $\frac{5}{7} = \frac{x}{6}$ of $\frac{3}{4}$. This may also be written thus:

$$\left.\begin{matrix}2:3\\5:7\end{matrix}\right\} = \left\{\begin{matrix}x:6\\3:4\end{matrix}\right..$$

105. Inverse ratio. The ratio $2:3$ is called the *inverse* of the ratio $3:2$.

Problems formerly solved by compound proportion are now more frequently solved by analysis (see Smith's *Intermediate Arithmetic*, p. 169). Where it is necessary to distinguish the ordinary proportion of four terms from compound proportion, the former is called *simple proportion.* Compound proportion may be omitted without interfering with the subsequent work.

106. Illustrative problems. 1. In the compound proportion here given, find the value of x.

$$\left.\begin{matrix}2:3\\5:7\end{matrix}\right\} = \left\{\begin{matrix}x:6\\3:4\end{matrix}\right.$$

1. Writing this in the more familiar form of fractions, we have

$$\frac{2}{3} \text{ of } \frac{5}{7} = \frac{x}{6} \text{ of } \frac{3}{4}.$$

2. If we divide these equals by $\frac{3}{4}$, or multiply by $\frac{4}{3}$, they will still be equal. Therefore

$$\frac{4}{3} \text{ of } \frac{2}{3} \text{ of } \frac{5}{7} = \frac{x}{6} \text{ of } \frac{\cancel{3}}{\cancel{4}} \text{ of } \frac{\cancel{4}}{\cancel{3}}.$$

3. Since we now have $\frac{1}{6}$ of x, x equals 6 times as much. Therefore

$$6 \times \tfrac{4}{3} \text{ of } \tfrac{2}{3} \text{ of } \tfrac{5}{7} = x.$$

Therefore $$\tfrac{80}{21} = x.$$

Such results should always be checked by being substituted for x in the compound proportion. Thus, $\frac{2}{3}$ of $\frac{5}{7} = \frac{1}{6}$ of $\frac{80}{21}$ of $\frac{3}{4}$, for each reduces to $\frac{10}{21}$.

2. If 4 loads of hay will last 6 horses 8 weeks, 8 loads will last how many horses 3 weeks?

1. Considering only the number of loads in relation to the number of horses, the ratio of the latter equals the ratio of the former, for twice as many horses will need twice as many loads, and so on. Therefore

$$x \text{ horses} : 6 \text{ horses} = 8 \text{ loads} : 4 \text{ loads, or } x : 6 = 8 : 4.$$

2. Considering only the number of weeks in relation to the number of horses, the ratio of the latter equals just the inverse ratio of the former; for if we have twice as many horses, the hay will last only half as long, the number of weeks diminishing in the same ratio that the number of horses increases. Therefore

$$x \text{ horses} : 6 \text{ horses} = 8 \text{ weeks} : 3 \text{ weeks, or } x : 6 = 8 : 3.$$

3. Finally, considering both the loads and the time, we have

$$x : 6 = \begin{cases} 8 : 4 \\ 8 : 3 \end{cases},$$

or, in the more familiar form of fractions,

$$\frac{x}{6} = \frac{8}{4} \text{ of } \frac{8}{3}, \quad \text{whence } x = \overset{2}{\cancel{6}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{4}} \text{ of } \frac{8}{\cancel{3}} = 32,$$

the number of horses.

107. We may simplify the actual work by noticing that, as in simple proportion,

In any proportion the product of the means equals the product of the extremes.

Hence, in the above example, after reasoning out the form of the proportion, we have simply to write

$$4 \times 3 \times x = 6 \times 8 \times 8.$$

$$x = \frac{\overset{2}{\cancel{6}} \times \overset{2}{\cancel{8}} \times 8}{\cancel{4} \times \cancel{3}} = 32.$$

WRITTEN EXERCISE

1. If 4 bbl. of flour last 6 persons 12 mo., how many barrels will last 15 persons 18 mo.?

2. If 75 hens eat 16 bu. of feed in 12 wk., how many bushels will 30 hens eat in 5 wk.?

3. If a farm 130 rd. by 80 rd. costs $1750, how much will one 118 rd. by 50 rd. cost at the same rate?

4. If the total cost of feeding 75 hens for 7 wk. is $10.50, what will it cost to feed 40 hens for 9 wk.?

5. If 4 five-cent loaves of bread last 6 persons 2 da., how many ten-cent loaves will last 5 persons a week?

6. If $800 yields $324 interest in 6 yr. 9 mo., how much interest will $500 yield in 4 yr. 5 mo. at the same rate?

7. In a sewing guild 18 girls make 40 garments in 4 hr. At this rate, how many garments can 30 girls make in 3 hr.?

8. If the value of the eggs laid by 75 hens in 12 wk. is $67.50, what is the value of the eggs laid by a third as many hens in 5 wk.?

9. If 5 boys, working 8 hr. a day, can build a cabin in the woods in 12 da., how many days will it take 15 boys working 2 hr. less a day?

10. If 5 girls in a sewing class make 20 aprons in 3 da., how many girls will it take to make 32 aprons in 4 da.? to make 16 aprons in 6 da.?

11. A stone wall is to be constructed around a rectangular piece of land 2000 ft. by 1075 ft. Ten men, working 8 hr. a day, build 890 ft. in 3 da. The number of men is then diminished 50%, and the remainder work 9 hr. a day. How long will it take to complete the wall?

12. If 25 children use 10 boxes of crayons, 200 in a box, in 4 mo., how long will 6 boxes, 100 in a box, last 40 children?

13. If 2 pumps, working 10 hr. a day for 4 da., can raise 120 T. of water, how many tons can 3 pumps raise in 5 da. of 8 hr. each?

14. If 5 men, working 14 da., 6 hr. a day, can excavate a cellar, how long will it take if there are 7 more men, and they work 1 hr. longer each day?

15. If it will take 5 boys 14 da., working 6 hr. a day, to clear a stony field for a ball ground, how long will it take 12 boys, working 8 hr. a day?

16. Two men rented a pasture for $83. The first puts in 6 horses for 4 mo., the second puts in 5 horses for $3\frac{1}{2}$ mo. What is the share to be paid by each?

17. If it takes 6 boys 15 da., working 4 hr. a day, to build a playhouse, how many boys should be able to do the work in 12 da., working 3 hr. a day?

18. If 9 girls in a manual training class made 12 baskets in 4 da., working 45 min. a day, how many baskets can 15 girls make in 6 da., working an hour a day?

19. If a railroad charges $9 for transporting 3 T. of goods 360 mi., how much should be charged for transporting $7\frac{1}{2}$ T. of goods 280 mi. at the same rate?

20. If 5 girls, earning money for Christmas gifts, dressed 14 dolls in 4 da., working 6 hr. a day, how many dolls could 8 girls dress in 3 da. of 5 hr. each, at the same rate?

21. Of 2 boys raising eggs for the market, one has 40 hens and the other 75. If the 40 hens lay 1440 eggs in 12 wk., how many will the 75 hens lay in half that time, at the same rate?

Farm Problems

WRITTEN EXERCISE

Estimate the answers, putting the estimates in writing for comparison with the correct answers. Some estimates should be exact.

1. A meadow is $19\frac{1}{2}$ rd. long and $9\frac{1}{4}$ rd. wide. What is its area in acres?

2. A square bin has a capacity of 2352 cu. ft. The bin is $5\frac{1}{3}$ ft. deep. What is its length? (Given that $441 = 21 \times 21$.)

3. If 1 qt. of corn will plant 230 hills, what will the corn cost for 2760 hills @ 25¢ a quart?

4. If 1 qt. of beans will plant 150 hills, what will the beans cost for 1950 hills @ 35¢ a quart?

5. What is the cost of seed for 7500 celery plants, allowing $\frac{1}{2}$ oz. to 1000 plants, @ 22¢ an ounce?

6. What is the cost of seed for 3500 cauliflower plants, allowing 1 oz. to 1000 plants, @ \$2.30 an ounce?

7. Find the cost of oats required to sow 15 acres, allowing $2\frac{1}{4}$ bu. to the acre, the oats being worth 90¢ a bushel.

8. Find the cost of beans required to plant 5 acres, allowing 12 qt. to the acre, the beans being worth \$1.50 a peck.

9. Find the cost of peas required to plant 5 acres, allowing $1\frac{1}{2}$ bu. to the acre, the peas being worth \$3.50 a bushel.

10. Find the cost of corn required to plant 20 acres, allowing 8 qt. to the acre, the corn being worth \$2 a bushel.

11. Find the cost of potatoes required to plant 12 acres, allowing 10 bu. to the acre, the potatoes being worth \$1.15 a bushel.

12. If a tree sparrow eats an ounce of weed seed every 4 days, how many pounds will 500 such birds eat in May?

13. If a tree sparrow eats in a day enough seeds to produce 75 weeds, a dozen such birds will destroy in a week enough seeds to produce how many weeds?

14. If a woodpecker eats on an average 1690 insect pests in a day, how many will 250 woodpeckers eat during June?

15. If a woodpecker eats on an average 10% less insect pests in May than in June, how many will 50 woodpeckers eat during May? (See Ex. 14.)

16. The king bird eats 89% animal food. Out of 11 lb. 8 oz. of food eaten by these birds, how many ounces are vegetable food?

17. Of the food of bobolinks, 63% is insects. How much insect food is consumed by them while eating 7 lb. $6\frac{2}{5}$ oz. of other (vegetable) food?

18. Of the food of blackbirds, 57% is weed seed. When these birds have consumed $14\frac{1}{3}$ lb. of other food, how many pounds of weed seed have they eaten?

19. Out of 5 lb. of food eaten by orioles, $12\frac{4}{5}$ oz. consist of vegetable food, the remainder being insects injurious to plant life. What per cent of the food is injurious to plants?

20. It is estimated that the offspring of a pair of English sparrows would, were there no deaths, amount in ten years to 275 million. Estimating 75% of the food of these birds to be grain, and allowing $\frac{1}{2}$ oz. of food to each bird daily, how many pounds of grain would this offspring consume in a day? in 365 da.?

21. If a heaped barrel of potatoes contains 3 bu., how many such barrels will contain 567 bu.? How many bushels in 297 such barrels?

22. The 2,568,391,056 bu. of Indian corn produced in this country in one year was 26% more than the number of bushels of wheat. How much wheat was produced?

23. The average price of hay in Kansas recently was only \$4.06 a ton, which was $23\frac{1}{5}$% as much as in Massachusetts. What was the price in Massachusetts, and why was there this great difference?

24. When the hay crop of the country is 60 million tons, worth \$7.50 a ton, what is the value of the whole crop? If the crop of New York, the chief hay-producing state, is worth 10% of this amount, what is its value?

25. In one of the states 2.2 million acres produced 74.8 million bushels of oats in one year, worth \$18,700,000. What was the average yield per acre, the average price per bushel, and the average value of the crop per acre?

26. When the annual products of the 155,000 farms in Minnesota were worth \$170,500,000, what was the average per farm?

27. Of these farms, if 6,500,000 acres produced 97,500,000 bu. of wheat, what was the average yield per acre? What was the total yield worth at 60¢ a bushel?

28. Some of this grain went to Duluth, where an elevator distributed 20,000 bu. of wheat to a ship in one hour. Allowing 62 lb. to the bushel, how many tons did it distribute in a minute?

29. If one elevator spout carries $8602\frac{14}{93}$ bu. of wheat in an hour, how long will it take a 6-spout elevator to unload a train of 40 cars, averaging 20 tons to a car, allowing 62 lb. to the bushel, no allowance being made for moving the cars?

ORAL EXERCISE

1. At $26 a ton, what is the cost of 200 lb. of bone meal? of 50 lb.?

2. At $26 a ton, what is the cost of 100 lb. of cotton-seed meal? of 500 lb.?

3. At $12.50 a ton, what is the cost of 100 lb. of acid phosphate? of 400 lb.?

4. At $1.35 per 100 lb., what is the cost of a ton of bone meal? of 300 lb.?

5. At $48 a ton, what does 1 lb. of nitrate of soda cost? What is the cost of 100 lb.?

6. At $125 a ton, what is the cost of 200 lb. of sulphate of copper? of 100 lb.?

7. At $2.50 per 100 lb., what is the cost of a ton of nitrate of soda? of 250 lb.?

8. At $2.20 per 100 lb., what is the cost of a ton of muriate of potash? of 500 lb.?

9. At $2.60 per 100 lb., what is the cost of a ton of sulphate of potash? of 500 lb.?

10. If nitrate of soda contains 15% nitrogen, what is the weight of nitrogen in a ton?

11. At $3.30 per 100 lb., what is the cost of a ton of sulphate of ammonia? of 600 lb.?

12. If acid phosphate contains 14% phosphoric acid, what is the weight of phosphoric acid in a ton?

13. A certain fertilizer for tobacco land is made of 900 lb. cotton-seed meal @ $1.30 per 100 lb., 100 lb. nitrate of soda @ $50 per ton, 250 lb. sulphate of potash @ $2.50 per 100 lb., 750 lb. acid phosphate @ 60¢ per 100 lb. What is the cost of each ingredient?

WRITTEN EXERCISE

1. To lessen potato scab the seed potatoes are soaked in a solution of $\frac{1}{2}$ pt. of formalin to 15 gal. of water. What is the per cent of formalin in the mixture? What is the per cent of water?

2. Bordeaux mixture, for spraying, contains $1\frac{1}{4}\%$ by weight of copper sulphate, $1\frac{1}{4}\%$ of unslacked lime, and the rest water. How many pounds of each in a barrel containing 250 lb. of the mixture?

3. Half of a peach tree was sprayed with Bordeaux mixture and the other half was not. From the sprayed half 284.8 lb. of fruit were taken, and from the other half 95% less. How many pounds were taken from the tree?

4. If only the fiber (lint) is taken from the land, a cotton crop removes from the soil only 8.6% as much plant food as a wheat crop. The wheat crop takes 32.4 lb. a year from each acre. What is the weight of plant food that a cotton crop takes each year from a 50-acre field?

5. It is estimated that a chickadee will destroy 30 egg-laying cankerworm moths during each of the 25 days that these moths, which are wingless, crawl up orchard trees. This would result in the destruction of how many eggs by a single chickadee in an orchard, if each moth would have laid 185 eggs?

6. A farmer by placing a burlap band around an apple tree caught 96 larvæ of the coddling moth in a week. Suppose each of these had emerged from its cocoon as a moth, had deposited an egg on each of 294 apples, each egg hatching into an apple-tree worm and destroying the apple, how many apples would have been saved by destroying these 96 larvæ?

7. If bone meal is 4% nitrogen, and nitrate of soda is 15% nitrogen, how many pounds of nitrogen in a ton of each? in 1500 lb.?

8. In a certain fertilizer (plant food) for land on which corn is grown, $43\frac{3}{4}\%$ is acid phosphate, $47\frac{1}{2}\%$ is cotton-seed meal, and the rest is kainit. How many pounds of each to a ton?

9. In a certain fertilizer for land on which cotton is grown, $53\frac{3}{4}\%$ is acid phosphate and the rest is 32 parts fish scrap to 5 parts muriate of potash. How many pounds of each to a ton?

10. A farmer used the following fertilizer for an acre of grapes: 300 lb. acid phosphate @ $11.50 a ton, the same weight of bone meal @ $26.50 a ton, and the same weight of muriate of potash @ $42.50 a ton. What did the fertilizer cost?

11. A Virginia tobacco grower used for his land a fertilizer composed of 35% cotton-seed meal, 5% nitrate of soda, and the rest of sulphate of potash and acid phosphate in the ratio of 1 to 3. How many pounds of each in 750 lb.? in 1 T.? in $1\frac{1}{2}$ T.?

12. A farmer mixed a ton of fertilizer for his corn land, using half cotton-seed meal @ $24.60 a ton and the rest acid phosphate and muriate of potash in the ratio of 19 to 1. The acid phosphate cost $11.50 a ton, and the muriate of potash $42.50 a ton. What did the fertilizer cost?

13. A cotton planter mixed a ton of fertilizer for his land, using half acid phosphate @ $11.50 a ton and the rest cotton-seed meal and muriate of potash in the ratio of 37 to 3. The cotton-seed meal cost $24.60 a ton and the muriate of potash $42.50 a ton. What did the fertilizer cost?

14. A market gardener used the following fertilizer for 3 acres of vegetables: 400 lb. nitrate of soda @ \$48 a ton, 900 lb. cotton-seed meal @ \$24.60 a ton, 900 lb. acid phosphate @ \$11.70 a ton, 900 lb. bone meal @ \$26.10 a ton, and $33\frac{1}{3}\%$ less muriate of potash than bone meal @ \$2.15 per 100 lb. What did the fertilizer cost per acre?

15. A gardener used 700 lb. of fertilizer per acre, composed of 1 part nitrate of soda to 2 parts each of acid phosphate, bone meal, and muriate of potash. The nitrate of soda cost \$48 a ton, the acid phosphate \$11.50 a ton, the bone meal \$2.15 per 100 lb., and the muriate of potash \$42.50 a ton. What did the fertilizer cost for 5 acres?

16. Of an Iowa farm of 160 acres, 10% was swampy. Before this part was properly drained it yielded some hay, averaging in value \$3.25 to the acre. After being drained and tiled it yielded grain to the value of \$12 to the acre. What was the per cent of increase in productiveness of this part of the farm after it was drained?

17. By a proper selection of seed corn for three years, a farmer was finally able to grow 80 bu. of corn on one acre. On an adjoining acre with similar soil where seed was taken at random, the yield was $33\frac{3}{4}$ bu. If he had followed the first plan instead of the second for all of his 52 acres, what would have been the per cent of gain?

18. On a piece of average ground a farmer counted 22 heads of smutty wheat out of a total of 176 heads. If the yield was 16 bu. of good wheat to the acre, and the tract contained 42 acres, what was the farmer's loss in yield? By treating a certain area with formalin he killed all the smut spores, and by selecting his seed from these healthy plants he afterwards grew all of his wheat free from disease. What was now the yield per acre?

19. A cow gives 875 lb. of milk in a certain month. The milk tests 4.3% butter fat. How many pounds of butter fat does she produce in this month?

20. If the butter fat is sufficient for $\frac{1}{8}$ more than its weight of butter, how many pounds of butter could be made from the milk mentioned in Ex. 19?

21. If the butter mentioned in Ex. 20 is sold at 24¢ a pound, what is the value of the butter produced in this month from the milk of this cow?

22. A man has a dairy that produces milk averaging 3.75% butter fat during a certain month. The butter fat weighs $787\frac{1}{2}$ lb. How many pounds of milk are produced?

23. The weight of the solid matter in the milk of a certain dairy during a summer was $12\frac{1}{2}$% of the total weight, and the butter fat was 25% of the solids. The butter fat was what per cent of the weight of the milk?

24. The weight of the solid matter in the milk of a certain dairy during June was 13.2% of the total weight, and the butter fat was $\frac{1}{4}$ of the solids. How many pounds of butter fat to a ton of milk?

25. A farmer bought 24 cattle and sold them a year and a half later for $1200. He estimated his net gain at $10 a head, 20% of which was due to extra care in feeding. What would have been the average selling price per head if he had not taken this extra care?

26. A farmer has two cows, one supplying 1000 lb. of milk in a certain month, testing 3% butter fat, and the other 800 lb., testing 4% butter fat. If the butter fat is sufficient for $116\frac{2}{3}$% of its weight in butter, and butter is worth 23¢ a pound, which cow pays the farmer the more for that month, the feed costing the same? How much more?

27. The books of a creamery show the following record for seven of its patrons, A, B, C, D, E, F, G, for six days. The figures represent the pounds of cream delivered each day, and the per cent of butter fat in the cream, as shown by tests.

	A	B	C	D	E	F	G
Feb. 2.	27	21	41	18	24	59	78
" 3.	26	28	42	19	34	67	81
" 4.	23	30	45	16	38	72	90
" 5.	25	32	47	20	39	76	91
" 6.	28	35	50	21	38	72	89
" 7.	23	39	52	20	39	75	92
Per cent of butter fat	29%	34%	27%	30%	26%	35%	32%

How many pounds of cream were supplied by each patron? Of this amount how many pounds were butter fat? If this butter fat was sufficient for $116\frac{2}{3}\%$ of its weight in butter, and the patrons received 24¢ per pound of butter, what did each receive for the six days?

28. One day's report of a creamery operating five stations shows the pounds of milk and cream received, as follows:

	Sta. 1	*Sta. 2*	*Sta. 3*	*Sta. 4*	*Sta. 5*
Milk	1032	1152	864	720	1008
Cream	500	480	384	288	360

a. If $16\frac{2}{3}\%$ of the milk is cream, how many pounds of cream has each station?

b. If 25% of the cream is butter fat, how many pounds of butter fat has each?

c. If the butter fat is sufficient for $116\frac{2}{3}\%$ of its weight in butter, how many pounds of butter does each produce?

d. What is the butter of each worth at 23¢ a pound?

e. Show that the total number of pounds of butter is 819.

29. A certain creamery uses 6400 lb. of milk in a week. The skim milk amounts to 80% of the whole milk, and tests $\frac{3}{8}$% butter fat. How many pounds of butter fat are lost in the skim milk? If this would suffice for $16\frac{2}{3}$% more than its weight in butter, how much butter is lost?

30. A cow gives 850 lb. of milk in a certain month. After the butter fat is extracted the milk weighs 816 lb. What per cent of butter fat did the milk yield? How many pounds? How many pounds of butter could be made from it, the butter weighing $116\frac{2}{3}$% as much as the fat? What would this butter be worth at 25¢ a pound?

31. An Illinois farmer made a fertilizer for his corn-fields, using 95 parts acid phosphate, 100 parts cotton-seed meal, and 5 parts muriate of potash. He then decided to add enough acid phosphate to 100 lb. of the mixture to make it 50% of the total. How much did he add?

If he adds x lb., then 50% of $(100 + x)$ lb. $= 47\frac{1}{2}$ lb. $+ x$ lb. Hence $2\frac{1}{2} = \frac{1}{2}x$, and $x = 5$.

32. A Louisiana planter has some fertilizer for his cotton fields, consisting of $62\frac{1}{2}$% acid phosphate, 30% dried blood, $7\frac{1}{2}$% muriate of potash. He finds that his land needs more acid phosphate, and decides to add enough of it to the mixture so that it shall be 70% of the total. How many pounds of acid phosphate must he add to a ton of the fertilizer?

33. A North Carolina farmer finds that a fertilizer that he has been using is too rich in acid phosphate. The formula used is 25% dried blood, 5% nitrate of soda, 20% sulphate of potash, and the rest acid phosphate. How many pounds less of acid phosphate must he use to every 400 lb. of sulphate of potash, so that the acid phosphate shall be 45% of the mixture?

CHAPTER II

I. BUSINESS APPLICATIONS

GOING INTO BUSINESS

108. How a boy may go into business. If a boy on leaving school goes into some trade, he will start with very low wages. He may be employed to go on errands, to wrap up goods, or to deliver parcels. He will soon show whether he has any elements of success. If he shirks work, indulges in too much talk, watches the clock for closing time, and wastes what money he receives, he will either be "out of a job" or be left in the lowest kind of place.

But if he works his best, exerts himself to be helpful in every way, is always a gentleman, and takes an interest in improving his work, he will soon begin to work his way up. If he saves money, shows himself a good manager, earns a reputation for absolute truth, and has a circle of customers who know they can always rely upon him, he will probably succeed in business for himself. These are the qualities that make great merchants and manufacturers. The world is waiting for such men.

109. How a girl may go into business. The girl who has to support herself may begin as cash girl or clerk in a store and work up to the position of buyer in some department. If she is in a factory, a position as overseer is among the possibilities. If she is a stenographer, good positions are always awaiting capable women, often demanding considerable knowledge of the business with which they are connected. If she marries, some knowledge of business is equally as important, for many homes are made unhappy because of lack of this knowledge.

WRITTEN EXERCISE

1. If a man's salary was decreased $2\frac{1}{2}\%$ and was then $1170 a year, how much was it before the decrease?

2. If a man's salary was increased 5% and was then $1207.50 a year, how much was it before the increase?

3. If you work 312 days in a year, at $1.75 a day, and save $91, what per cent of your income do you spend?

4. An agent's bill for goods bought, plus his commission of 2%, is $1377. What did the goods cost?

5. If you worked at a salary of $1.25 a day, and spent 80% of your income for living expenses, and saved $75 in a year, how many days did you work?

6. If you should work 310 days in a year, and spend $82\frac{1}{2}\%$ of your income for living expenses, how much would you have to earn a day in order to save $108.50 a year?

7. A boy is offered a salary of $7.50 a week for the first year in a store, $33\frac{1}{3}\%$ more the second year, and 50% beyond that thereafter. What will be the total salary for 3 years, allowing 52 weeks to the year?

8. If a carpenter earns $3 a day and works 300 days in one year, what will be the alteration in annual income if his daily wages are increased 10% the following year and the working days are decreased 10% in number?

9. A boy who has been working this year at $25 a month is offered either an increase of 20% for next year or a salary of $7 a week. Which will bring the more income, and how much more per year? (Use 52 wk.)

10. A girl who has been working in a factory at $21.67 a month, is offered an increase of 10% where she is or a salary of $5.60 a week elsewhere. Which will bring the more income, and how much more per year? (Use 52 wk.)

11. A boy went to work at 90¢ a day. The second year his wages were increased 20%, the third year they were 42¢ a day more than the second, and the fourth they were increased $33\frac{1}{3}$%. At 300 working days to the year, what was his total income for each year?

12. A girl entering a trade school finds that graduates from the dressmaking department receive on an average $4.60 a week the first year; those from the millinery department, 5% less; those from the embroidery department, 5% more than the dressmakers; and those from the operating department, $66\frac{2}{3}$% as much as the last two together. Find the average wages of each, and tell which department the girl probably entered. (Use 52 wk.)

13. A girl leaving the public school finds she can enter a city shop at a salary of $3 a week the first year, with $16\frac{2}{3}$% more the second year, and a $14\frac{2}{7}$% increase the third year. Instead of this she enters a trade school for a year, tuition free. She then receives a salary of $5 a week the first year and 20% more the second year. Counting 50 working weeks a year, how much more does she receive in three years by the plan she follows after leaving the public school than she would have received without the trade-school training?

BANK ACCOUNTS

110. A bank account essential. One thing essential to any one who hopes to succeed is a bank account. Any one may "open an account," as it is called, as soon as he begins to save even a small amount. In most parts of the country small sums are usually deposited in savings banks.

111. The savings banks. To deposit money a person simply goes to the bank, says that he wishes to open an account, and leaves his money with the officer in charge. The officer gives him a book in which the amount is written, and the depositor writes his name in a book of signatures, for identification. When he wishes to draw out any money, he takes his book to the bank, states how much he wishes, signs a receipt (in some parts of the country he makes out a check), has the amount entered in his book, and the money is paid to him if it does not exceed his deposits, — his "balance," as they say. If he forms the habit of depositing money when he has any to spare, and never drawing except in an emergency, he will be surprised to see how it accumulates.

WRITTEN EXERCISE

1. What does 25 ct. saved a day amount to in a year?

2. How much will 15 ct. a day, 310 days a year, amount to in 10 years, not counting interest?

3. If a boy, beginning when he is 16, saves 25 ct. a day for 300 days a year, and deposits it in a savings bank, how much will he have when he is 21, not counting interest?

4. If a father gives his daughter $1 when she is 1 yr. old, $2 on her next birthday, and so on until she is 21, depositing it for her in a savings bank, how much will she have when she is 21, not counting interest?

5. A man saves on an average 25¢ a day the first year he works on a certain farm, and 120% as much the second year, depositing it in a savings bank. If he works 306 days each year, how much does he save in the two years?

6. A farmer's savings-bank deposits average $25 a month during a certain year, and 97% as much during the next year. The year following he deposited 110% as much as he did the second year. How much did he deposit during the three years?

7. If a man saved $1 a week during the year in which he was 21, and increased his savings each year by 10% on the amount of the preceding year, and worked 50 weeks a year, how much would he save during the year in which he was 29 years old?

8. A merchant saves $375 the first year he is in business. The second year he increases his savings one third. The third year they are only 85% as much as the second year. The fourth year they increase 30%. How much does he save in the four years?

9. A man works on a salary of $15 a week for 50 weeks in a year. His expenses are $15 a month for house rent, 60% as much for clothing, 300% as much for food as for clothing, and 20% as much for other necessary expenses as for food. How much of his salary can he reserve for the savings bank?

10. A clerk in a store had a salary of $12 a week two years ago, and a commission of 2% on the goods sold by him. That year he worked 48 weeks and sold $2350 worth of goods. Last year his salary was increased 25%, his commissions remaining the same. He worked 49 weeks and sold $2750 worth of goods. How much was his income increased?

112. How savings banks pay interest. Savings banks usually pay interest every six months. This is added to the principal, and the amount draws interest.

113. Compound interest. When the interest is added to the principal as it becomes due, and the amount draws interest, the owner is said to receive *compound interest.*

Compound interest is no longer allowed on notes. But if any one collects his interest when due, and at once reinvests it, of course he practically has the advantage of compound interest. The method of finding compound interest is substantially the same as that used in simple interest.

114. Illustrative problem. Required the amount of \$2000 invested in a savings bank at 4% annually, the interest compounded semiannually, for 2 years.

\$2000	= first principal
.02	
40	= first interest
2000	
2040	= amt. after 6 mo.
.02	
40.80	= int. second 6 mo.
2040	
2080.80	= amt. after 1 yr.
.02	
41.62	= int. third 6 mo.
2080.80	
2122.42	= amt. after $1\frac{1}{2}$ yr.
.02	
42.45	= int. fourth 6 mo.
2122.42	
\$2164.87	= amt. after 2 yr.

The simple interest for the same length of time is \$160, \$4.87 less than the compound interest.

ORAL EXERCISE

Find the amount at compound interest:

1. \$1000, 2 yr., 4%.
2. \$1000, 2 yr., 5%.

In Ex. 1 we have \$40 interest each year, plus 4% of \$40.

3. \$2000, 2 yr., 4%.
4. \$2000, 2 yr., 5%.
5. \$3000, 2 yr., 3%.
6. \$3000, 2 yr., 2%.
7. \$5000, 2 yr., 4%.
8. \$5000, 2 yr., 3%.
9. \$8000, 2 yr., 5%.
10. \$10,000, 2 yr., 4%.

WRITTEN EXERCISE

Find the amount at simple interest, and at interest compounded annually:

1. \$2500, 4 yr., 5%.
2. \$4000, 3 yr., 6%.
3. \$3500, 3 yr., 4%.
4. \$2250, 4 yr., 3%.
5. \$2750, 4 yr., 3%.
6. \$1750, 3 yr., 3½%.
7. \$625.50, 4 yr., 3½%.
8. \$10,000, 6 yr., 4%.
9. \$425.50, 4 yr., 4½%.
10. \$275.50, 3 yr., 4½%.

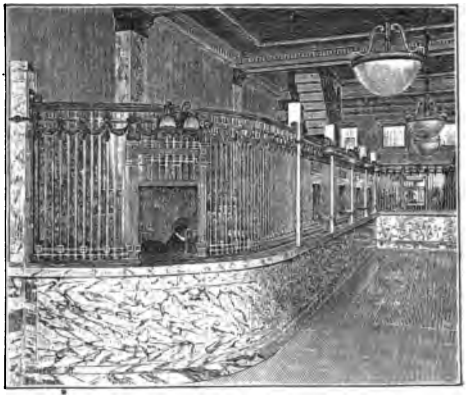
Find the amount of principal and interest, the interest being compounded semiannually:

11. \$200, 3 yr., 4%.
12. \$300, 2 yr., 4%.
13. \$650, 2 yr., 6%.
14. \$700, 3 yr., 3%.
15. \$800, 3 yr., 4%.
16. \$500, 2 yr., 4%.
17. \$1000, 2 yr., 4½%.
18. \$1000, 3 yr., 4%.
19. \$2000, 2 yr., 3½%.
20. \$3000, 2 yr., 2½%.

21. Which brings the more interest in 2 yr., on \$1250, 4% in a savings bank, compounded semiannually (the money being deposited at the beginning of the year), or 4½% simple interest?

22. If a man on January 1 deposited \$2000 in a savings bank, and left it for 2 yr., at 4%, the interest being compounded semiannually, how much less interest would he receive than by loaning it at 5% simple interest?

23. If a man deposits in a savings bank \$200 January 1, \$300 February 1, \$100 May 1, \$400 August 1, and \$350 November 1, and the rules of the bank allow 4% interest on all of these sums, compounding it on July 1 and January 1, how much will the man have to his credit on the January 1 following these deposits?



115. Banks of deposit. When a man has money enough ahead to pay his bills by checks, he will need to have an account with an ordinary bank, sometimes called a *bank of deposit.*

Such banks do not pay interest on small accounts, the deposit being a matter of convenience and safety. If a man wishes to open an account, he sometimes has to give references, for banks do not wish to do business with unreliable people.

116. Deposit slips. When a man deposits money he makes out a *deposit slip,* as here shown, and leaves it at the bank.

SECOND NATIONAL BANK

BOSTON, MASS.

Deposited for the account of

....

Date..19....

Bills and small coin . .		
Gold		
Silver		
Check on....................Bank		
" " .... "		
Total		

ORAL EXERCISE

Tell the sum of each of the following lists of deposits:

1.	2.	3.	4.
$15.50	$12.75	$23.80	$12.00
3.75	2.25	12.70	13.75

WRITTEN EXERCISE

Make out deposit slips for the following deposits, naming some bank in your town:

1. Bills, $254; silver, $40; checks on First National Bank, $87.50; Traders Bank, $127.50.

2. Bills, etc., $423.57; gold, $275; silver, $135.75; check on Cotton Exchange Bank, $342.60.

3. Bills, etc., $135.50; checks on Chemical National Bank of New York, $325; Third National, $63.73.

4. Bills, etc., $1726.45; gold, $125; silver, $100; checks on Traders Bank, $335.50; Second National, $175.40.

5. Bills, etc., $262.75; checks on Garfield National, $96.50, $200; Jefferson and Lee National, $325, $46.50.

6. Gold, $50; checks on First National, $27.62, $41.75, $32.80; Lincoln Trust Company, $37.42, $21.85.

7. Bills, etc., $146.73; silver, $275; checks on Miners National Bank, $43.50; Rocky Mountain National, $250.

8. Bills, $145; silver, $350; gold, $480; checks on Merchants National Bank, $255; Farmers Trust Company, $162.50; Second National Bank, $275.50.

9. A merchant deposited $175.80 in cash to-day, and a check for 25% of a debt of $176 due him, and a check in payment for 18¾ yd. of velvet at $2.16 a yard less 33⅓% on account of a bargain sale. Make out a deposit slip.

10. A jeweler received cash for the following: 3 doz. forks @ $16.50; 4½ doz. teaspoons @ $15; a watch @ $32.75; and 2 clocks @ $6.75. He also received checks on the First National Bank for the following: 2½ doz. table spoons @ $19.50; 3 doz. dessert spoons @ $17.75; and 9 nut crackers @ $9 a dozen. Make out a deposit slip.

117. Check books. A *check book* is given the depositor. Each page has one or more *checks* and *stubs*.

Stub	*Check*
No. 275	No. 275 CHICAGO, ILL., ...[*Date*].....19.....
Date........................	FIRST NATIONAL BANK OF CHICAGO
To J. H. Smith	Pay to the order of
For Bal. acct.	John H. Smith 15.50
Amt. $15.50	Fifteen and $\frac{50}{100}$ ~~~~~~~~~~ Dollars
	Robert J. Brown

118. Payee. The person to whom a check is payable is called the *payee.* A check is usually made payable to:

1. "Self," in which case the drawer alone can collect it.
2. The order of the payee, when the payee must indorse it.
3. The payee or bearer, when any one can collect it.
4. "Cash," in which case any one can collect it.

ORAL EXERCISE

Given the following deposits and checks, tell the balance:

	Deposits	*Checks*		*Deposits*	*Checks*		*Deposits*	*Checks*
1.	$25	$15	2.	$37	$19	3.	$48	$20
	70	27		41	32		27	15
	32	42		20	15		13	17
4.	$62	$37	5.	$75	$62	6.	$50	$65
	48	41		25	17		40	30
	17	20		32	40		75	50

WRITTEN EXERCISE

1. If your deposits in a bank have been $14.75, $32, $15, $25, $50, $75, $1.75, and you have drawn checks for $9.50, $18.75, $30, what is your balance?

2. A man earning $15 a week deposits $12 every Saturday, and on each Monday gives a check for $6 for his board. What will be his balance in 6 months (26 weeks)?

3. If your deposits in a bank have been $32.75, $49.63, $28, $14.90, $25, $10, $30, $42.75, $71.25, and you have drawn checks for $5.75, $13.50, $32, $4.50, $36.50, $52.80, what is your balance?

4. A farmer having $275.60 in the bank deposits during the next six months $23.50, $17.42, $75, $32.40, and the proceeds of the sale of 7 cows @ $37.50 less 5%. He gives checks for $17.92, $8.50, $72.80, $9.50, $6.75, $13.90, and $26.50. What is his balance?

5. A merchant's deposits this week have been $127.42, $135, $72.50, $265.75, $327.40, $182.50, and he has drawn checks for $236.50, $15.75, $27.90, $241.60, and $29.50. He has also paid by check a bill for $176.70 less 10%, and another for $75.25 less 10%, 3%. At the beginning of the week he had $475.80 in the bank. What is his balance at the end of the week?

6. A merchant's deposits this week have been $261.50, $392.75, $62.40, $112.30, $98.76, $115, and he has drawn checks for $42.80, $174.95, $162.30, $81.50, $15, $27.40, and $37.50. He has also given a check for $125 plus 6 months' interest at 5%, and one in payment of a bill for $248.50 less 8%, 5%. At the beginning of the week he had $692.80 in the bank. What is his balance at the end of the week?

119. Borrowing money. If the time comes when a man wishes to start in business, and he has saved some money and has an unquestionable reputation for honesty, he will be able to borrow more money for buying a business or purchasing goods. Suppose he needs $1000. A bank will be found that has heard enough about his standing to lend him the money if he has a good *indorser.*

120. Indorser. One who writes his name across the back of a note is called an *indorser.* The indorser is responsible for the note if the maker does not pay it.

If Richard Roberts will indorse for John Brown, Mr. Brown makes out a note like this:

BOSTON, MASS., *May 15*, 19*06*

Two months after date, *without grace, I* promise to pay to the order of *Richard Roberts* $*1000.00*

One thousand Dollars

at **The First National Bank, Boston, Mass.**

Value received

Residence *7030 Beacon St.* *John Brown*

Due *July 15*

It is not a very direct way of borrowing from the bank, but it is a common one. The original idea was that Mr. Brown borrowed of Mr. Roberts, and Mr. Roberts sold the note to the bank, thus getting the money to lend Mr. Brown. But now it means simply that Mr. Roberts agrees to pay if Mr. Brown does not. Banks often take notes payable directly to the cashier.

Some states allow three "days of grace," that is, two months means two months + three days; but the custom is rapidly going out of use. Teachers should use them where it is the custom.

121. Bank discount. The note described on page 149 does not mention interest. This is because the borrower, Mr. Brown, pays the interest to the bank in advance.

Interest paid on a note in advance is called *bank discount.* The note is then said to be *discounted.*

122. Illustrative problem. What is the discount on the $1000 note on page 149 at 6% a year?

The discount on $1000 for 2 mo. @ 6% a year is $10.

The rate of discount is understood to be by the year, like the rate of interest, unless the contrary is stated.

123. Proceeds. The balance after deducting the discount from the face of a note is called the *proceeds.*

Thus, the proceeds on the above note are $1000 − $10 = $990.

124. Usual time for bank notes. Notes are usually made for 30, 60, or 90 days, and are then renewed if desired.

In states where days of grace are allowed, teachers should see that they are included.

ORAL EXERCISE

State the discount on the following:

1. $50, 2 mo., 6%.
2. $75, 2 mo., 6%.
3. $125, 2 mo., 6%.
4. $650, 2 mo., 6%.
5. $100, 1 mo., 6%.
6. $250, 1 mo., 6%.
7. $400, 3 mo., 6%.
8. $600, 3 mo., 6%.
9. $600, 2 mo., 5%.
10. $1200, 3 mo., 5%.

State the proceeds on the following notes, discounted as specified:

11. $100, 2 mo., 6%.
12. $300, 2 mo., 6%.
13. $600, 1 mo., 5%.
14. $600, 3 mo., 5%.
15. $1000, 3 mo., 6%.
16. $1000, 3 mo., 5%.

WRITTEN EXERCISE

Find the discount and proceeds on the following:

1. $875, 30 days, 6%.
2. $975, 90 days, 5%.
3. $425, 30 days, 6%.
4. $2568, 60 days, 5%.
5. $1250, 60 days, 6%.
6. $1500, 90 days, 5%.
7. $2565, 90 days, 5%.
8. $3250, 90 days, 5½%.
9. $4250, 30 days, 4½%.
10. $325.50, 30 days, 6%.
11. $427.50, 60 days, 5%.
12. $237.50, 30 days, 5%.

13. Make out a 60-day note for $750, dated to-day, payable to John Doe's order at some bank in your town or city. Discount it at 5%.

14. Make out a 30-day note for $675, dated to-day, payable to Richard Roe's order at some bank of which you know. Discount it at 6%.

15. Make out a 90-day note for $1250, dated to-day, payable to James Jameson's order at some bank of which you know. Discount it at 6%.

16. A man's bank account shows deposits, $37.50, $75, $82.50, $100, $50, $300, $40, $125, $500; drawn by checks, $13.75, $62, $5, $125.50, $82.75. He needs $2000 to start him in business, and wishes to keep about $500 in the bank. How much must he borrow, to the nearest $100?

17. If he makes out a note for this amount for 90 days at 6%, how much discount must he pay? What are the proceeds? What are they for 60 days?

18. A merchant has to pay a debt of $1275 less 10%. He has in the bank $672.80, and has $127.50 in cash in his safe. He wishes to leave about $300 in the bank and about $65 in his safe. How much must he borrow, to the nearest $50? Discount the note for 30 days at 6%.

125. Discounting notes. If a dealer buys some goods for the fall trade, but does not wish to pay for them until after the holidays, he may buy them on credit, giving his note. The manufacturer may need the money at once, in which case he will indorse the note and sell it to a bank for the sum less the discount.

126. Illustrative problem. If you give a manufacturer your note dated September 1, due on January 1, for $500, with interest at 5%, and he, needing the money, discounts it at the bank on September 16 at 6%, what are the proceeds?

Face of the note	= $500.
Interest 4 mo., 5%	= 8.33
Amount at maturity	= $508.33
Discount 107 da., 6%	= 9.07
Proceeds,	$499.26

Banks usually compute the discount period in days. In some parts of the country both the first and last days are included in the discount period, thus making it 108 days in the above example. Teachers should be guided by the local custom.

ORAL EXERCISE

State the following:

1. 5% of $70.
2. 5% of $75.
3. 5% of $65.
4. 4% of $55.
5. 4% of $35.
6. 4% of $95.
7. 6% of $75.
8. 6% of $85.
9. 6% of $65.
10. 3% of $125.
11. 3% of $225.
12. 3% of $365.
13. 2% of $42.50.
14. 2% of $63.50.
15. 2% of $27.50.
16. 4% of $20.25.
17. 4% of $15.25.
18. 4% of $41.25.
19. 5% of $15.20.
20. 5% of $20.40.
21. 5% of $30.60.
22. 6% of $25.50.

WRITTEN EXERCISE

Find the discount and proceeds:

	Face	*Date*	*Due*	*Interest*	*Date of Discount*	*Rate of Discount*
1.	$300	July 1	July 31	4%	July 16	6%.
2.	$800	Sept. 1	Nov. 1	5%	Sept. 11	6%.
3.	$550	Oct. 1	Jan. 1	4%	Dec. 2	6%.
4.	$325	July 1	Dec. 1	6%	Nov. 1	5%.
5.	$1000	Nov. 1	Dec. 1	6%	Nov. 1	6%.
6.	$1250	Aug. 15	Jan. 1	5%	Sept. 1	5%.

Exs. 7–32 draw no interest.

7. $3750, 60 days, 5%.
8. $4225, 60 days, 5%.
9. $9500, 63 days, 6%.
10. $7500, 63 days, 5%.
11. $4225, 93 days, 5%.
12. $5575, 93 days, 6%.
13. $7500, 33 days, 5%.
14. $4750, 33 days, 6%.
15. $42.50, 30 days, 6%.
16. $27.60, 30 days, 6%.
17. $35.50, 30 days, 6%.
18. $28.75, 30 days, 6%.
19. $125.50, 60 days, 6%.
20. $275.25, 60 days, 6%.
21. $375.50, 90 days, 6%.
22. $450.75, 90 days, 6%.
23. $457.50, 30 days, 5%.
24. $296.50, 30 days, 5%.
25. $287.60, 90 days, 5%.
26. $375.40, 90 days, 5%.
27. $2750, 93 days, 4½%.
28. $3275, 93 days, 4½%.
29. $4760, 63 days, 5½%.
30. $2745, 63 days, 5½%.
31. $1545, 33 days, 4½%.
32. $1575, 33 days, 3½%.

33. The discount on a note for 90 days at 6% is $18.75. What is the face of the note that is discounted?

34. A dealer buys $1750 worth of goods, giving his note on October 4, for 90 days, at 5%. On October 10 the holder of the note discounts it at 6%. Find the proceeds.

PARTIAL PAYMENTS

127. Partial payments. If a dealer holds a note against one of his customers, and it bears interest, and partial payments are made from time to time, the amount due on the day of settlement is found in the following way:

1. *Add the interest to the principal whenever the payment (or sum of the payments) equals or exceeds the interest.*

2. *Then deduct the payment (or payments) and continue as before.*

This is the *United States Rule of Partial Payments*, the legal one in most states. Where other rules are legal, teachers should explain the law and require the problems solved accordingly.

128. Illustrative problem. A note for $1000, at 5%, is dated January 1, 1906. The following payments are indorsed (written across the back, as is the custom): July 1, 1906, $10; January 1, 1907, $40; July 1, 1907, $20; January 1, 1908, $130. How much is due July 1, 1908?

The interest on July 1, 1906, is $25, and the payment is $10. Hence the payment cannot be deducted. The reason is easily seen, for if we should take $1025 – $10 = $1015 as the new principal, we should be drawing interest on more than the $1000.

First principal,	$1000
Int. for 1 yr.,	50
First amount, January 1, 1907,	$1050
1st and 2d payments,	50
Second principal, January 1, 1907,	$1000
Int. for 1 yr.,	50
Second amount, January 1, 1908,	$1050
3d and 4th payments,	150
Third principal,	$900
Int. for 6 mo.,	22.50
Due July 1, 1908,	$922.50

WRITTEN EXERCISE

1. A note for $375, at 5%, has a payment of $18.75 indorsed annually at the close of each year from its date, for 3 years. How much is due at the end of the fourth year?

2. A note for $500, at 6%, is dated January 1, 1906, and has the following partial payments indorsed: July 2, 1906, $100; January 1, 1907, $100. How much is due July 1, 1907?

3. A note for $300, at 6%, is dated April 1, 1906, and has the following partial payments indorsed: September 1, 1906, $50; February 16, 1907, $75; January 1, 1908, $100. How much is due July 1, 1908?

4. A note for $1800, at 5%, is dated April 16, 1906, and has the following partial payments indorsed: July 2, 1906, $500; October 1, 1906, $250; February 1, 1907, $100; May 1, 1907, $375. How much is due September 5, 1907?

5. A note for $750, at 4%, is dated June 15, 1906, and has the following partial payments indorsed: March 6, 1907, $200; September 6, 1907, $150; January 17, 1908, $250. How much is due on the day of settlement, May 14, 1908?

6. A note for $600, at 5%, is dated July 2, 1906, and has the following partial payments indorsed: January 2, 1907, $50; February 11, 1907, $75; July 8, 1907, $200; September 10, 1907, $80; November 2, 1907, $50. How much is due January 2, 1908?

7. A note for $1200, at 4%, is dated May 1, 1906, and has the following partial payments indorsed: July 2, 1906, by labor, $5; July 9, 1906, by labor, $2; September 1, 1906, $93; May 1, 1907, $450; November 1, 1907, $500. How much is due January 1, 1908?

8. A note for $750, at 6%, is dated February 2, 1907, and has the following partial payments indorsed: June 2, 1907, $150; July 17, 1907, $75; February 2, 1908, $100. How much is due August 2, 1908?

9. A note for $575, at 6%, is dated January 15, 1906, and has the following partial payments indorsed: May 7, 1906, $30; August 13, 1906, $50; December 18, 1906, $75. How much is due January 30, 1907?

10. A note for $625, at 5%, is dated November 13, 1906, and has the following partial payments indorsed: February 14, 1907, $35; April 17, 1907, $75.50; October 23, 1907, $50; March 19, 1908, $135. How much is due September 23, 1908?

11. A note for $175, at 6%, is dated December 9, 1907, and has the following partial payments indorsed: February 4, 1908, $25; May 27, 1908, $63.50; September 2, 1908, $15; February 25, 1909, $20. How much is due April 22, 1909?

12. A note for $675.50, at 5%, is dated August 5, 1907, and has the following partial payments indorsed: January 7, 1908, $50; August 5, 1908, $50; March 18, 1909, $35; July 15, 1909, $75; November 11, 1909, $130. How much is due January 7, 1910?

13. A man owes another $750 for a village building lot, and he gives his note at 6%, due on demand, with the understanding that he should pay part of it by labor. The note is dated April 15, 1908, and the following partial payments are indorsed: July 22, 1908, 4 days' labor @ $3; September 16, 1908, $3\frac{1}{2}$ days' labor @ $3; January 21, 1909, $4\frac{1}{2}$ hours' labor @ 50¢; March 11, 1909, $11\frac{1}{2}$ days' labor @ $3; July 15, 1909, 37 days' labor @ $3; September 16, 1909, cash, $350. How much is due January 1, 1910?

129. Merchants' rule. Where a note or account runs a year or less, and partial payments have been made, business men often compute the balance due by means of a rule sometimes called *The Merchants' Rule*, as follows:

1. *Find the amount of the principal at the time of settlement.*

2. *Find the amount of each payment from the time it was made until the time of settlement.*

3. *From the amount of the principal subtract the amount of the payments.*

The rule, not being as fair for longer periods as the United States Rule, is not legal. But for short periods it is easier and gives nearly the same result.

130. Illustrative problem. A note for $100 made January 1 has the following payments indorsed: March 1, $10; April 1, $25. Find the balance July 1, allowing 6% interest.

Amount of $100 for 6 mo.		$103
" " $10 " 4 "	$10.20	
" " $25 " 3 "	25.38	
		35.58
Balance,		$67.42

WRITTEN EXERCISE

Notes for the following amounts have indorsed the payments indicated. Find the balance at the date of settlement, using the Merchants' rule, and allowing 6% interest:

1. $875, April 1. Indorsements: $300, May 1; $125, June 15; $275, July 25. Settled November 20.

2. $250, February 1. Indorsements: $30, March 1; $70, May 15; $50, July 10. Settled September 15.

3. $750, March 1. Indorsements: May 10, $100; July 12, $75; September 4, $40. Settled October 15.

TRADE DISCOUNT

131. Advantage of trade discount. Dealers wish to buy their goods at as great a discount as possible. When they pay cash for them they get the best price, because a special discount is usually allowed for cash.

132. Several discounts. We have already met cases in this series of arithmetics in which more than one discount was allowed. Sometimes more than two are allowed, new discount lists being sent to retail dealers as the cost of production changes.

ORAL EXERCISE

1. What is the cost of goods listed at \$300, 5% off?
2. What is the cost of goods listed at \$600, at $\frac{1}{3}$ off?
3. What is the cost of goods listed at \$250, 10% off?
4. If on a bill of goods amounting to \$250 a discount of 20%, 10% is allowed, what is the net price?

$\$250 - \$50 = \$200$; $\$200 - \$20 = \$180$.

State the cost of goods listed as follows, less the discount:

5. \$250, 10%, 4%.
6. \$200, 5%, 1%.
7. \$300, 10%, 1%.
8. \$250, 20%, 4%.
9. \$100, 10%, 10%.
10. \$500, 20%, 6%.
11. \$200, 15%, 10%.
12. \$400, 25%, 10%.
13. \$1250, 20%, 3%.
14. \$660, $33\frac{1}{3}$%, 5%.
15. \$400, $12\frac{1}{2}$%, 2%.
16. \$1000, 15%, 2%.
17. \$300, $33\frac{1}{3}$%, 10%.
18. \$600, $33\frac{1}{3}$%, 10%.
19. \$600, $16\frac{2}{3}$%, 10%.
20. \$500, 20%, $12\frac{1}{2}$%.
21. \$1000, 10%, 10%.
22. \$540, $16\frac{2}{3}$%, 10%.

133. Sample price list. The following is a price list of certain school supplies, with the discounts allowed to schools and dealers where the prices are not net:

Rulers,	$0.35	per	doz., net		
Composition books,	4.50	"	gross,	less	5%
Thumb tacks,	0.40	"	100,	"	40%
Drawing pencils,	4.71	"	gross,	"	20%
Drawing paper 9 × 12,	1.10	"	package,	less	$\frac{1}{10}$
Pens,	0.61	"	gross,	less	$\frac{1}{4}$, 10%
Tubes of paste,	4.05	"	"	"	10%, 5%
Penholders,	3.20	"	"	"	12%, 5%
Drawing compasses,	1.65	"	doz.,	"	10%, 5%

WRITTEN EXERCISE

1. How much must your school pay for $\frac{1}{2}$ gross of penholders? for $\frac{1}{4}$ gross? for a gross?

2. The school wishes to buy 5 packages of drawing paper and 300 thumb tacks. What will they cost?

3. What will 8 gross of pens and $\frac{1}{2}$ gross of composition books cost? 2 gross of pens and 1 doz. rulers?

4. There are 40 pupils in a class, and each needs drawing compasses and a ruler. What will they cost the school? Suppose there were 60 pupils?

5. If a dealer sells pens at a cent apiece, what does he gain per gross? If he sells penholders at 2 ct. each, what does he gain per gross?

6. A dealer buys a gross of rulers and a gross of drawing pencils. He sells both at 5 ct. each. What does he gain on the lot? Suppose he buys 2 gross of each?

7. If a dealer buys a gross of tubes of paste for mounting pictures, and sells the tubes at 5 ct. each, what does he gain on the purchase? How much on 3 gross?

134. Ordering goods. The following is a model order:

WOOD & ROBERTS
BOOKSELLERS

MEMPHIS, TENN., Feb. 15, 19 07

Messrs. Ginn & Company, Publishers,
378 Wabash Ave., Chicago, Ill.

Dear Sirs: Please send at once, by express,
75 Smith's Practical Arithmetics,
60 " Primary Arithmetics.

Yours truly,
WOOD & ROBERTS

135. Model bill. The following is the bill that Ginn & Company would send in reply:

Messrs. Wood & Roberts,
Memphis, Tenn. Feb. 16, 19 07

Bought of GINN & COMPANY
Educational Publishers
378 WABASH AVE., CHICAGO

Terms of this Invoice: *Net Cash*

75 Smith's Practical Arithmetics,	$.65	$48.75	
60 " Primary Arithmetics,	.35	21.00	
		$69.75	
	1/6	11.62	
			$58.13

It is customary to state per cents like $16\frac{2}{3}\%$, $12\frac{1}{2}\%$, 20%, and 25% in the common-fraction form, as above.

WRITTEN EXERCISE

Write both orders and bills for the following goods bought of yourself, by the person named or by some one you know:

1. Bought 2 doz. tennis rackets @ $21; 8 doz. balls @ $3; $\frac{3}{4}$ doz. tennis nets @ $19.20. Discount 12%, 5%, 2%.

2. Henry James, Des Moines, Iowa: $\frac{1}{2}$ doz. rugs @ $84; 750 yd. carpet @ 82¢; 3 doz. hassocks @ $6. Discount $\frac{1}{8}$.

3. Bought 12 doz. tumblers @ $1.05; 35 doz. dinner plates @ $2.50; $1\frac{1}{2}$ doz. sugar bowls @ $9.60. Discount $\frac{1}{8}$.

4. Jones & Co., 3497 Wabash Ave., Chicago: 2 bbl. P.R. molasses @ $12; 200 lb. Rio tapioca @ 6 ct.; 150 lb. Mocha coffee @ 20 ct. Discount 6%.

5. Bought 5 doz. bottles of ink @ $3; 300 lb. paper @ 30 ct.; 2 doz. bottles of mucilage @ $4; 6 gross pencils @ $2.75. Discount 3%, 2%, 1%.

6. George Lloyd, Lincoln, Neb.: 18 brass bedsteads, No. 142, @ $17.25; 15 woven wire mattresses, No. 16, @ $3.45; 3 sideboards, No. 196, @ $17.25. Discount $\frac{1}{8}$.

7. Newton & Co., 2831 Spring St., New York: 21 Ohio steers, 1226 lb. (average), @ $5.10 (per 100 lb.); 20 do. (ditto = the same), 1247 lb., @ $5.10; 20 do., 1112 lb., @ $4.95. Discount $\frac{1}{10}$.

8. Sherman & Culver, New York: 20 Illinois steers, 1157 lb. (see Ex. 7), @ $4.60; 28 do., 996 lb., @ 4.12\frac{1}{2}$; 20 Kentucky steers, 1230 lb., @ $4.80; 7 Indiana do., 1078 lb., @ $4.70. Discount $\frac{1}{10}$.

9. R. B. Homer, Buffalo: 75 180-lb. bags of Western potatoes @ $1.48; 50 150-lb. do. @ $1.28; 125 168-lb. do. @ $1.42; 1620 lb. Jersey potatoes in bulk @ $1.47 per 180 lb. Discount $\frac{1}{8}$.

SIMPLE ACCOUNTS

136. The debit column. In his account a man charges his income to himself, and places it in the left column. He is debtor (*Dr.*) to himself for this column.

137. The credit column. His expenditures he credits to his account, and places them in the column marked *Cr.*

19.....		Dr.		Cr.	
Jan. 7	Cash on hand	7	45		
" 7	Carfare .10, pencil .05				15
" 8	Drawing book .15, ink .10				25
" 9	Geography			1	12
" 12	Paper .10, ruler .05				15
" 14	Balance			5	78
		7	45	7	45

WRITTEN EXERCISE

1. Make out the following account for a day. Cash on hand, $174.30; Receipts, mdse., $12.50, $6.75, $0.42, $17.30, $9.50, $42.75; Expenses, Perry & Co. bill, $75.82.

2. Make out the following account for a week. Cash on hand, $21.30; Receipts, from apples sold, $7.50; Eggs, $3.25; Poultry, $6.90. Disbursements, Horseshoeing, $0.75; Repairs to wagon, $1.20; Shingles, $4.20; Seed, $3.50.

3. Make out the following account for a week. Cash on hand, $1.75; Receipts, weekly allowance, $1. Expenses, Writing paper, 10 ct.; Pencils, 8 ct.; Ink, 10 ct.; Compasses, 10 ct.; Arithmetic, 90 ct.; Penholder, 3 ct.; Eraser, 5 ct.

Pupils should be required to make out imaginary accounts.

Make out the following accounts for last week, inserting the proper year, month, and day of the month:

4. Dr., Monday, Cash on hand, $1.05; Allowance for house expenses, $6; Thursday, Sewing for Mrs. Graham, $1. Cr., Tuesday, Groceries, $1.35; Meat, 85¢; Wednesday, Meat, $1.08; Thursday, Groceries, $2.30; Saturday, Meat, $1.12.

5. Dr., Monday, Cash on hand, $17.50; Eggs sold, $4.20; Tuesday, Corn sold, $26.50; Poultry sold, $6.75; Thursday, 3 T. 750 lb. hay sold @ $8. Cr., Tuesday, 4 days' wages, John Cobb, @ $2.50; Wednesday, Grocery bill, $7.28; Friday, Repairs, $1.60.

6. Dr., Monday, Cash on hand, $26.70; Saturday, Wages, $15. Cr., Monday, Allowance for house, $7.50; Carfare, 10¢; Tuesday, Carfare, 10¢; Wednesday, Rent, $20; Carfare, 10¢; Thursday, Carfare, 10¢; Friday, Shoes, $3; Saturday, Suit for Rob, $6.40.

7. Dr., Monday, Cash on hand, $9.75; Allowance for house expenses, $10; Thursday, Sale of old suit, $1.25. Cr., Tuesday, Meat bill, 95¢; Carfare, 15¢; Grocer, $1.68; Wednesday, Meat bill, $1.30; Grocer, $2.15; Thursday, Laundry, 60¢; Gas bill, $1.80; Friday, Ice, $1.60; Grocer, $3.60; Meat bill, $1.42; Broom and ironing board, $1.10.

8. Dr., Monday, Cash on hand, $278.50; Sales, $72.80; Tuesday, Sales, $98.75; Wednesday, Sales, $126.40; Sale of old show case, $6.50; Thursday, Sales, $82.75; Friday, Sales, $110.62; Saturday, Sales, $96.30. Cr., Monday, Gorham bill, $168.40 less 10%; Tuesday, Whiting bill, $68.90 net; Barton bill, $137.50 less 6%; Saturday, Frank's wages, $12.50; Morgan's wages, $15; Hulbert's wages, $14.

PARTNERSHIP

138. Partitive proportion. If partners invest equal sums and contribute equally in work, the profits are divided equally. Otherwise partitive proportion (page 119) is employed.

139. Partners usually have an annual settlement. Money withdrawn by any partner during the year is charged to him with interest at the settlement.

140. Illustrative problem. Brown, Edgcomb, and Thomas form a partnership on February 1, Brown putting in \$2500, Edgcomb \$4000, and Thomas \$3500. What is the share of each in \$6000 profits on the following February 1?

1. Since the total capital is \$10,000, and Brown put in \$2500,

   Brown's share is $\frac{2500}{10000}$, or $\frac{5}{20}$,
   Edgcomb's " " $\frac{4000}{10000}$, " $\frac{8}{20}$,
   Thomas's " " $\frac{3500}{10000}$, " $\frac{7}{20}$.

2. Therefore Brown receives $\frac{5}{20}$ of \$6000, or \$1500

Brown receives	$\frac{5}{20}$	of \$6000,	or	\$1500
Edgcomb "	$\frac{8}{20}$	" "	"	\$2400
Thomas "	$\frac{7}{20}$	" "	"	\$2100
The total being	1			\$6000

ORAL EXERCISE

Separate into two parts having the given ratio:

1. \$150, 1 : 2.
2. \$500, 2 : 3.
3. \$400, 3 : 1.
4. \$500, 1 : 3.
5. \$250, 2 : 3.
6. \$350, 3 : 4.
7. \$180, 4 : 5.
8. \$330, 4 : 7.
9. \$560, 3 : 5.
10. \$660, 8 : 3.
11. 140 ft., 5 : 9.
12. 132 yd., 7 : 5.
13. 600 ft., 8 : 7.
14. 450 yd., 7 : 8.
15. 420 rd., 10 : 11.
16. 260 mi., 11 : 15.

WRITTEN EXERCISE

1. X, Y, and Z invest $345, $625, and $730, respectively. They make $153. What is the share of each?

2. A, B, and C invest $2200, $3350, and $1650, respectively. They make $1440. What is the share of each?

3. Roberts, Jacobs, and Jameson invest $2700, $3250, and $2050, respectively. They make $2400. What is the share of each?

4. A, B, and C pay $195 irrigation taxes for their farms. A has 250 acres, B 180 acres, C 220 acres. What is the share of each?

5. X, Y, and Z pay $1085 for some water power. X uses 30 horse power, Y 45 horse power, Z 80 horse power. What should each pay?

6. Three men rent a summer cottage. The first occupies it 5 weeks, the second 4 weeks, and the third 3 weeks. At $264 for the season, what should each pay?

7. Day and McFarlane pasture some cattle in a field, Day putting in 62 head for 7 weeks, and McFarlane 48 head for 5 weeks. The bill for pasturage being $101.10, how much should each pay?

8. Two contractors for rock excavating in a tunnel run 16 and 21 drills respectively, getting compressed air from the same engine. If the expense of furnishing the compressed air is $555, what is the share of each?

9. Two farmers whose lands join have a windmill and tank in common. They pay for the annual repairs according to the greatest number of head of cattle kept by each at any time during the year. If they have 37 and 48 head respectively, and the repairs amount to $25.50, what is the share of each?

10. Messrs. Brown and Jones form a partnership, Brown furnishing $4200 and Jones furnishing $3300. After a year Brown puts in $500 more. At the end of 2 years they sell out for $9300. How much should each receive?

11. Ayres and Ives go into partnership, Ayres putting in $3500 and Ives $6500. Ives is to give all of his time to the business and take out $2000 before any division of profits. If they make $2500 this year, to how much of it is each entitled?

12. Glover and Staughton go into partnership, Staughton putting in $7000 and Glover $9000. Glover doing no work, it is agreed that Staughton shall take $2000 a year from the profits before a division. The profits last year were $6400. What was the share of each?

13. Webb, Bull, and Smith form a partnership, Webb putting in $8000, Bull $3000, and Smith $5000. It is agreed that Bull shall contribute no work, but that Webb shall receive $1500 and Smith $900 before the rest of the profits are divided. The year's profits were $7200. How much did each receive?

14. Messrs. Davids, Glover, and James buy a house for $9500. Davids puts in $3200, and Glover and James put in the rest, Glover putting in half as much as James. They rent the house this year for 10% of its cost, and pay $15 for insurance, $40 for taxes, and $40 for repairs. What is the share of each in the net receipts?

15. Wood, Wallace, and Harris own a spring, and pay for repairs to the spring house and piping according to the amount of water used by each, as shown by their water meters. If the respective amounts used during a given period are 34,000 gal., 26,500 gal., and 42,500 gal., what is the share of each in repairs amounting to $24.72?

EXCHANGE

141. Paying bills at a distance. If a man owes money to some one living in another place, he may send it by a registered letter. He will be more likely, however, to pay his bill by a check, a money order, or a draft.

142. Exchange. The payment of money by means of checks, money orders, or drafts is called *exchange.*

143. Paying by check. If a man sends his check for the amount, the one to whom he owes the money will deposit it in the bank where he keeps his account. This bank will send it to the debtor's bank for collection, and will probably charge a small amount for the trouble.

144. Paying by money order. A money order may be purchased at the post office, or from an express company, or it may, at considerable expense, be telegraphed. Postal or express orders may be sent to the creditor, who can then obtain the money at his post or express office. The extra cost of postal money orders is as follows:

For sums not exceeding \$2.50, 3 cents; above this, not exceeding, \$5, 5 cents; above this, not exceeding \$10, 8 cents; above this, not exceeding \$20, 10 cents; above this, not exceeding \$30, 12 cents; above this, not exceeding \$40, 15 cents; above this, not exceeding \$50, 18 cents; above this, not exceeding \$60, 20 cents; above this, not exceeding \$75, 25 cents; above this, not exceeding \$100, 30 cents.

ORAL EXERCISE

Referring to the above list, state the cost of money orders for:

1. \$31.50.	2. \$52.75.	3. \$92.30.	4. \$16.30.
5. \$19.90.	6. \$86.50.	7. \$69.95.	8. \$90.50.

145. Paying by bank draft. One of the most common methods of paying a debt in another place, particularly debts of large size, is by means of the bank draft.

No. *48104*

MERCHANTS NATIONAL BANK OF AUSTIN

AUSTIN, TEXAS, *July 6,* 19*07*

Pay to the order of..........*John Roberts*........................\$*78.*^{75}

Seventy-eight and $\frac{75}{100}$ ~~~~~~~~~~~~~~~~~~~~~ Dollars

To The Chemical National Bank,
New York City

A. B. Smith
Cashier

A draft is therefore the same as a check, except that it is made by the cashier of some bank and is drawn on another bank.

Banks usually charge a slight premium on the face of the draft. Thus, a \$250 draft at 0.1% premium would cost \$250 + 0.1% of \$250 = \$250.25.

If John Roberts, who purchased the above draft, owed Robert Jones the money, he would indorse it, thus:

Pay to the order of Robert Jones

John Roberts

It might also have been made payable directly to the order of Robert Jones in the first place, but this is not the custom.

Drafts on large money centers are usually cashed for customers of the bank without any discount.

146. Illustrative problem. What would the above draft cost at 0.1% premium?

0.1% of \$78.75 = \$0.08, to the nearest cent.

The bank, however, would probably charge 10¢ to make a convenient amount. Therefore it would cost \$78.85.

WRITTEN EXERCISE

1. What will a draft for \$300 cost at $\frac{1}{8}\%$ premium?

2. What will a draft for \$3200 cost at 0.1% premium? for \$2500 at $\frac{1}{20}\%$ premium?

3. What will a postal money order for \$37.50 cost? for \$62.75? for \$14.30? for \$86.50? for \$75.40?

4. What will a New Orleans merchant pay for a draft on Chicago for \$2750 at 40¢ premium per \$1000?

5. When the government charges 30¢ per \$100 for a money order, what per cent premium does it charge?

6. A draft cost a merchant \$2752.75, including 0.1% premium. What was the face of the draft? What was the premium? Write the draft.

7. A draft cost a merchant \$3751.50, including the premium of 40¢ per \$1000. What was the face of the draft? What was the premium?

8. If a man owes to different jobbers from whom he buys goods \$250, \$150, \$100, and \$350, what will drafts for these amounts cost at 0.1% premium?

9. Which is cheaper for you, if you owe \$75 for some goods, to send a money order, or a draft for which you have to pay 15¢ premium? How much cheaper?

10. It costs 8¢, besides postage, to register a letter. If it is not delivered the government will pay the loss, up to \$25. If you owe \$18, is it cheaper for you to send it by registered letter or by money order? How much cheaper?

11. If you owe \$100 to a manufacturer at a distance, from whom you have bought goods, how will you make the payment? Tell why you will make it in that way, and how much it will probably cost. (The premium on drafts may be taken at the common rate of 0.1%.)

147. Commercial drafts. A creditor sometimes *draws* directly on a debtor, the draft being of this form:

> DAYTON, OHIO, *Feb. 15,* 19*10*
>
> At sight pay to the order of
>
> *The First National Bank of Dayton* $*735.80*
>
> *Seven hundred thirty-five and* $\frac{80}{100}$ Dollars
>
> To *The National Cash Register Co.*
>
> *James X. Wye,* *Per J. H. P.*
>
> *Cleveland, Ohio*

148. Parties to a draft. Here the National Cash Register Company is the *drawer;* Wye is the *drawee;* The First National Bank is the *payee.*

149. The Register Company deposits this draft with the First National Bank. The bank sends it to some bank in Cleveland. The Cleveland bank sends a messenger to Mr. Wye for the money, and, having collected it, sends the money (or its equivalent) to the Dayton bank. The Dayton bank then notifies the company that it is paid, and the amount, less some slight commission, the *proceeds* of the draft, is added to the company's account.

WRITTEN EXERCISE

1. Brown & Co. draw on J. H. Brownson for $750. The banks charge 0.1% for collection. What are the proceeds?

2. The Electric Company draws on Mr. X for $550. The banks charge 0.1% for collection. What are the proceeds?

3. The Arithmetic Publishing Company draws on R. T. Jewett for $150. The banks charge 0.2% for collection. What are the proceeds? Write the draft.

4. Robertson Bros. draw on J. P. Shipley for \$37.50. The charges for collection are 10¢. This is what per cent of the face?

5. M. D. St. John collects a \$150 debt through the bank, the proceeds being \$149.70. What is the bank's rate for collecting this debt?

6. S. L. James of Des Moines draws on L. D. Richards of Cedar Rapids, Iowa, for \$75. The bank charges $\frac{1}{8}$% for collecting. What are the proceeds?

7. M. T. Snell of Cleveland is the drawee of a \$200 draft; The Farmers Trust Co. of Detroit is the payee; The World Soap Co. is the drawer. Write the draft.

8. A. B. Stanley owes M. S. Stanton for 2 doz. suits @ \$113.50; $1\frac{1}{2}$ doz. suits @ \$130; 2 doz. overcoats @ \$139; $7\frac{1}{2}$ doz. pairs trousers @ \$40. Stanton draws on Stanley for the money, and the bank charges him 50¢ for collecting. What rate is this?

9. Suppose William Bentley of Winnipeg, Manitoba, owes you \$250, and you wish to draw upon him for this amount. Write out a draft, payable to the order of some bank near your home. If the bank charges $\frac{1}{10}$% for collecting, what are the proceeds?

10. R. H. Dudley owes Cayley & Co. for 4 dressers @ \$14, and 6 washstands @ \$3.50, all less 15%, and a bedroom set at \$34.55 net. They draw on him for the amount, the bank charging 10¢ for collecting. What is the net amount received by Cayley & Co.?

11. R. J. Doane of Montreal owes A. D. Kane of Pittsburg for 10 steel girders @ \$50, and tells the latter to draw upon him for the amount. Mr. Kane keeps his account at the Iron Exchange Bank. Make out a draft for Mr. Kane. What is the bank's charge for collecting, at $0.1\frac{1}{2}$%?

150. The rates of exchange. A money order is always sold at a slight advance over its face, and usually a bank draft costs more than its face. In each case the variation from the face is called the *rate of exchange.*

151. Premium. If the rate of exchange is added to the face, exchange is *at a premium,* as we have already seen.

152. Par. If there is no rate of exchange, exchange is *at par.*

153. Discount. If the rate of exchange is subtracted from the face, exchange is *at a discount.*

154. For small sums, say for $500 or less, New York, Chicago, or Philadelphia exchange usually sells at a premium of about 0.1%. This is to pay the bank for its trouble and for the expense of shipping the money when its balance at these cities gets low. Banks usually buy such drafts at their face value, thus making no charge for cashing them.

155. But on large sums the rate of exchange varies. If the Chicago banks owe the New York banks $2,000,000, they must send that amount by express, an expensive proceeding. If a man in Chicago at that time wished to buy a draft on New York for $30,000, they would charge him more than usual because they would have to express that much more to New York. But if a man in New York wished to buy a draft on Chicago, he might buy it for less than $30,000 because the bank would get its money at once and the risk and expense of transmitting it would be saved.

156. The premium or discount is usually quoted as a certain per cent of the face of the draft, but sometimes as so much on $1000. In the latter case the quotation of $\frac{1}{4}$% premium is the same as that of $2.50 premium.

The explanation of the *Clearing House*, a place where bank officials of a city meet daily to exchange drafts and checks, and to pay balances due one another, is too technical for most classes, and if given at all should be explained verbally by the teacher.

WRITTEN EXERCISE

Find the cost of the following drafts:

1. \$3756.70, at par.	2. \$3500, 0.1% discount.
3. \$750, 0.2% premium.	4. \$6750, $\frac{1}{20}$% premium.
5. \$2450, 0.1% premium.	6. \$17,500, 0.2% discount.

7. What is the cost of a draft on San Francisco for \$5200 at $\frac{3}{4}$% discount? Write the draft.

8. What must be paid in Memphis for a draft on Chicago for \$3400, exchange being at $\frac{1}{4}$% premium?

9. A draft for \$4800 was bought for \$4794. Was exchange at a premium or a discount? What was the rate?

10. When a Boston draft for \$35,000 can be bought in New Orleans for \$34,930, is exchange at a premium, at par, or at a discount? What is the rate?

11. If J. R. Glover draws on J. B. Thornton for \$250, and the banks charge 0.1% for collection, what are the net proceeds that Glover will receive? Write the draft.

12. My agent in Toronto sells a house for me for \$2500. He charges 2% commission, and the bank charges \$2.50 premium for a draft for any amount between \$2000 and \$2500. What sum does he remit to me?

13. A telegraphic money order costs twice the rate for a ten-word message, plus 1% premium on the face. A ten-word message from Kansas City to Albany costs 50 ct. What will a telegraphic money order for \$375 cost?

14. Mr. Edgcomb of Denver owes Mr. Nourse \$3243.24 in that city. Mr. Nourse has gone to New York on business and asks that the money be sent, less the cost of exchange. Exchange being at 0.1% premium, what is the face of the draft?

157. Foreign exchange. If a man buys foreign goods, he often has to send money abroad.

158. Table of English money:

12 pence (d.) = 1 shilling (s.) = \$0.243 +.
20 shillings = 1 pound (£) = \$4.8665.

We commonly think of the pound as about \$5, the shilling as about 25¢, and the penny as about 2¢. Canada uses the same table of money as the United States.

159. Table of French money:

100 centimes (c.) = 1 franc (fr.) = \$0.193.

We commonly think of the franc as 20¢.
This system is used in several European countries.

160. Table of German money:

100 pfennigs (pf.) = 1 mark (M.) = \$0.238.

We commonly think of the mark as 25¢, and 4 pf. as 1¢.

ORAL EXERCISE

Taking the above approximate values, state about how much the following sums represent in our money:

1. £7.	2. £6 5s.	3. £9 4s.	4. £8 10s.
5. £50.	6. £10 16s.	7. £12 2s.	8. £15 12s.
9. £150.	10. £50 4s.	11. £60 7s.	12. £90 9s.
13. 50 fr.	14. 75 fr.	15. 125 fr.	16. 300 fr.
17. 800 fr.	18. 1200 fr.	19. 80 M.	20. 100 M.
21. 240 M.	22. 640 M.	23. 840 M.	24. 1200 M.

25. £8 4s. 6d.	26. £7 2s. 4d.	27. £5 7s. 9d.
28. 50 fr. 50 c.	29. 75 fr. 25 c.	30. 80 fr. 20 c.
31. 8 M. 20 pf.	32. 10 M. 40 pf.	33. 80 M. 4 pf.

34. Express \$40 approximately as German money; as English money; as French money.

WRITTEN EXERCISE

Express as pence:

1. £3.	2. £2 4s.	3. £3 5s. 6d.
4. 3s. 8d.	5. 18s. 3d.	6. £5 1s. 2d.

Express as shillings and decimals:

7. 8s. 8d.	8. 9s. 3d.	9. £3 6s. 9d.

Express as pounds and decimals:

10. £2 2s.	11. £3 4s.	12. £6 4s. 6d.

Express as centimes:

13. 25 fr.	14. 37 fr.	15. 35 fr. 30 c.

Express as francs and decimals:

16. 275 c.	17. 475 c.	18. 1275 c.

Express as marks and decimals:

19. 350 pf.	20. 480 pf.	21. 9275 pf.

Express as pfennigs:

22. 175 M.	23. 200 M.	24. 10 M. 75 pf.

Taking £1 *as equal to* $4.87, *express in our money:*

25. £75.	26. £68.	27. £16 10s.

Taking £1 *as equal to* $4.87, *express in English money:*

28. $38.96.	29. $24.35.	30. $43.83.

Taking 1 M. *as equal to* 23.8¢, *express in our money:*

31. 75 M.	32. 125 M.	33. 3750 M.

Taking 1 M. *as equal to* 23.8¢, *express in German money:*

34. $9.52.	35. $14.28.	36. $71.40.

Taking 1 fr. *as equal to* 19.3¢, *express in our money:*

37. 85 fr.	38. 230 fr.	39. 750 fr.

161. Bills of exchange. Foreign drafts are also called *bills of exchange.*

162. Rate of exchange. The rate of foreign exchange varies continually, depending on the demand.

Thus, when English exchange is quoted at 4.90 (that is, a draft for £1 costs $4.90) it is above par, for 4.8665 is par.

163. Foreign quotations. English exchange is quoted at dollars to the pound, as 4.92.

French exchange is quoted either at francs to the dollar, 5.14 meaning that $1 will buy a draft for 5.14 francs, or at cents to the franc, 19.7 meaning that a draft for 1 fr. costs 19.7 ct.

German exchange is quoted at cents to the 4 marks, 97 meaning that a draft for 4 M. costs 97 ct., or at cents to the mark, 24.1 meaning that a draft for 1 M. costs 24.1 ct.

164. At present much of the foreign exchange for small sums is carried on by post-office or express money orders.

165. Newspaper quotations. Exchange is often quoted thus:

	Demand	*60 Days*	*90 Days*
Sterling	4.83	4.79	4.77
Francs	5.20	5.21	5.23
Marks	$94\frac{5}{8}$	$94\frac{3}{8}$	$93\frac{7}{8}$

This means that a £1 draft on England (sterling) will cost $4.83 if payable on demand (at sight), $4.79 if payable 60 days after sight, or $4.77 90 days after sight. The quotations for the following examples may be taken as above or from a newspaper.

ORAL EXERCISE

State the cost of demand drafts for:

1. £10.	2. £5.	3. £2.	4. £1000.
5. 5.20 fr.	6. 52 fr.	7. 104 fr.	8. 5200 fr.
9. 40 M.	10. 400 M.	11. 4000 M.	12. 40,000 M.

166. Illustrative problems. 1. What will a 60-day draft for £40 cost?

1. £1 costs \$4.79.
2. £40 cost 40 times \$4.79 = \$191.60.

2. What will a demand draft for 75 M. cost?

1. 4 M. cost $\$0.94\frac{5}{8}$, 1 M. costs $\frac{1}{4}$ of $\$0.94\frac{5}{8}$.
2. 75 M. cost 75 times $\frac{1}{4}$ of $\$0.94\frac{5}{8} = \$17.74$.

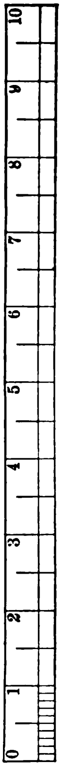
3. What will a 90-day draft for 125 fr. cost?

1. 5.23 fr. cost \$1, 1 fr. costs $\frac{\$1}{5.23}$.
2. 125 fr. cost $\frac{\$125}{5.23} = \$23.90$.

WRITTEN EXERCISE

1. Find the cost of a 60-day draft for £90 10s.
2. Find the cost of a demand draft for 305 fr. 50 c.
3. Find the cost of a 90-day draft for 750 M.; 620 M.
4. How large a sterling 90-day draft will \$71.55 buy?
5. How large a demand draft on Paris will \$125 buy?
6. How large a 60-day draft on Leipzig will \$76 buy?
7. A merchant buys Scotch tweeds in London to the amount of £127 10s. What will a 60-day draft cost?
8. A city library buys books in Paris to the amount of 578 fr. 31 c. How much will a draft for this amount cost when exchange is quoted at 5.21?
9. I have bought 400 M. worth of books in Leipzig. Which is better for me, to buy a demand draft or an express money order @ 24¢ to the mark?
10. I have bought a painting in Florence for 670 lire (lērā, francs), and 7 lire for packing. If exchange on Florence is 5.18, how much will a draft for this amount cost?

METRIC SYSTEM



10 centimeters = 1 decimeter

167. Where used. If a man buys foreign goods, except from England, the measures will probably be a more convenient system than ours, invented in France about 1800, and now used in a large part of the civilized world. It is easily learned, it is much more simple than our system, and we need it in all scientific work and in our newspaper reading. It is called the *Metric System.*

168. Meter. The unit of length is the *meter.* It is nearly 39.37 in. long, or nearly one ten-millionth of the distance from the equator to the pole.

169. Liter. The unit of capacity is the *liter* (lētēr), a cube $\frac{1}{10}$ of a meter on an edge. It is nearly 1 qt.

170. Gram. The unit of weight is the *gram.* It is practically the weight of a cube of water $\frac{1}{10}$ of a meter on an edge. It equals nearly 15.4 grains.

171. The prefixes. The tables are easily learned when the prefixes are known.

Just as 1 mill = $\frac{1}{1000}$ of a dollar,
so 1 millimeter = 0.001 of a meter.
Just as 1 cent = $\frac{1}{100}$ of a dollar,
so 1 centimeter = 0.01 of a meter.
Just as decimal means tenths,
so 1 decimeter = 0.1 of a meter.

	THE PREFIX	MEANS	AS IN	WHICH MEANS	
From the Greek	myria-	10,000	myriameter	10,000	meters.
	kilo-	1000	kilogram	1000	grams.
	hekto-	100	hektoliter	100	liters.
	deka-	10	dekameter	10	meters.
		1		1	
From the Latin	deci-	0.1	decimeter	0.1	of a meter.
	centi-	0.01	centigram	0.01	of a gram.
	milli-	0.001	millimeter	0.001	of a meter.

172. Table of length:

A myriameter	=	10,000	**meters.**
A **kilometer** (km.)	=	1000	"
A hektometer	=	100	"
A dekameter	=	10	"
Meter (m.)			
A decimeter (dm.)	=	0.1	of a meter.
A **centimeter** (cm.)	=	0.01	"
A **millimeter** (mm.)	=	0.001	"

In the tables the most important names are in heavy type.

173. Approximate values. The meter is about 39.37 in., $3\frac{1}{4}$ ft., or a little over a yard; the kilometer is about 0.6 of a mile.

174. Abbreviations. The abbreviations in this book are recommended by various scientific associations. Some, however, use Km., Dm., dm., for kilometer, dekameter, and decimeter.

ORAL EXERCISE

Express as meters and decimals:

1. 1 km.	2. 65 km.	3. 225 dm.
4. 375 dm.	5. 700 dm.	6. 750 km.
7. 3275 cm.	8. 4550 cm.	9. 6500 cm.
10. 120,000 mm.	11. 216,500 mm.	12. 100,575 mm.

Express approximately as meters (1 m. = $3\frac{1}{4}$ ft. = 39 in.):

13. 13 ft.	14. 39 ft.	15. 65 ft.
16. 3.9 in.	17. 78 in.	18. 390 in.
19. 325 ft.	20. 650 ft.	21. 0.39 in.
22. 9 ft. 9 in.	23. 975 ft.	24. 3 ft. 3 in.

Express approximately as feet, inches, or miles:

25. 8 m.	26. 24 m.	27. $\frac{1}{6}$ km.	28. $\frac{2}{3}$ km.
29. 100 km.	30. 300 km.	31. 60 km.	32. 0.5 m.
33. 30.5 km.	34. 20.5 km.	35. 100 m.	36. 1200 km.

WRITTEN EXERCISE

Express as meters and decimals:

1. 275.3 mm.	2. 476.4 cm.	3. 293.8 dm.

Express as kilometers and decimals:

4. 4862 m.	5. 12,758 cm.	6. 628,341 mm.
7. 47 mi.	8. 26.5 mi.	9. 10,560 ft.

Express as miles:

10. 751 km.	11. 286 km.	12. 34.9 km.
13. 5280 m.	14. 2976 m.	15. 14,781 dm.

Express as feet, taking $3\frac{1}{4}$ ft. = 1 m.:

16. 17 m.	17. 64 m.	18. 108 m.
19. 6894 cm.	20. 2986 cm.	21. 81,296 mm.

Express as inches, taking 39.37 in. = 1 m.:

22. 47 m.	23. 324 cm.	24. 4680 mm.
25. 34.5 m.	26. 2.83 cm.	27. 3000 mm.

28. Express the diameter of a 7-cm. gun in inches.

29. A certain hill is 203 m. high. Express this in feet.

30. A certain tower in Paris is 37.5 m. high. Express this in feet.

31. The distance from Dieppe (Dē-ep′) to Paris is 209 km. Express this in miles.

32. The distance between two places in Germany is 178 km. Express this in miles.

33. The distance from Paris to Cologne is 306 mi. What is the railway fare at 12 centimes per kilometer?

34. The distance from Paris to Brussels is 326 km., and it takes 7 hours to make the trip by railway. What is the average number of miles an hour?

175. Table of square measure:

A square myriameter	=	100,000,000 square meters.
" kilometer (km^2.)	=	1,000,000 " "
" hektometer	=	10,000 " "
" dekameter	=	100 " "
Square meter (m^2.)		
A square decimeter (dm^2.)	=	0.01 of a square meter.
" centimeter (cm^2.)	=	0.0001 " "
" millimeter (mm^2.)	=	0.000001 " "

The abbreviation sq. m. is often used for m^2.

176. Land measure. The square dekameter is also called an **are** (är); and since there are 100 square dekameters in 1 hm^2., a square hektometer is called a **hektare** (ha.). The hektare equals 2.47 acres = nearly $2\frac{1}{2}$ acres.

ORAL EXERCISE

Express as square meters:

1. 2 km^2. 2. 30,000 cm^2. 3. 5 square dekameters.
4. 1000 dm^2. 5. 100,000 cm^2. 6. 2 square hektometers.

Express as square dekameters:

7. 5000 m^2. 8. 2 km^2. 9. 5 square hektometers.

Express as hektares:

10. 10,000 m^2. 11. 1 km^2. 12. 5 square hektometers.

Express as hektares, taking 1 ha. = $2\frac{1}{2}$ A.:

13. 5 A. 14. 25 A. 15. 100 A. 16. 50 A.

WRITTEN EXERCISE

Express as square centimeters:

1. 750 km^2. 2. 37 m^2. 3. 4296 mm^2. 4. 6.25 m^2.
5. France has an area of 322,335 km^2. How many acres?

177. Table of cubic measure:

A cubic myriameter	=	10^{12} cubic meters.	
" kilometer	=	10^9 " "	
" hektometer	=	1,000,000 " "	
" dekameter	=	1000 " "	
Cubic meter (m^3.)			
A cubic decimeter (dm^3.)	=	0.001	of a cubic meter.
" centimeter (cm^3.)	=	0.000001	" "
" millimeter (mm^3.)	=	0.000000001	" "

178. The stere. The cubic meter is also called a **stere** (stĕr, st.), a unit used in measuring wood.

ORAL EXERCISE

Express as cubic meters:

1. 17 st. 2. 5000 dm^3. 3. 2,000,000 cm^3.
4. 1000 dm^3. 5. 2 cubic dekameters.

Express as cubic decimeters:

6. 7 m^3. 7. 8 st. 8. 5000 cm^3.
9. 19 m^3. 10. 26 st. 11. 10,000 cm^3.

12. How many centimeters in a meter? square centimeters in a square meter? cubic centimeters in a cubic meter? cubic decimeters in a cubic dekameter?

13. Estimate the length, width, and height of your schoolroom, in meters. How many cubic meters in the room?

WRITTEN EXERCISE

Express as cubic meters:

1. 19.75 cubic dekameters. 2. 427,653.84 mm^3.
3. 37.5 st. + 98.9 st. + 764 st. + 27.43 st. + 196.8 st. + 37 st.
4. 0.000001 cubic hektometer. 5. 0.0000001 km^3.
6. $34\frac{3}{8}\frac{1}{4}$ cu. ft. (calling $3\frac{1}{4}$ ft. equal to 1 m.).

179. Table of weight:

A metric ton (t.)	=	1,000,000	grams.
A quintal (q.)	=	100,000	"
A myriagram	=	10,000	"
A kilogram (kg.)	=	1000	"
A hektogram	=	100	"
A dekagram	=	10	"
Gram (g.)			
A decigram	=	0.1	of a gram.
A centigram (cg.)	=	0.01	"
A milligram (mg.)	=	0.001	"

The metric ton is nearly the weight of 1 m^3. of water at its greatest density; the kilogram, of 1 dm^3.; and the gram, of 1 cm^3.

180. Approximate values. A kilogram is about $2\frac{1}{5}$ lb. A 5-ct. piece weighs 5 g. A metric ton is nearly 2204.6 lb.

ORAL EXERCISE

Express as grams:

1. 147 cg. 2. 3400 mg. 3. 5200 cg.
4. 348 kg. 5. 2950 kg. 6. 1728 kg.

Express as centigrams:

7. 121 g. 8. 3 kg. 9. 19 kg.
10. 20 mg. 11. 135 mg. 12. 6800 mg.

Express as milligrams:

13. 3.75 g. 14. 4.25 g. 15. 55.2 cg.

Express as kilos (kilograms):

16. 17 t. 17. 3.5 t. 18. 4.25 t.
19. 18,000 cg. 20. 15,000 mg. 21. 2460 g.

Express as pounds:

22. 25 kg. 23. 30 kg. 24. 50 kg.
25. 500 kg. 26. 100 kg. 27. 125 kg.

WRITTEN EXERCISE

Express as kilos (kilograms):

1. 374 lb.	2. 352 oz.	3. 3275 g.
4. 15,428 lb.	5. 48.4 lb.	6. 7275 g.
7. 1 T. 204 lb.	8. 300,000 cg.	9. 173.8 lb.

10. What is the weight of 35 dm^3. of water in pounds?

11. How many 5-ct. pieces will it take to weigh a kilo?

12. Express a gram as a fraction of an ounce; of a pound.

13. In traveling on the continent of Europe, 25 kg. of baggage is usually transported free. How many pounds?

14. What is the weight, in metric tons, of water in a tank 2.5 m. by 3 m. by 1.5 m.? 3.4 m. by 6 m. by 4.25 m.?

15. What is the weight, in metric tons, of water in a tank 5.2 m. by 3.4 m. by $1\frac{3}{4}$ m.? 2.8 m. by 4 m. by 0.75 m.?

16. Steel being 7.8 times as heavy as water, what is the weight of a bar of steel 7.8 cm. wide, 3.1 cm. thick, and 1.5 m. long?

The number 7.8 is called the *specific gravity* of steel. From Exs. 18 and 19 the specific gravity of lead is 11.3, and of gold 19.3.

17. The specific gravity of granite being 2.7, what is the weight of a block of granite 1.2 m. by 0.75 m. by 0.25 m.?

18. Lead being 11.3 times as heavy as water, what is the weight of a bar of lead 3.3 cm. square on the end, and 25.8 cm. long?

19. Gold being 19.3 times as heavy as water, how many grams does a cube of gold 3 cm. on an edge weigh? Express the result also as a fraction of a kilogram.

20. A man traveling abroad steps on a penny-in-the-slot weighing machine and finds he weighs 75 kg. How many pounds does he weigh?

181. Table of capacity:

A hektoliter (hl.)	=	100 liters.	
A dekaliter	=	10	"
Liter (l.)			
A deciliter	=	0.1	of a liter.
A centiliter	=	0.01	"
A milliliter (ml.)	=	0.001	"

182. Approximate values. A liter is the same as 1 dm^3. Hence 1 l. of water weighs 1 kg. A liter is practically the same as our quart.

ORAL EXERCISE

Express as liters:

1. 5000 ml.	2. 3 hl.	3. 17 hl.
4. 2500 ml.	5. 7.5 hl.	6. 2.25 hl.
7. 5 gal.	8. 8 pt.	9. $12\frac{1}{2}$ gal.

Express as hektoliters:

10. 250 l.	11. 375 l.	12. 125.5 l.
13. 2575 l.	14. 3500 l.	15. 625.3 l.
16. 300 qt.	17. 2600 qt.	18. 50 gal.

Express as quarts:

19. 175 l.	20. 16.25 l.	21. 4 hl.
22. 20.75 l.	23. 12.50 hl.	24. 0.8 hl.

Express in kilograms the weight of the following amounts of water:

25. 25.5 l.	26. 18.75 l.	27. 16 hl.
28. 17.50 l.	29. 26.50 hl.	30. 0.01 hl.

31. How many decimeters in a meter? How many cubic decimeters in a cubic meter? How many liters in a cubic meter? A cubic meter of water weighs how many kilos?

WRITTEN EXERCISE

1. Milk weighing 1.032 times as much as water, how many kilos does 7 l. weigh?

2. Alcohol being 0.83 as heavy as water, find the weight of 7 l. Express the result in kilos; in pounds.

3. The Eiffel Tower in Paris is 300 m. high. Express this as inches; as feet; as a fraction of 1 km.

4. Gold being 19.3 times as heavy as water, how much does 17.5 cm^3. weigh? Answer in grams; in grains.

5. Cork being one fourth as heavy as water, how much does 9 dm^3. of cork weigh? Answer in kilos; in pounds.

6. The specific gravity of silver (see page 184, Ex. 16) being 10.5, what is the weight of a piece of silver 12.5 cm. by 7.2 cm. by 3.1 cm.?

7. The specific gravity of petroleum being 0.7, what is the weight of 7.34 dm^3. of petroleum?

8. The specific gravity of alcohol being 0.79, what is the weight of alcohol in a tank 5 dm. by 3 dm. by 2.5 dm.?

9. The specific gravity of copper being 8.9, what is the weight of a piece of copper 1 m. by 2 dm. by 9 cm.?

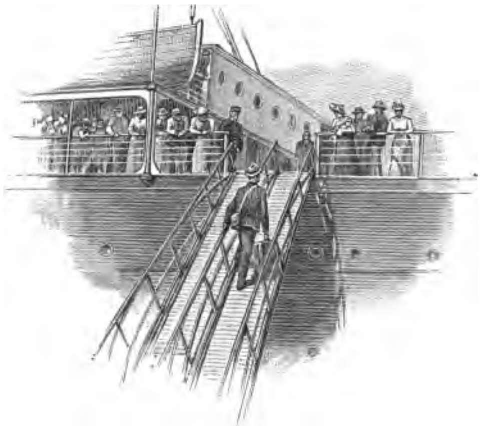
10. One piece of land is 20 m. long and 30 m. wide, and another is 20 yd. long and 30 yd. wide. Find the area of the first in hektares and the second in acres.

11. Samples of merchandise may be sent abroad by mail if they do not weigh more than 350 g. Express this as a fraction of a pound; as ounces; as a fraction of 1 kg.

12. Find in kilos the weight of water in a tank 3 m. by 2 m. by 5 m.; also the weight in pounds of water in a tank 3 ft. by 2 ft. by 5 ft. (1 cu. ft. of water weighs 62.5 lb. Notice the simplicity of the metric system.)

Mr. Field's Trip to Europe

Mr. and Mrs. Field took a vacation trip to Europe, landing at Liverpool, traveling through England, France, Germany, Switzerland, and Italy, and sailing home from Naples. After leaving England they found the Metric System used by every one, and of course they did not meet our system of money until they landed in New York again. They found that Switzerland and France used the same system of money, and that Italy had the same values, but



called the franc a *lira* (lē-rä; plural, *lire*, lē-rā). In making estimates in our own system they allowed \$5 to £1, 25¢ to 1s., 2¢ to 1d., 25¢ to 1 M., 20¢ to the franc or lira, $1\frac{1}{10}$ yd. or $3\frac{1}{4}$ ft. to 1 m., $\frac{3}{5}$ mi. (0.6 mi.) to 1 km., and $2\frac{1}{5}$ lb. (2.2 lb.) to 1 kg.

ORAL EXERCISE

1. At \$90 a steamer ticket each way, less 10% on the return portion of the ticket, what did both tickets cost?

2. On the voyage to Liverpool he gave £3 5s. for stewards' fees. How many dollars did he give?

3. He landed in Liverpool at 2 P.M., and at once telegraphed his brother in San Francisco and his sister in New York. Allowing an hour for delay, at what time did the messages reach their respective destinations?

4. He paid 21s. 9d. a ticket from Liverpool to London. How much did he pay for the two tickets? How many dollars?

5. He paid the cab driver in London 2s. 6d. to take them to their hotel. How much is this in our money?

6. He paid 12s. a day for each, at one of the smaller hotels. They remained there a week. How much did it cost the two? How much in our money?

7. He paid 33s. each for two tickets to Paris. How much did he pay for both? How much in our money?

8. At Paris he paid the cab driver 2 fr. 50 c. to take them to their hotel. How much is this in our money?

9. He paid 14 fr. a day for each, at a hotel near the Arch of Triumph, and they spent 11 days in Paris. How much was the hotel bill? How much in our money?

10. The Arch of Triumph is 50 m. high. How many feet high? It is at the end of a beautiful avenue 2.2 km. long. Express this distance in miles.

11. Not far from his hotel is the Eiffel Tower, 300 m. high. Express this height in feet.

12. The distance from Paris to Cologne is 510 km. Express this distance in miles.

13. He bought two tickets at 44 fr. each. How much did they cost? How much in our money?

14. They reached Cologne at 5 : 30 P.M. (Mid-European time, 15° E.). What time (standard) was it then in England? in Boston? in Chicago? in Portland, Oregon?

15. They spent two days in Cologne, paying 12 M. each per day. How much was the hotel bill? How much in our money?

16. At Cologne Mrs. Field did some shopping, spending 150 M. How much is this in our money?

17. Among her purchases was some silk for a gown. She bought 18 m. How many yards?

18. From Cologne they went up the Rhine to Mainz, a distance of 150 km. How many miles?

19. The tickets from Cologne to Mainz were 15 M. 50 pf. each. What did the two cost? How many dollars?

20. They spent two days at Mainz, paying 14 M. each per day. How much was the hotel bill? How many dollars?

21. They then went to Lucerne, a distance of 400 km. How many miles?

22. The tickets to Lucerne cost 32 M. apiece. How much did the two cost? How many dollars?

23. They spent two weeks in Switzerland, their expenses averaging 22 fr. 50c. apiece per day. How much were they for two? How many dollars? How many dollars for the two weeks?

24. Their tickets from Lucerne right through to Naples, including Florence and Rome, cost 100 fr. apiece. How much for the two tickets? How many dollars?

25. They spent three weeks in Italy, their expenses averaging 20 fr. apiece per day. How much were they for two? How many dollars?

26. Their other purchases and extras in England amounted to £8, in France to 125 fr., in Germany to 120 M., in Switzerland to 60 fr., and in Italy to 80 lire. Express each in dollars.

WRITTEN EXERCISE

Taking the expenses as stated in the Oral Exercise, find the cost of the trip taken by Mr. and Mrs. Field.

TAXES

183. What are taxes? Whatever a man's occupation, one of his expenses will be *taxes*, money paid for the support of the village, town, city, county, or state.

184. Expenses of our government. The expenses of the United States government vary from year to year, but they average about $1,300,000 a day, or $473,500,000 a year. Some items of our income and expenditures are as follows, varying from year to year:

Income:

Customs (duties on imported goods)	$275,000,000
Internal revenue (tobacco, etc.)	275,000,000
Sale of public lands	7,500,000
Miscellaneous	36,000,000

Expenditures:

War Department	$112,000,000
Navy Department	88,000,000
Pensions	138,000,000
Indians	10,000,000
Salaries, diplomatic service, etc.	150,000,000

WRITTEN EXERCISE

1. When our income is $575,000,000 and our customs receipts are $242,000,000, these receipts are what per cent of the income?

2. Taking our expenditures as $488,000,000 a year, what per cent of this goes to the War Department, as above stated? to the Navy Department? to pensions?

3. The colleges and universities of the country cost $23,850,000 a year, and are not supported by the government. This is what per cent of the $488,000,000 in Ex. 2?

4. Our income from public lands is what per cent of that from our customs?

In this and Exs. 5–8, use the items as given on page 190.

5. Our public lands receipts are what per cent less than the income from internal revenue?

6. Our income from internal revenue is what per cent greater than that from our public lands?

7. What per cent of our internal revenue would be necessary to pay the Navy Department expenditures?

8. What per cent of our customs receipts would be necessary to pay the War Department expenditures?

9. In a certain year the government paid $69,210,000 for carrying the mails, and 33% as much for salaries to postmasters. How much did it pay for both of these items?

10. The income of the Post-Office Department through the New York and Brooklyn offices in a certain year amounted to $16,206,000. This was what per cent of the total income ($146,000,000) of the department that year?

11. In a certain year the government received $135,810,015 from taxes on spirits, 33% as much from taxes on tobacco, and 10% more on fermented liquors than on tobacco. What were its total receipts from these three sources?

12. In a certain year the internal revenue receipts of the country amounted to $233,000,000. The largest amount paid by any state was 23%, paid by Illinois. What was this amount? What did the rest of the country pay?

13. In a certain year the government received $143,820,000 from the sale of postage stamps and money orders, which was 94% of the expenses of the Post-Office Department. How much deficiency did Congress have to vote to the Post-Office Department that year?

185. Our post-office system. We do not often think of our post-offices as places where we all pay taxes (when we buy postage stamps). The department is nearly supported by such receipts. In the following miscellaneous problems the annual income may be taken as $150,000,000, and the number of pieces of mail matter handled as 9 billion, but the figures vary from year to year.

WRITTEN EXERCISE

1. The average number of mistakes reported against clerks who handle the mails is only 5 out of 57,500 pieces. What is the per cent of errors?

2. If the post offices issue 46 million domestic money orders a year, amounting to $358,800,000, what is the average amount of each money order?

3. If the receipts of the New York post office amount to $13,650,000 a year, this is what per cent of our total annual postal income as stated above?

4. How much does it cost to register a letter or parcel? If the post offices transmit 22,831,400 registered letters and parcels annually, what is the income from this source?

5. If we have 75,000 post offices, what is the average annual income for each? If the sum of the salaries of the postmasters is $22,200,000, what is their average salary?

6. If the number of postal routes is 35,000, averaging 14.5 miles each, what is their total length? If the average number of annual trips over each route is 936, what is the total distance traveled?

7. If the contractors who carry the mail receive over the stage mail routes 6.58¢ per mile traveled, how much will a contractor receive a year who travels 25 miles daily, except 52 Sundays?

186. Our army and navy. It is necessary for us to keep a navy and a small army of sufficient strength to protect us from foreign attack. It is an expense and therefore is considered under taxes, with certain review problems.

WRITTEN EXERCISE

1. Two of our naval cruisers cost together $6,810,000, one costing $11\frac{79}{161}\%$ more than the other. What did each cost?

2. Gun metal is composed of 11 parts of copper to 2 parts of tin. How much tin must be added to 2607 lb. of copper to make gun metal?

3. The bronze trimmings of the guns are composed of 17 parts of copper to 4 parts of tin. How much copper must be added to 108 lb. of tin to make bronze?

4. A certain battle ship carries 2000 short tons of coal. How many pounds is this? If it is $12\frac{332}{389}\%$ of the loaded weight of the ship, what is this weight?

5. The speed of a 19-knot battle ship is $20\frac{5}{6}\%$ less than that of a fast mail boat. How long would it take the mail boat to overtake it, giving the battle ship 120 knots the start?

6. The battle ships *Connecticut, Kansas, Louisiana, Minnesota,* and *Vermont* cost $4,200,000 each. The amount paid for these five ships would send how many men through an agricultural or trade school, at $1400 each?

7. If our government pays for the War and Navy Departments $207,000,000 a year, this is how many times the $9,000,000 annually paid by our people to educate their children in colleges and universities? The tuition is what per cent of the war and navy expenses?

8. If our swiftest torpedo boat makes 31.395 knots an hour, which is $36\frac{1}{2}\%$ faster than our best cruiser, what is the speed of the latter?

9. The 7-in. guns are protected by 6.9-in. steel armor, which is $68\frac{32}{101}\%$ as thick as that protecting the 12-in. guns. How thick is the latter?

10. A 12-in. gun weighs 64 T. 719 lb. and fires a 1000-lb. projectile with 487 lb. of powder. How many pounds does it weigh when loaded? (Always use the 2000-lb. ton unless otherwise directed.)

11. The cruiser *Minneapolis* is $21\frac{1}{19}\%$ faster than the 19-knot battle ship *Georgia.* At these rates, if the battle ship has 288 knots the start, how long will it take the cruiser to overtake it in a race?

12. One of our battle ships has four 12-in. guns, or 20% as many as it has 7-in. guns. The rest of its guns are 3-in. The 12-in. and 7-in. guns together are $54\frac{6}{11}\%$ of the total number. How many 7-in. and 3-in. guns are there?

13. A shot from a certain 12-in. gun exerts enough force at the muzzle to lift a weight of 46,240 tons to the height of 1 ft. This is 70% more force than that exerted by a certain 10-in. gun. Find the force exerted by the latter.

14. A 16-in. gun fires a projectile over 5 ft. long, weighing 1 T. 400 lb., with 576 lb. of smokeless powder. At \$1 a pound for such powder, and 5¢ a pound for the projectile, how much does it cost the government to fire such a gun once? to fire it a dozen times?

15. One ship fired a signal to another, which was answered as soon as it was heard. The first ship heard the answering gun $18\frac{1}{4}$ sec. after the first gun was fired. What was the distance between the ships, sound traveling (at the temperature then observed) 1142 ft. per second?

187. Tariff. The United States collects a large part of its income by a tax on goods brought into the country. This income is called *customs revenue, tariff,* or *duty.*

188. Customhouse. Customs revenue is collected at *customhouses.* These are situated at *ports of entry.*

189. Classes of goods. Goods imported may be:

1. On the *free list,* and not subject to duty, as raw silk.

2. Subject to *ad valorem* (on the value) duty, a certain per cent on the value at the place of purchase, as clocks, on which the duty is 40% *ad valorem.*

3. Subject to *specific* duty, a certain amount per bushel, etc., as potatoes, on which the duty is 25¢ a bushel.

4. Subject to both *ad valorem* and *specific* duty, as velvet carpets, the duty being 60¢ per square yard plus 40%.

ORAL EXERCISE

State the duty on the following:

1. 750 T. of hay, duty $4 per ton.
2. $275 worth of goods, duty 10%.
3. $484 worth of goods, duty 25%.
4. $800 worth of goods, duty 40%.
5. $325 worth of goods, duty 30%.
6. $700 worth of goods, duty 60%.
7. $500 worth of goods, duty 35%.
8. $2000 worth of goods, duty 45%.
9. $88.40 worth of goods, duty $12\frac{1}{2}$%.
10. $60.30 worth of goods, duty $16\frac{2}{3}$%.
11. 600 lb. of goods, duty 22¢ per pound.
12. 1100 lb. of goods, duty 44¢ per pound.
13. 6000 sq. ft. of plate glass, duty 8¢ per square foot.

WRITTEN EXERCISE

1. What is the duty on \$4350 worth of bronzes at 45%? Is this specific or *ad valorem?*

2. What is the duty on 3500 bu. of barley at 30 ct. per bushel? Is this specific or *ad valorem?*

3. What is the duty on 15 sets of Thackeray's works at £7 a set, allowing \$4.87 to the pound, the duty being 25%?

4. What is the duty on 2 doz. sets of Scott's works at £4 10s. a set, allowing \$4.87 to the pound, the duty being 25%?

5. What is the duty on 2000 yd. of tapestry Brussels, 27 in. wide, invoiced at \$1.50 a yard, at 28 ct. per square yard and 40% *ad valorem?*

6. What is the duty on \$3500 worth of ready-made clothing at 50% *ad valorem,* and \$1800 worth of silk dresses at 60% *ad valorem?*

7. A man bought a painting in Rome, paying 750 lire (francs) for it. Allowing 19.3 ct. to the lira, what did it cost in our money, and what was the duty at 25%? What was the total cost, including the duty?

8. A jeweler bought an invoice of Swiss watches, paying 9875 fr. for them in Geneva. Allowing 19.3 ct. to the franc, what was the cost in our money, and what was the duty at 40%? What was the total cost, including the duty?

9. A lady traveling in Europe bought \$650 worth of jewelry (duty 60%), \$250 worth of silk dresses (duty 60%), \$75 worth of engravings (duty 25%), and a 10 × 12 Oriental rug costing \$700 (duty 10 ct. per square foot and 40%). What duty did she have to pay, if \$100 worth of goods (jewelry in this case) was admitted duty free?

190. Table of customs duties. The following is a table of certain dutiable articles:

Books in English	25%	Barley, 48 lb. to the bu.	30¢ per bu.
Bronzes	45%	Blankets	22¢ per lb. + 30%
Cheese	6¢ per lb.	Flannel cloth	22¢ per lb. + 30%
Cotton handkerchiefs	45%	Fruits, preserved	1¢ per lb. + 35%
Diamonds, cut and set	60%	Glass, plate	8¢ per sq. ft.
Hay	$4 per ton	Knit woolen	44¢ per lb. + 50%
Paintings	20%	Matches	8¢ per gross
Potatoes	25¢ per bu.	Perfumery	60¢ per lb. + 45%
Watches	40%	Soap, toilet	15¢ per lb.

WRITTEN EXERCISE

Find the duty on the following:

1. A $1500 diamond necklace.
2. An oil painting worth $1250.
3. A bronze statue costing $2500.
4. A car load of barley weighing 5 tons.
5. A shipment of cheese weighing 650 lb.
6. A dozen Swiss watches worth $45 each.
7. An English encyclopedia valued at $150.
8. A case of matches containing 1000 gross.
9. A 350-lb. bale of flannel cloth worth $400.
10. A shipment of toilet soap weighing 500 lb.
11. A 150-lb. box of knit woolen goods valued at $425.
12. A hundredweight of preserved fruits valued at $40.
13. Three hundred pieces of plate glass, each $16'' \times 24''$.
14. Fifty tons of hay and a car load of 250 bu. of potatoes.
15. A bale of blankets weighing 225 lb. and worth $300.
16. A hundred dozen cotton handkerchiefs @ 60¢ a dozen.

191. State and local taxes. State and local taxes are usually a certain per cent levied on the property of business concerns, on land, on money, and on other property.

192. Assessors. The property to be taxed is valued by officers called *assessors.*

193. Assessed valuation. The value placed upon property for taxation is called the *assessed valuation.*

194. Rate of taxation. Upon the assessed valuation a certain *rate of taxation* is fixed.

The words *rate of taxation* are often used to designate the *number of mills* of tax on each dollar of valuation.

Thus, a tax of $5\frac{1}{4}$ mills means $5\frac{1}{4}$ mills on a dollar.

Male citizens over 21 yr. of age pay a *poll* (head) *tax* in parts of the country.

This is a fixed sum, usually about $1.

195. Illustrative problem. For example, if a village with an assessed valuation of $3,200,000 must raise $16,800, what is the rate of taxation?

$$\begin{array}{r} \$0.005\tfrac{1}{4} \\ 32000\cancel{0}\cancel{0})\overline{\$168\cancel{0}\cancel{0}} \\ \underline{160000} \\ 8000 \quad \tfrac{8}{32}=\tfrac{1}{4} \end{array}$$

$16,800 ÷ 3,200,000 = $0.005¼.

ORAL EXERCISE

State the tax on the following amounts:

1. $1000 @ 5 mills.
2. $1000 @ 4 mills.
3. $2000 @ 6 mills.
4. $5000 @ 5 mills.
5. $5000 @ 4 mills.
6. $7500 @ 4 mills.
7. $7000 @ 5 mills.
8. $1000 @ $4\frac{3}{4}$ mills.
9. $7500 @ 6 mills.
10. $2000 @ $5\frac{1}{2}$ mills.
11. $1000 @ $3\frac{1}{2}$ mills.
12. $10,000 @ $5\frac{3}{4}$ mills.

WRITTEN EXERCISE

1. What is the tax on $6500 at $6\frac{1}{4}$ mills on a dollar?

2. The rate of taxation being $1\frac{1}{3}\%$, how much must a company pay whose property is assessed at $75,000?

3. A town has an assessed valuation of $4,500,000, and it has to raise $24,750 by taxation. What is the rate?

4. The rate of taxation being $7\frac{1}{2}$ mills, how much tax must a man pay on $8500, together with $1 poll tax?

5. A town has an assessed valuation of $720,000, and it wishes to levy a tax of $5400 for highways and schools. What must be the rate?

6. What is the total tax of a man whose property is assessed at $8400, the rate being 1 mill for state purposes, 3 mills for county purposes, $\frac{1}{2}$ mill for the town, and 2 mills for school purposes, together with $1 poll tax?

Given the following assessed valuations and tax levies, find the rates of taxation:

7. $4,200,000, $16,800.
8. $2,900,000, $8700.
9. $3,700,000, $20,350.
10. $7,875,000, $23,625.
11. $9,250,000, $41,625.
12. $14,500,000, $87,000.

Given the following tax levies and rates of taxation, find the assessed valuation:

13. $13,750, 5 mills.
14. $21,000, 6 mills.
15. $57,750, 7 mills.
16. $73,000, 8 mills.

Given the following assessed valuations and rates, find the tax levies:

17. $4,230,000, $4\frac{1}{2}$ mills.
18. $6,350,000, $7\frac{1}{2}$ mills.
19. $7,250,000, $5\frac{1}{2}$ mills.
20. $12,780,000, $6\frac{1}{2}$ mills.

196. Tax table. Collectors make out a *tax table* like this:

TAX TABLE. RATE $5\frac{1}{2}$ MILLS ON $1

	0	1	2	3	4	5	6	7	8	9
0	0000	0055	0110	0165	0220	0275	0330	0385	0440	0495
1	0550	0605	0660	0715	0770	0825	0880	0935	0990	1045
2	1100	1155	1210	1265	1320	1375	1430	1485	1540	1595
3	1650	1705	1760	1815	1870	1925	1980	2035	2090	2145
4	2200	2255	2310	2365	2420	2475	2530	2585	2640	2695
5	2750	2805	2860	2915	2970	3025	3080	3135	3190	3245
6	3300	3355	3410	3465	3520	3575	3630	3685	3740	3795
7	3850	3905	3960	4015	4070	4125	4180	4235	4290	4345
8	4400	4455	4510	4565	4620	4675	4730	4785	4840	4895
9	4950	5005	5060	5115	5170	5225	5280	5335	5390	5445

Here the first figure of the number of dollars assessed is given at the left, and the second one at the top. Thus, at $5\frac{1}{2}$ mills on $1, the tax on $100 is $0.55; on $250, $$1.37\frac{1}{2}$.

197. Collector's commission. The law allows the collector a commission on the tax collected, usually about 1%.

198. Illustrative problem. What must a man pay on $8250, at $5\frac{1}{2}$ mills on $1, the collector's commission being 1%?

By the table, the tax on $8200 is $45.10
" " " 50 " 0.28 (really $27\frac{1}{2}$¢)
" " " $8250 " $45.38
Add collector's 1% on the *tax* .46 (really 45.38¢)
$45.84

WRITTEN EXERCISE

Find the tax at $5\frac{1}{2}$ mills, as in § 198:

1. $6500. 2. $8750. 3. $4875. 4. $7675.
5. $12,000. 6. $16,500. 7. $24,400. 8. $23,500.
9. $31,100. 10. $23,250. 11. $18,875. 12. $45,250.

FIRE INSURANCE

199. Insurance. An agreement to compensate any one for some specified loss is called *insurance.*

200. Policy. The written agreement of an insurance company to pay a certain amount in case of loss is called a *policy.* The insurance companies are often called *underwriters.*

201. Face of policy. The amount specified to be paid in case of loss is called the *face of the policy.*

202. Premium. The cost of insurance is called the *premium.* The *rate of premium* is sometimes stated as a certain sum for each $100, and sometimes as a certain rate per cent.

Houses are usually insured for three or five years, business property is usually insured for one.

203. Illustrative problem. What is the premium for insuring a store against loss by fire, for $4000 at $1.20 a year?

1. Since the rate is $1.20 on $100, it is $0.012 on $1.
2. 4000 times $0.012 equals $48, the premium.

ORAL EXERCISE

1. What is the premium on a $650 policy at 2%?

2. What is the premium on a $3000 policy at $1\frac{1}{2}$%?

3. A farmer insured his growing crop of wheat for $800 at 4%. What was the premium?

4. A building which cost $24,000 is insured for $\frac{5}{8}$ of its cost. What is the face of the policy?

5. A man paid $20 for insuring his house, the rate being 1%. What was the face of the policy?

6. A schoolhouse is insured at 1%, the premium being $50. The face of the policy is $\frac{5}{8}$ of the value of the building. Required the value.

State the premiums on the following policies at the rates specified:

7. \$1000, $1\frac{1}{4}\%$.
8. \$725, 4%.
9. \$2000, $1\frac{3}{4}\%$.
10. \$825, 4%.
11. \$2000, $2\frac{1}{2}\%$.
12. \$500, $2\frac{1}{2}\%$.
13. \$3000, $1\frac{1}{3}\%$.
14. \$700, $2\frac{1}{2}\%$.
15. \$4000, $2\frac{3}{4}\%$.
16. \$900, $1\frac{1}{2}\%$.
17. \$2500, $1\frac{1}{5}\%$.
18. \$650, $1\frac{1}{2}\%$.
19. \$3500, 1.2%.
20. \$1250, 4%.
21. \$4250, \$2.
22. \$6250, \$2.
23. \$1750, \$2.
24. \$3750, \$2.
25. \$12,500, \$1.
26. \$12,000, \$3.
27. \$25,000, \$3.
28. \$15,000, \$2.
29. \$10,000, \$1.75.
30. \$50,000, \$1.50.

State the faces of the policies, given the following premiums and rates:

31. \$70, 2%.
32. \$90, 2%.
33. \$17, $\frac{1}{2}\%$.
34. \$50, $2\frac{1}{2}\%$.
35. \$25, $2\frac{1}{2}\%$.
36. \$13.75, 1%.
37. \$17.50, 1%.
38. \$18.50, $\frac{1}{2}\%$.

State the rates, given the following faces of policies and premiums:

39. \$1500, \$15.
40. \$2000, \$10.
41. \$2500, \$50.
42. \$3500, \$35.
43. \$5000, \$25.
44. \$7500, \$150.
45. \$6000, \$60.
46. \$6500, \$130.

47. A building worth \$8000 is insured for $\frac{3}{4}$ of its value at 1%. What is the premium?

WRITTEN EXERCISE

1. At $0.95, what is the premium on a $2500 policy?

2. At $1.10, what is the premium on a $2800 policy?

3. At $1.15, what is the premium on a $3750 policy?

4. A building worth $12,000 is insured for $\frac{2}{3}$ of its value at 2%. What is the premium?

5. If you insure the contents of your store for $6000, what is the premium, at $1.25?

6. The premium for insuring some property at $1.50 is $52.50. What is the face of the policy?

7. A man insured his factory, valued at $135,000, for $\frac{4}{5}$ of its value, at $1.90. What was the premium?

8. A dealer insured his stock of goods, valued at $14,000, for $\frac{7}{8}$ of its value, at $1.65. What was the premium?

9. If a 3-year policy for $1750 costs $27, what is the rate of premium for the 3 years? What is the rate per year?

10. If a 3-year policy for $3000 costs $36, what is the rate of premium for the 3 years? What is the rate per year?

11. A merchant insured his stock for $\frac{3}{4}$ of its value, at $1\frac{3}{4}$%. The premium was $131.25. What was the value of the stock?

12. A manufacturer insured his factory for $\frac{5}{8}$ of its value, at $2. The premium was $210. What was the face of the policy? the value of the factory?

13. A factory worth $33,000 is insured for $\frac{5}{6}$ of its value, at 2%. In case of total loss how much would the owner lose, including his premium paid?

14. A man insured his library for its full value, $2500, at $1.25. What was the premium? If a fire destroyed $\frac{3}{8}$ of the library, how much could he recover?

15. Which is the cheaper, a 3-year policy on $3500 at 1½%, or an annual policy renewed for the same period, at 60¢ a year? How much cheaper?

16. If Mr. Wood insures his stock of goods, invoiced at $28,000, for 80% of its value, at 1¼%, and the property is entirely destroyed by fire, what is his loss, including the premium paid?

Include the premium in Exs. 17, 18.

17. A dealer insured his stock of goods, valued at $21,800, for ⅞ of its value, at 1.3%. The property was entirely destroyed by fire. What was his loss?

18. If Mr. Baldwin insures his stock of goods, invoiced at $37,500, for 80% of its value, at 95¢, and the property is entirely destroyed by fire, what is his loss?

19. Suppose a company willing to take a risk covering the value of a stock of goods and the premium paid, and the face of the policy is $25,500, the rate being 2%, what is the value of the stock of goods?

20. A dealer pays $21 premium for insuring a shipment of grain at 1¾%. What is the face of the policy? When this grain is placed in the elevator it is again insured for the same amount at ¼%. What is the premium?

21. A manufacturer of especially inflammable goods finds that the insurance rate is 6% a year. Rather than pay such a rate, he could afford to have his stock burn once in how many years, not counting interest?

22. If Roberts & Eastman insure their business block for $12,000, at $1.15; the first-floor contents for $8000, at $1.25 (on account of the difficulty of removing goods); and the contents of the other floors for $3000, at $1.40, what is the total amount of premiums paid?

MARINE INSURANCE

204. Marine insurance. Insurance against loss by navigation is called *marine insurance.*

The value placed on the goods is the same as the face of the policy unless the contrary is stated.

WRITTEN EXERCISE

1. A man paid $36 for insuring some goods from Philadelphia to Marseilles, the rate being ½%, less 20% of the premium. What was the face of the policy?

2. A new ocean-going tug running between Norfolk, Va., and Eastport, Me., is insured at 5% a year, the premium being $2245. What is the face of the policy?

3. A man pays $24,793.35 for a cargo of hides in Galveston, which included the premium at ½% net for shipping the same to New York. What was the amount of premium?

4. What is the premium on a shipment of $10,200 worth of boots and shoes by steamer from New York to Manila, the rate being 1¼%, less 20% (see Ex. 1)? How much is it by sailing vessel, the rate being 2%, less 20%?

5. What is the premium on an under-deck shipment of $9000 worth of lumber from Georgia to New York, the rate being ¾%, less 20%? What is it for a shipment made on deck, the rate being 2¼%, less 20%? How much premium is saved by taking the under-deck shipment?

6. What is the premium on a shipment of $8460 worth of typewriters by passenger steamer from New York to Liverpool, the rate being 0.2% net? How much by a tramp steamer, the rate being ⅜%, less 20%? How much by a sailing vessel, the rate being 1¼%, less 20%?

LIFE INSURANCE

205. Why a man should insure his life. If a man is careful as to the future and considers the fact that in later years there will probably be one or more persons dependent upon him, he will wish to insure his life. The earlier this is done the lower will be the premium and the sooner he will cease to make payments if he takes a 10-, 15-, or 20-payment policy.

Premiums are stated at so much on $1000. They vary with the age, being smaller for young persons.

The best plan is to take a policy in a company of unquestionable standing, and never forfeit it by nonpayment of premiums.

ORAL EXERCISE

State the premiums on the following policies, the premium on $1000 being given:

1. $15,000, $26.
2. $3500, $25.
3. $2000, $21.30.
4. $4500, $30.
5. $3000, $22.10.
6. $5500, $30.
7. $4000, $22.50.
8. $12,000, $25.
9. $5000, $28.40.
10. $2500, $22.20.
11. $10,000, $27.65.
12. $20,000, $31.20.

State the faces of the policies, given the following premiums and the premiums on $1000:

13. $131, $26.20.
14. $175, $25.
15. $54.20, $27.10.
16. $49, $24.50.
17. $96.60, $32.20.
18. $63.90, $21.30.

State the premiums on $1000, given the following policies and premiums:

19. $3000, $84.
20. $4000, $27.

206. Kinds of policies. Four of the leading kinds of policies are:

1. *Ordinary life*, the insured agreeing to pay a certain premium, usually annually, for life.

The rate is always given as the cost of $1000 worth of insurance. That is, the rate $25.50 means that the annual premium on a policy for $1000 is $25.50. Some insurance companies allow *dividends* each year, thus reducing the premium slightly.

2. *Limited life*, the premiums being payable for some fixed number of years, as twenty, the policy then being called *paid up* (no more premiums being due), but the face not being paid until the death of the insured.

Naturally the premiums are higher on this form.

3. *Endowment*, the premiums being paid for some fixed number of years, as ten, fifteen, or twenty, at the end of which time the face of the policy will be paid to the insured.

Since this may be paid during the lifetime of the insured, the premiums are also higher than on an ordinary life policy.

4. *Term insurance*, the premiums being paid for a specified length of time and the face of the policy being payable if the insured dies within the term of insurance.

Thus, a person may insure his life or his health for thirty days, as in certain forms of accident insurance. He may also insure his life for a certain number of years only. The premium on this form of policy is low, since the company may not have to pay the face at all.

There are various other forms of policy, but most of them are modifications of the above types. Some are arranged so that the face will not be paid on the death of the insured, but a certain amount will be paid annually (an annuity) during the lifetime of the one for whose benefit the policy was written.

WRITTEN EXERCISE

1. What is the premium on a $5000 policy at $27.39 on $1000?

2. What annual premium must a man pay on a 10-year endowment policy for $4000 at $102.60 per thousand? What would he pay in the 10 years?

3. If a young man takes out a $5000 20-payment policy at $27.39 per thousand, how much will he have paid when the policy matures (that is, after 20 payments)?

4. What is the premium on a $5000 20-payment policy of such a kind that the rate is $34.20 on a thousand? What are the total premiums for the 20 years?

5. What is the premium on a $7500 20-payment policy of such a kind that the rate is $32.50 on a thousand? What are the total premiums for the 20 years?

6. If a man takes out a $2500 policy in one company at $26.40 per thousand, and a $3500 policy in another at $23.50 per thousand, what are his annual premiums?

7. A young man takes out a $10,000 policy on the 25-payment plan, at $26.36 per thousand. What is the annual premium? How much are the premiums for the 25 years?

8. A man takes out a $5000 policy, paying $26.40 a thousand. He dies just before the eighth annual payment is due. How much does his estate receive above what he paid to the company?

9. A young man took out a $5000 20-payment policy 20 years ago, paying $136.95 a year. How much has he paid in the 20 years? Estimating that the insurance company has had the use of all this amount for the equivalent of 10 years, at 4%, what is the total amount received by the company?

10. A man pays $61.50 annually on a $2500 policy. What is the rate of premium per thousand dollars?

11. At the age of 21 the annual premium on $1000 of term insurance for 10 years is $11, and the dividends decrease each premium, after the first, 15% of this sum. What is the total net cost of $10,000 of such insurance for 10 years?

12. At the age of 21 the annual premium on an ordinary life policy is $19.12 per thousand. What would be the total amount of premiums paid on $5000 in 20 years? What would be the net amount if the company allowed $198 per thousand in dividends during this period?

13. How much more would the policy of Ex. 12 cost if taken out at the age of 32, the premium then being $25.09, and the dividends for 20 years being $237 per thousand?

14. At the age of 21 the annual premium on a 20-payment limited life policy is $28.98, and the dividends allowed by a certain company amount to $222 per thousand in 20 years. What is the net cost of $3000 of such insurance before it is paid up?

Of course the insured also loses the interest on the premiums paid, but this item is not required in the example.

15. How much more would the policy of Ex. 14 cost if taken out at the age of 50, the premium then being $54.65, and the total amount of the dividends for 20 years being $625 per thousand?

16. At the age of 21 the annual premium on a 20-year endowment policy is $48.48, and the dividends amount to $353 per thousand during this period. What is the difference between the amount paid the company (less dividends) and the amount received in return on a $5000 policy?

II. BUSINESS ARITHMETIC COMPLETED. MENSURATION

CORPORATIONS

207. Corporation. The laws of our various states permit a number of persons who wish to go into business together to organize as one body, called a *corporation.*

208. Capital. The members contribute the money to start the business. This is called the *capital.*

209. Shares of stock. The capital is divided into *shares,* usually of $100 each, although sometimes less. Each person who owns one or more shares is a *stockholder,* and receives a *certificate of stock* stating how many shares he owns.

For example, if the capital is $100,000, and the shares are $100 each, there are 1000 shares. If a man has 50 shares, he owns $5000 worth of stock, or 5% of the whole corporation.

210. Directors and officers. The stockholders elect a few of their number to have general direction of the company. These are called *directors,* and they elect the officers.

211. Dividends. The earnings of the company, after paying the expenses and providing for a bank account sufficient for probable needs, are divided into *dividends,* the directors deciding on the *rate of dividend.* These dividends are sent to the stockholders, by means of checks.

For example, if a company with a capital of $100,000 earns $6000 beyond all expenses, it may declare a 6% dividend. Then a man who owns $5000 worth of stock will receive a check for $300.

ORAL EXERCISE

State the dividends to be paid on the following:

1. $3000, 5%. 2. $7000, 4%. 3. $12,000, $5\frac{1}{2}$%.
4. $2500, 3%. 5. $8000, $1\frac{1}{4}$%. 6. $20,000, $2\frac{1}{2}$%.

WRITTEN EXERCISE

1. How many shares in a company with $1,500,000 capital? (In these exercises let one share be $100.)

2. A company divides $50,000 in dividends, and stockholders receive $4 per share. What is the capital?

3. A company with $3,000,000 capital declares a 5% dividend. What does the holder of 100 shares receive?

4. A company with $500,000 capital divides $45,000 in dividends. What does the holder of 30 shares receive?

5. How much does the holder of 30 shares of a certain railway stock receive when a $4\frac{1}{2}$% dividend is declared?

6. A man receives from the treasurer of a certain company $25 every three months, as dividends on his 20 shares of stock. What is the rate of quarterly dividends?

7. A company with a capital of 50 million dollars declares a dividend of $2\frac{3}{4}$% every six months. How much money does it distribute among its stockholders annually?

8. A company with $250,000 capital declares four dividends a year, each of $1\frac{1}{4}$%. What are the total annual dividends? What is the annual income of a man who owns 50 shares?

9. A company with $350,000 capital declares two dividends a year. A stockholder who owns 20 shares receives an annual income of $110. What is the rate of the semiannual dividends?

10. A company with a capital of $250,000 has earned $15,000 this year above all expenses. It decides to save $2500 of this for emergencies, and to divide the rest in dividends. What is the rate?

212. Above and below par. If stock is paying a high rate of dividend, that is, more than can be received from good ordinary investments, people will be so anxious to buy it that they will pay more than $100 for a $100 share. It is then said to be *above par*. If a $100 share can be bought for just $100, the dividends are about on a par with other investments, and the stock is said to be *at par*. If the dividends are low, say 1% or 2%, people will not be anxious to buy the stock, and it will sell *below par*.

213. Buying stock. Stocks are usually bought and sold through a *broker*, generally at a *stock exchange*, a kind of auction room for such business.

214. Brokerage. The broker charges *brokerage* or *commission*, usually $\frac{1}{8}$% *of the par value*, making this charge both for buying and for selling.

215. Meaning of stock quotations. A quotation of $118\frac{3}{4}$ means that $100 worth of stock will cost $118.75 besides $\$\frac{1}{8}$ (or $12\frac{1}{2}$ ct.) brokerage. The buyer then pays $\$118.87\frac{1}{2}$ per share, while the seller, who must also pay his broker, receives $\$118.75 - \$0.12\frac{1}{2} = \$118.62\frac{1}{2}$ per share.

In stock quotations fractions are always expressed in halves, fourths, or eighths. Fractions of a share cannot be bought on the stock exchange. The standard amount bought is 100 shares, although smaller lots are often purchased.

216. Illustrative problems. 1. What is the cost of 10 shares of stock quoted at $137\frac{1}{8}$, brokerage as usual?

One share costs $\$137\frac{1}{8} + \$\frac{1}{8}$ brokerage, or $\$137\frac{1}{4}$.
10 shares cost 10 times $\$137\frac{1}{4}$, or $1372.50.

2. What is the amount received from the sale of 100 shares of stock quoted at $96\frac{5}{8}$, allowing the usual brokerage?

One share brings $\$96\frac{5}{8} - \$\frac{1}{8}$ brokerage, or $\$96\frac{1}{2}$.
100 shares bring 100 times $\$96\frac{1}{2}$, or $9650.

ORAL EXERCISE

State the cost of 10 shares of stock quoted as follows, adding the brokerage in each case:

1. $94\frac{7}{8}$.	2. $73\frac{7}{8}$.	3. $69\frac{7}{8}$.	4. $84\frac{3}{8}$.
5. $114\frac{7}{8}$.	6. $63\frac{3}{8}$.	7. $47\frac{3}{8}$.	8. $82\frac{5}{8}$.
9. $106\frac{3}{8}$.	10. $127\frac{3}{8}$.	11. $75\frac{5}{8}$.	12. $109\frac{7}{8}$.

State the amount received from the sale of 10 shares of stock quoted as follows, allowing for the brokerage:

13. $87\frac{1}{8}$.	14. $69\frac{1}{8}$.	15. $54\frac{1}{8}$.	16. $96\frac{5}{8}$.
17. $134\frac{1}{8}$.	18. $68\frac{5}{8}$.	19. $72\frac{5}{8}$.	20. $107\frac{1}{8}$.

State the cost of the following, including brokerage:

21. 100 shares @ $47\frac{7}{8}$.	22. 100 shares @ $34\frac{7}{8}$.
23. 50 shares @ $119\frac{7}{8}$.	24. 50 shares @ $123\frac{7}{8}$.

WRITTEN EXERCISE

Find the cost of the following, including brokerage:

1. 75 shares @ $127\frac{1}{4}$.	2. 25 shares @ $136\frac{1}{2}$.
3. 60 shares @ $141\frac{3}{4}$.	4. 125 shares @ $62\frac{5}{8}$.
5. 150 shares @ $109\frac{1}{8}$.	6. 250 shares @ $172\frac{3}{8}$.

Find the amount received from the sale of the following:

7. 80 shares @ $134\frac{1}{4}$.	8. 500 shares @ 64.
9. 175 shares @ 169.	10. 250 shares @ 178.
11. 140 shares @ $147\frac{3}{4}$.	12. 120 shares @ $142\frac{1}{2}$.

13. A man paid, including brokerage, $2047.50 for 30 shares of stock. At what price was it quoted?

14. A man received for some stock, less the brokerage, $5195, when it was quoted at 130. How much did he sell?

217. Newspaper quotations. Daily newspapers give the stock quotations. The following examples may be solved by using the newspaper quotations, or the following:

At., Top. & S. F.	$85\frac{7}{8}$	N. Y. Central	$138\frac{1}{8}$
Balt. & Ohio	$95\frac{1}{8}$	Penn. R.R.	$139\frac{1}{2}$
Can. Pac.	$130\frac{1}{4}$	Southern Pac.	$62\frac{3}{4}$
Ill. Cent.	$152\frac{1}{8}$	Union Pac.	$113\frac{1}{4}$
Louisv. & Nash.	$132\frac{3}{8}$	Western Un. Tel.	93
Missouri Pac.	$101\frac{5}{8}$	Wisconsin Cent.	20

In the following examples add $\frac{1}{8}\%$ to the quotation if you are buying, and subtract $\frac{1}{8}\%$ if you are selling, to pay the broker.

218. Illustrative problem. If a man buys 10 shares of Union Pacific when quoted at $113\frac{1}{4}$ and sells it when quoted at $115\frac{3}{4}$, how much does he gain?

1. He buys it at $113\frac{1}{4} + \frac{1}{8}$ (brokerage), or $113\frac{3}{8}$.
2. He sells it at $115\frac{3}{4} - \frac{1}{8}$ (brokerage), or $115\frac{5}{8}$.
3. Therefore he gains $2\frac{1}{4}$, or \$2.25 a share.
4. On 10 shares he gains 10 times \$2.25, or \$22.50.

$$\begin{array}{r} \$115\frac{5}{8} \\ 113\frac{3}{8} \\ \hline \$2\frac{1}{4} \\ 10 \\ \hline \$22.50 \end{array}$$

WRITTEN EXERCISE

1. A man buys 50 shares of Atchison, Topeka & Santa Fé stock as quoted, and sells at $89\frac{1}{8}$. What is his gain?

2. A man bought 50 shares of Wisconsin Central when it was quoted at $25\frac{7}{8}$, and sold it at the above quotation. How much did he lose?

3. If we buy 20 shares of Baltimore & Ohio at 90, and 100 shares of Canadian Pacific at 131, and sell at the above quotations, what do we gain or lose?

4. Find the cost of 50 shares of Pennsylvania R.R., 100 shares of Southern Pacific, 20 shares of Union Pacific, and 10 shares of Western Union Telegraph Co.

Find the gain or loss on buying 50 shares of the following stocks as quoted on the preceding page, and selling at the prices here given:

5. Ill. Cent. 153.
6. Ill. Cent. $149\frac{7}{8}$.
7. Can. Pac. 125.
8. Union Pac. 115.
9. Can. Pac. $130\frac{1}{2}$.
10. Penn. R.R. $137\frac{1}{8}$.
11. Southern Pac. $63\frac{1}{4}$.
12. Union Pac. $114\frac{1}{2}$.
13. N. Y. Central $136\frac{1}{4}$.
14. Penn. R.R. $139\frac{3}{4}$.
15. At., Top. & S.F. $86\frac{1}{2}$.
16. Balt. & Ohio $97\frac{1}{4}$.
17. Louisv. & Nash. 134.
18. Missouri Pac. $100\frac{1}{8}$.
19. Western Un. Tel. $87\frac{1}{2}$.
20. Wisconsin Cent. $18\frac{1}{4}$.

21. A man bought 40 shares of stock and sold it when quoted at $127\frac{1}{8}$. He gained $80 on the deal. What was the quoted price when he bought it?

22. A man bought 150 shares of New York Central stock when quoted as on page 214. He sold it so as to gain $600. What was the quoted price when he sold it?

23. If a dealer buys 100 shares of Illinois Central, 50 shares of Louisville & Nashville, and 25 shares of Missouri Pacific at the quotations on page 214, what does it all cost?

24. A man bought some Illinois Central stock when quoted as on page 214. He sold it when quoted at $150\frac{3}{8}$, and lost $500 by so doing. How many shares did he buy?

25. A man bought some Canadian Pacific stock when quoted as on page 214. He sold it when quoted at $132\frac{1}{2}$, and made $400 by so doing. How many shares did he buy?

26. A man buys 100 shares of New York Central as quoted on page 214, holds it a year, receives 5% in dividends (5% on the par value), and sells it at $141\frac{5}{8}$. Money being worth 4% a year, what does he gain?

219. Bonds. When corporations wish to borrow any considerable amount of money they issue *bonds.*

A written or printed promise to pay a certain sum at a specified time, signed by the maker and bearing his seal. is called a *bond.*

220. Bonds secured by a mortgage. Bonds are secured by a *mortgage,* an agreement that the owners of the bonds may sell the property if the bonds and the interest are not paid.

221. Difference between stocks and bonds. Bonds differ from stocks in this way: the stockholders of a railway are the owners; the bondholders are the ones to whom the road owes money. Bonds bear a fixed rate of interest, but the income of stocks depends on the earnings of the company after the interest on the bonds and the running expenses have been paid.

222. Computing dividends. Stock dividends and bond incomes are always computed on the par value.

223. Illustrative problems. 1. How much is the income on \$5000 worth of 4% bonds?

$$4\% \text{ of } \$5000 = \$200.$$

2. What is the income on 50 shares of railway stock at 5%?

1. 50 shares have a par value of \$5000.
2. 5% of \$5000 = \$250.

3. If I buy a 5% bond at $124\frac{7}{8}$, what is the rate of income on my investment?

1. I pay $\$124\frac{7}{8} + \$\frac{1}{8}$ (brokerage) = \$125 for a \$100 bond.
2. My income is 5% of \$100, or \$5.
3. Since $\quad x\% \text{ of } \$125 = \$5,$

therefore $\quad x\% = \$5 \div \$125 = .04.$

4. Hence my income is 4% on my *investment* (but 5% on the *par value*), not considering the date of maturity.

WRITTEN EXERCISE

1. What will 75 shares of stock cost when quoted at $96\frac{1}{8}$? at $102\frac{5}{8}$? at $68\frac{1}{2}$? at $99\frac{7}{8}$? (Remember the $\frac{1}{8}\%$.)

2. I buy a 6% bond at $149\frac{7}{8}$. What is the rate of income on the money invested? (Remember the $\frac{1}{8}\%$.)

3. Which gives the better income, 5% stock bought when quoted at $139\frac{7}{8}$ or $3\frac{1}{2}\%$ bonds bought when quoted at $99\frac{7}{8}$?

$5 ÷ $140, compared with $3.50 ÷ $100.

4. Which gives the better rate of income, a 6% bond at 120 or a 5% promissory note, leaving out the question of brokerage in this example?

$6 ÷ $120, compared with $5 on $100.

5. A man buys some stock that regularly pays 7% dividends, when quoted at $199\frac{7}{8}$. What is the rate of income on the money invested?

6. When United States 4% bonds are quoted at $116\frac{7}{8}$, what rate of income does a purchaser receive on his investment, not considering the question of the date of maturity of the bonds?

7. If you had some money to invest, which would you prefer, a stock that regularly pays 8%, quoted at $159\frac{7}{8}$, a 5% bond at $109\frac{7}{8}$, or a $5\frac{1}{2}\%$ promissory note, the security being equally good?

Compare $8 return on $160 invested, $5 return on $110 invested, and $5.50 return on $100 invested. The $8 on $160 gives an income of $\frac{8}{160}$ or 5%. What per cent do the others give?

8. If you had some money to invest, which would you prefer, a stock that regularly pays 7%, quoted at $149\frac{7}{8}$, a 4% bond at $89\frac{7}{8}$, or a $4\frac{1}{2}\%$ promissory note, the security being equally good?

Find which pays the better per cent, and how much, not considering the brokerage, in Exs. 9–21:

9. A 5% bond at 121, or a 4% bond at 97.

10. A 5% bond at 115, or a 4% bond at 92.

11. A 5% bond at 130, or a 6% bond at 156.

12. A 3% bond at 102, or a 5% bond at 170.

13. A 6% bond at 130, or a 5% bond at 108.

14. A 6% bond at 140, or a 5% bond at 117.

15. A 3% bond at 92, or a $3\frac{1}{2}$% bond at 107.

16. A $4\frac{1}{2}$% bond at par, or a 5% bond at 111.

17. A 5% bond at 20 above par, or a 4% bond at 4 below par.

This means a 5% bond at 120, or a 4% bond at 96.

18. A 6% bond at 30 above par, or a 4% bond at 14 below par.

19. A 5% bond at 10 above par, or a 4% bond at 12 below par.

20. A stock paying regularly 7%, at 140, or one paying regularly 5%, at 95.

21. A stock paying regularly 6%, at 137, or one paying regularly 5%, at 114.

22. An investor buys 100 shares of stock at $110\frac{1}{4}$, holds the stock 2 years, receives 8 quarterly dividends of $1\frac{1}{2}$%, and sells it at $113\frac{3}{8}$. Is this better than to have invested his money for the same time at 5%? State the two incomes.

23. A man buys 100 shares of stock when quoted at $74\frac{3}{8}$, holds the stock two years, receives 8% in dividends, and then sells it at 72. Compare his gain or loss with that resulting from investing his money in a savings bank so that it earns 3% compound interest.

BUYING PRODUCE

224. Large dealings in produce. A man's work may lead him into large dealings in the products of the soil. These products are bought and sold in large quantities in great auction houses, known by various names, such as the Board of Trade, the Produce Exchange, and the Cotton Exchange.

Teachers should not require these business customs to be memorized but should seek to make the transactions seem real and legitimate.

225. Buying grain. As a rule, not less than 1000 bu. of grain are sold at a time on the Board of Trade at Chicago, the great center for such transactions. The broker charges $\frac{1}{8}$¢ per bushel for buying, and the same for selling. The quotations are by the number of cents to the bushel and always vary by multiples of $\frac{1}{16}$¢ per bushel.

226. Buying pork. As a rule, not less than 250 bbl. of pork are sold on the exchanges. The broker's commissions are $2\frac{1}{2}$¢ per barrel for buying, and the same for selling. The quotations are by the number of dollars to the barrel and always vary by multiples of $2\frac{1}{2}$¢ per barrel.

In all of the following examples remember the broker's commissions as stated above.

WRITTEN EXERCISE

1. What is the cost of 3000 bu. of wheat quoted at $91\frac{7}{8}$¢?

2. If a man buys 5000 bu. of wheat at $89\frac{3}{16}$¢ and sells it at $91\frac{1}{8}$¢, how much does he gain?

3. If a dealer buys 6000 bu. of corn at $47\frac{3}{16}$¢ and sells it at $46\frac{7}{8}$¢, how much does he lose?

4. If a firm of produce dealers buys 750 bbl. of pork at 12.72\frac{1}{2}$, and sells it at $13.20, what is the gain?

227. Buying lard. This is one of the leading products of the central part of our country. As a rule, not less than 250 tierces are sold on the exchanges. A tierce is 340 lb. The broker charges $2\frac{1}{2}$¢ per tierce. The quotations are in dollars per tierce.

228. Buying cotton. This is the great product of the South. As a rule, not less than 100 bales are sold on the exchanges. A bale is considered as 500 lb. The broker charges $5 per 100 bales. The quotations are in cents per pound, and vary by hundredths of a cent.

229. Buying coffee. This is one of the chief imported products dealt in on the exchanges. As a rule, not less than 250 bags are sold. A bag is considered as 130 lb. The broker charges $10 per 250 bags. The quotations are in cents per pound.

In all of the examples remember the broker's commissions.

WRITTEN EXERCISE

1. What will 1250 bags of coffee cost at 8.42¢?
2. What will 750 tierces of lard cost at $7.02?
3. What will 3000 bales of cotton cost at 12.30¢?
4. What will 1000 tierces of lard cost at $$6.97\frac{1}{2}$?
5. What will 2500 bales of cotton cost at 11.75¢?
6. If a man buys 1500 tierces of lard at $$6.87\frac{1}{2}$, and sells it at $$6.82\frac{1}{2}$, how much does he lose?
7. A man buys 250 bags of coffee at 8.25¢, and sells it at 8.60¢. Does he gain or lose, and how much?
8. A cotton factory buys 500 bales at 10.61¢, 300 bales at 10.68¢, and 200 bales at 10.75¢. What is the cost?
9. A dealer buys 1200 bales of cotton at 11.50¢ and sells it at 11.61¢. Does he gain or lose, and how much?

INDUSTRIAL PROBLEMS

230. Problems concerning our various industries. These problems may be used when the industries are being studied in geography. In general they involve percentage.

WRITTEN EXERCISE

1. If when we had a population of 79,374,120 there were 661,451 persons engaged in the cloth industry, one person out of how many was engaged in this work?

2. There were 297,929 wage earners in the cotton industries of our country in a certain year, and they received \$84,909,765 a year. There were also 4713 salaried clerks and officials, receiving in all \$7,121,343 a year. What was the average income of each class per capita?

3. If our factories produce \$297,000,000 worth of woolen goods in a certain year, and the materials cost 61%, the labor 19%, the salaries of officers 2%, and the miscellaneous expenses 6% of this sum, how much is expended for each of these items, and how much is left for profit?

4. Of all the timber cut in this country in a certain year, 21½% was white pine, 9.8% was hemlock, 4.2% was spruce, 27.8% was yellow pine, and 12.8% was oak. We averaged 2,530,000 M ft. that year, B.M. How many feet of each of these woods were cut? How many of all the other woods?

5. This timber was worth, on an average, \$2.18 per M, standing, and \$6.28 when ready for the mill. How much did the cutting and logging add to the value of the total?

6. Out of this increase, \$1.76 per M went for wages, and 94¢ for other expenses, the rest being profit. What was the total profit?

7. If it costs $1.55 a ton to manufacture ice, and the wholesale price is $2 a ton, what is the per cent of profit to the manufacturer?

8. The value of our leather product in a certain year was $204,000,000, an increase of 18$\frac{26}{43}$% in 10 years. What was the value 10 years before?

9. The value of the shoes manufactured in this country in a certain year was $262,700,000, an increase of 18$\frac{1}{3}$% in 10 years. What was it 10 years before?

10. At the opening of the century the amount of money invested in this country in the production of leather was $174,000,000, an increase of 77$\frac{27}{49}$% over the amount 10 years before. What was the amount then?

11. In a certain year we produced 67,890,000 pairs of boots and shoes for men, 21,110,000 for boys, 65,000,000 for women, 42,000,000 for girls and children, 17,000,000 pairs of slippers, and 6,000,000 other pairs. The men's boots and shoes were what per cent of the total?

12. Some of the leather that year went into 2,895,700 dozen pairs of gloves, of which 87% were for men. How many gloves were produced for women and children?

13. In a certain year we had 134,000 wage earners engaged in making cotton cloth, and they received $46,900,000. Ten years before that we had 89,000 wage earners, and they received $33,820,000. Had the wages increased or decreased per capita, and how much?

14. When New York City had a population of 3$\frac{1}{2}$ millions it manufactured 411,000 tons of ice a year, while New Orleans, with a population of 300,000, manufactured 140,000 tons. The population and ice product of New Orleans are what per cents of those of New York? Why does the former manufacture relatively more?

15. In a year when our flour mills produced \$589,950,000 of flour, our meat products were worth $33\frac{1}{3}\%$ more. What was the value of the meat products?

16. In a year when our wool products amounted to \$446,740,000, this was 75% more than the value of our boot and shoe products. What was the value of the latter?

17. In a year when our mines produced 610,815,384 lb. of copper, the total output was worth \$73,297,846.08, which was 20% more per pound than it was worth five years before. What was it worth per pound then?

18. In a year when the wheat crop of the world amounted to 3,124,422,000 bu., we produced 22% of the total, and Canada produced 15% as much as we did. How many bushels did Canada and the United States each produce? How many together?

19. Suppose the grain elevators of Duluth to have a capacity of 34 million bushels. Estimating a bushel as $1\frac{1}{4}$ cu. ft., what is this capacity in cubic feet? (It would be interesting to see how many times the volume of your schoolroom this is.)

20. Of three great flour mills of Minneapolis, if one has a capacity of 28,000 bbl. a day, a second of $3\frac{4}{7}\%$ less, and a third of $33\frac{1}{3}\%$ less than the second, and all should run at their full capacity for 300 working days of a year, how many barrels would they produce?

21. In a certain state it takes 21 lb. of milk to make 1 lb. of butter, while it takes 22 lb. in a second state, 23 lb. in a third, and 24 lb. in a fourth. The first state takes what per cent less than the second? The second what per cent more than the first? The fourth what per cent more than the third? The third what per cent less than the fourth?

231. Our sugar industry. This country produces about 500,000 tons of sugar a year, the greater part being made from sugar cane, and the rest from beets. At present the chief beet-sugar producing states are California, Michigan, Colorado, and Utah. Our cane sugar comes from Louisiana.

WRITTEN EXERCISE

1. If 2457 tons of beet sugar are worth $275,184, what is the average price per ton? What would it be if the price were decreased $2\frac{1}{2}\%$?

2. If 23,241 tons of sugar beets are worth $92,964, what is the average price per ton? What would it be if the price were increased $7\frac{1}{2}\%$?

3. If we manufacture 5325 tons of maple sugar in a year and 300,000 tons of cane sugar, the amount of maple sugar is what per cent of that of cane?

4. In a year when we had 132,441 acres planted to sugar beets, the average yield was 6 tons to the acre. What was the value of the crop at $4.10 a ton?

5. When we produced 300,000 long tons of cane sugar and 195,000 long tons of beet sugar, how many pounds of each did we produce? Each was what per cent of the sum?

6. If the world's production of beet and cane sugar in one year is 9,835,392 tons, and the cane sugar is 839,595 tons more than one third of the total, how much beet sugar is produced?

7. If we have 1970 wage earners employed in making beet sugar, and their wages are $1,091,380, and if we pay 100% more to each salaried person employed in this industry than to the wage earner, and if these salaries amount to $356,776, how many salaried persons are employed?

PROBLEMS IN IRON WORKING

WRITTEN EXERCISE

1. What is the cost of 12′ 8″ of iron rod, $2\frac{1}{8}$ lb. to the foot, at $1\frac{3}{4}$¢ a pound?

2. What is the weight of a steel girder 26′ 6″ long, weighing $42\frac{1}{2}$ lb. to the foot?

3. The wooden pattern from which an iron casting is made weighs $6\frac{1}{4}\%$ as much as the iron. The pattern weighs $45\frac{5}{8}$ lb. How much does the casting weigh?

4. If steel rails weighing 80 lb. to the yard are used between New York and Chicago, a distance of 980 mi., how many tons will be required for a double-track road?

5. An iron tire expands $1\frac{9}{16}\%$ on being heated for shrinking on a wheel. A wooden wheel needs a tire 4′ 8″ in diameter. If the circumference is $3\frac{1}{7}$ times the diameter, how much longer will the tire be when thus heated?

6. In a certain blast furnace the casting machine turns out 20 pigs per minute, averaging in weight 110 lb. each. If this machine runs at this rate for 308 days, 16 hours a day, how many long tons of pig iron will it turn out?

7. A cellar window 3′ × 2′ is to be fitted with an iron grating. This is to be made by constructing an iron frame and then putting in cross pieces of iron rods 3″ between centers, the top and bottom ones being 3″ from the window casing. The iron frame weighs $2\frac{1}{4}$ lb. per running foot, and the rods weigh 13 oz. per running foot. What will the grating cost at $6\frac{1}{4}$¢ a pound, not allowing for corners nor for welding the joints? Draw to scale a plan of the grating.

Railway Problems

WRITTEN EXERCISE

1. If ties are 8″ wide, and are placed 18″ apart, how many ties are there to a mile?

2. We use 90 million new ties a year, averaging 8″ × 6″ × 8′ 6″. How many cubic feet of timber do we use?

3. The modern coal car has a capacity of 100,000 lb. At 35 cu. ft. to the ton (of 2000 lb.), what is the volume?

4. The Pikes Peak railway makes an ascent of 7552 ft. in a length of $8\frac{3}{4}$ mi. What is the average gradient (ratio of ascent to length)?

5. The rails are usually 30′ long and often weigh 80 lb. to the yard. If a man carries 160 lb., how many men will it take to carry a rail?

6. The standard gauge in America and England is 4′ $8\frac{1}{2}$″. Express this in meters, as used in most European countries, the meter being 39.37 in.

7. The standard American gauge (see Ex. 6) is $\frac{1}{5}$ more than the gauge of a certain mountain road. What is the gauge of the latter?

8. If the cost of maintaining the single track of a certain railway averages $846 a mile, what is the cost of maintaining its 450 mi. of double track?

9. In a year when our country had 1,189,000 men employed in the railroad business, 4% were employed as engineers. The average wages for engineers that year were $3.84 a day, which was 20% more than the average wages for conductors. There were 35,070 conductors employed that year. How much more was the total paid that year for engineers' wages than for conductors'?

232. The great ocean steamers. Some idea of the great shipping industry may be obtained by thinking that in one year 857,000 immigrants (steerage or third-cabin passengers) came to our country, besides the first- and second-cabin passengers. A large ocean steamer has a capacity of over 2,000,000 cu. ft., perhaps a thousand times the capacity of your recitation room.

WRITTEN EXERCISE

1. One of the fastest day's runs recorded for a steamer is 601 knots. How many statute miles is this?

2. A certain ocean steamer can carry 3192 persons, including the crew. The crew number 12% as many as the passengers. How many are there of each?

3. Prior to 1860 the best steamship record between New York and Queenstown was 9 da. 2 hr. When it was reduced to 5 da. 7 hr., what was the per cent of reduction?

4. To build one of the large Atlantic steamers, 1400 plates of steel were used in the hull alone. They weighed 4 long tons each. How many tons did they all weigh? How many pounds?

5. A steamer carries 350 first-cabin passengers, 48% as many in the second cabin, and 400% as many in the third cabin (steerage) as in the other two together. How many passengers does it carry?

6. A 20,000-ton boat carries 12 times as much freight as the old-style ocean steamers, makes 25% better time, and the expenses are only 4 times as much. In one year such a modern boat will do how many times the work of the old kind? At the same cost, it will carry how many times as much freight?

233. Problems in meteorology. Problems concerning rainfall, temperature, and the general state of the weather are called problems in *meteorology.* They are so related to agriculture as to have a place among industrial problems.

WRITTEN EXERCISE

1. What is the weight of an inch of rainfall upon an acre, taking the weight of 1 cu. ft. of water as 1000 oz.? What is it on a 200-acre farm? Answer in tons.

2. The lowest daily temperatures registered during a certain week in January, in Buffalo, were 7°, 3.6°, − 9° (9° below zero), 6°, − 3°, 0°, 8°. What is the average of these readings?

3. In our common Fahrenheit (F.) thermometer the freezing point of water, 32°, is the 0° in the Centigrade (C.) thermometer, and the boiling point, 212°, is the 100° in the Centigrade. Express 4° C. in Fahrenheit, and 62° F. in Centigrade. Draw each thermometer to scale.

4. A barometer registered 29.024 in. on Monday. In the next 4 days it rose 0.135 in., 0.044 in., 0.095 in., and 0.573 in. On Saturday and Sunday it fell 0.021 in. and 0.417 in. What was the Friday reading? the Sunday reading? What were the weather indications on Friday? on Sunday?

A rising barometer indicates fair weather. When the mercury falls rapidly a storm is indicated.

5. The mean (average) annual rainfall at Mobile is 62.2 in.; at Sacramento, 20.9 in.; at Denver, 14.5 in.; at Indianapolis, 43 in.; at Des Moines, 33.1 in.; at St. Paul, 27.5 in.; at St. Louis, 41.1 in.; and at Portland, Oregon, 46.8 in. Taking the weight of 1 cu. ft. of water as 62.5 lb., what is the weight of water annually falling on a square mile in each of these cities? Answer in tons.

6. If light travels 186,000 mi. per second, and it takes it $8.158\frac{1}{3}$ sec. to come from the sun to the earth, what is the distance traveled?

7. Water is composed of two gases, oxygen and hydrogen, 88.89% by weight being oxygen. What is the weight of the hydrogen in a cubic foot of water?

8. The air is composed of two gases, oxygen and nitrogen. In every cubic foot of air there are 345.6 cu. in. of oxygen. What per cent of the volume of the air is nitrogen?

9. A certain cirrus cloud is observed to be 6 mi. above the earth, or 240% higher than a certain rain cloud observed a few hours before. How high was the rain cloud?

10. At the temperature when sound travels 1120 ft. per second, what is the distance of a thunder cloud in which lightning is seen $17\frac{2}{5}$ sec. before the thunder is heard?

11. If a large drop of rain falls at the rate of 20 ft. per second, and a small one, blown by the wind, only 25% as fast, how long will it take the small one to reach the earth from a cloud $2\frac{1}{2}$ mi. high?

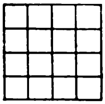
12. The wind pressure in a hurricane has been known to be as great as 49.2 lb. per square foot. In such a storm how many tons pressure on the side of a large office building 104 ft. long and 308 ft. high?

13. In a great storm the velocity of the wind often reaches 88 ft. per second. What is then its velocity per minute? In a hurricane it has been known to be $66\frac{2}{3}$% greater. What is then its velocity per hour?

14. When the mercury in the barometer is at 30 in., the pressure of air on every square inch of surface is 15 lb. What is the pressure on a pane of glass 2 ft. by 3 ft., when the increased air pressure forces the mercury up to 31 in.? Why does the glass not break?

POWERS AND ROOTS

234. Square numbers and square roots. If a square has a side 4 units, it has an area 16 square units. Therefore 16 is called the *square* of 4, and 4 the *square root* of 16.



235. Square roots of areas. Therefore, considering the *abstract* numbers representing the sides and area,

The side of a square is the square root of its area.

236. Writing squares and roots. The square of 4 is written 4^2; the square root of 16 is written $\sqrt{16}$.

237. Perfect squares. A number like 16 is a *perfect square*, but 10 is not a perfect square. We speak, however, of $\sqrt{10} = 3.16+$, because 3.16^2 nearly equals 10.

238. Square roots of perfect squares. Square roots of perfect squares may often be found by factoring.

$$\begin{array}{r} 3\,\underline{)441} \\ 3\,\underline{)147} \\ 7\,\underline{)49} \\ 7 \end{array}$$

For example, $\sqrt{441} = \sqrt{3 \times 3 \times 7 \times 7}$
$= \sqrt{21 \times 21} = 21.$

ORAL EXERCISE

State the square roots of the numbers in Exs. 1–8:

1. 64.
2. 9.
3. 81.
4. 49.
5. 121.
6. 144.
7. 1600.
8. 4900.

What are the sides of squares whose areas are as follows?

9. 64 sq. in.
10. 49 sq. ft.
11. 1.44 sq. in.
12. 0.25 sq. ft.
13. $\frac{1}{144}$ sq. yd.
14. $\frac{1}{121}$ sq. in.

State the perimeters of squares whose areas are:

15. 1.21 sq. ft.
16. 0.49 sq. ft.
17. 169 sq. in.

WRITTEN EXERCISE

By factoring, find the square roots of the following:

1. 625.	2. 324.	3. 484.	4. 729.
5. 576.	6. 2304.	7. 1296.	8. 1089.
9. 65.61.	10. 12.25.	11. 40.96.	12. 14,641.

Find the sides of squares whose areas are as follows:

13. 4.84 sq. in.	14. 1.96 sq. ft.	15. 2.25 sq. ft.
16. 4.41 sq. in.	17. 4356 sq. in.	18. 15,625 sq. in.
19. 5929 sq. yd.	20. 10.89 sq. yd.	21. 0.1225 sq. ft.

Find the perimeters of squares whose areas are as follows:

22. 6561 sq. ft.	23. 12,100 sq. in.	24. 11,025 sq. ft.
25. 19,600 sq. ft.	26. 146.41 sq. in.	27. 20,736 sq. in.
28. 16,384 sq. in.	29. 16,900 sq. ft.	30. 129,600 sq. in.

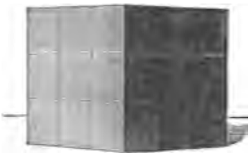
31. From the corner of a square piece of land containing 576 sq. rd. a small square lot containing 64 sq. rd. is cut out. Draw the plan of the lots and find the perimeter of each.

32. A square lot has an area of 169 sq. rd. How far is it around the lot? How far is it around a lot of four times this area? The second perimeter is how many times the first? Plot each lot to a scale.

33. A man has two adjacent building lots fronting on the street, each lot being square. The area of the two together is 89 sq. rd., that of the larger being 64 sq. rd. What is the frontage of the lots?

34. A square lot has an area of 289 sq. rd. How far is it around the lot? (Try the prime numbers between 10 and 20.) How far is it around a lot of nine times this area? The second perimeter is how many times the first?

239. Cube numbers and cube roots. If a cube has an edge 3 units, it has a volume 27 cubic units. Therefore 27 is called the *cube* of 3, and 3 the *cube root* of 27.



240. Cube roots of volumes. Therefore, considering the *abstract* numbers representing the edges and volume,

The edge of a cube is the cube root of its volume.

241. Writing cubes and roots. The cube of 3 is written 3^3; the cube root of 27 is written $\sqrt[3]{27}$.

242. Powers. Squares and cubes are called *powers.* We also have higher powers, like the fourth, fifth, and so on. Raising to powers is sometimes called *involution;* extracting roots, *evolution.*

243. Cube roots of perfect cubes. Cube roots of perfect cubes may often be found by factoring.

$$\begin{array}{r|l} 2 & 216 \\ 2 & 108 \\ 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

For example,

$$\begin{aligned} \sqrt[3]{216} &= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3} \\ &= \sqrt[3]{(2 \times 3) \times (2 \times 3) \times (2 \times 3)} \\ &= \sqrt[3]{6 \times 6 \times 6} = 6. \end{aligned}$$

ORAL EXERCISE

1. $\sqrt[3]{8}$.
2. $\sqrt[3]{125}$.
3. $\sqrt[3]{64}$.
4. $\sqrt[3]{1000}$.
5. $\sqrt[3]{27}$.
6. $\sqrt[3]{0.027}$.
7. $\sqrt[3]{0.125}$.
8. $\sqrt[3]{0.064}$.

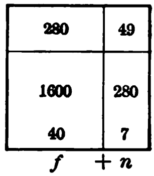
WRITTEN EXERCISE

1. $\sqrt[3]{1331}$.
2. $\sqrt[3]{729}$.
3. $\sqrt[3]{512}$.
4. $\sqrt[3]{1728}$.
5. $\sqrt[3]{15625}$.
6. $\sqrt[3]{2744}$.
7. $\sqrt[3]{4096}$.
8. $\sqrt[3]{5832}$.
9. What is the edge of a cube of volume 10,648 cu. in.?

244. Letters used to represent numbers. If we have two letters, like x and y, the product of their values is indicated by xy. If $x=5$ and $y=7$, then $xy=5\times 7=35$, $2\,xy=70$, $x^2=25$, and $y^2=49$.

This is all the work with letters necessary for the understanding of square root. If not already known, a few minutes of drill upon similar work will suffice.

245. Square on the sum of two lines. If we have two lines, f and n, and construct a square on their sum, we see by this figure that there are two squares and two rectangles, f^2, n^2, and fn, fn. Therefore



The square of the sum of two numbers equals the square of the first, plus twice the product of the first and second, plus the square of the second.

That is, $$(f+n)^2=f^2+2\,fn+n^2.$$

246. Illustrative problem. What is the square of 47?

$$\begin{aligned}47^2&=(40+7)^2=40^2+2\times 40\times 7+7^2\\&=1600+560+49=2209.\end{aligned}$$

ORAL EXERCISE

Square as in § 245:

1. 15.	2. 31.	3. 52.	4. 22.	5. 25.
6. 14.	7. 41.	8. 51.	9. 61.	10. 72.
11. $a+b$.	12. $m+n$.	13. $f+n$.	14. $a+x$.	
15. $x+y$.	16. $t+u$.	17. $2+y$.	18. $4+x$.	

State the square root of:

19. $c^2+2\,cd+d^2$.

20. $30^2+2\times 30\times 7+7^2$.

21. $t^2+2\,tu+u^2$.

22 $20^2+2\times 20\times 6+6^2$.

247. How to find a square root. Find $\sqrt{289}$.

Because 289 is not so readily factored as the numbers we have considered, we take another method of finding the square root.

Imagine a square containing 289 square units (Fig I).

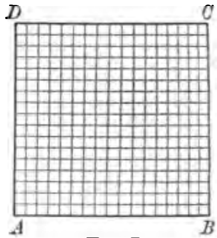


FIG. I

The greatest square of *tens* in 289 (Fig. I) is 100 (Fig. II), for $20^2 = 400$, and this is greater than 289.

Taking away 10^2, or 100, as marked off in Fig. III, we have left 189, as shown in Fig. IV. We may now lay Fig. IV lengthwise as in Fig. V.

Now we know that Fig. V has an area of 189 square units and a length a little over $(10 + 10)$ units (because of the little square). Therefore if we divide 189 by $10 + 10$, or 20, we shall find nearly the width PB, or the side SC of the little square $SCTQ$. Dividing, $189 \div 20 = 9$ (nearly). But this would make the total length RP (Fig. V) equal $10 + 9 + 10 = 29$, and the area would be $9 \times 29 = 261$, which is greater than 189. Therefore 9 is too large. In the same way, 8 is found to be too large, for it makes the area 224. But 7 is found to be just right, for $7 \times (10 + 7 + 10) = 7 \times 27 = 189$.

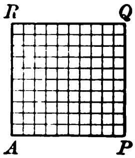


FIG. II

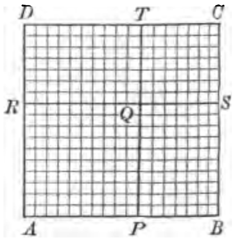


FIG. III

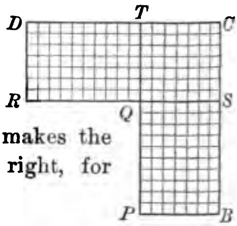


FIG. IV

Therefore $\sqrt{289} = 17$.

Again we see the truth of § 245, for $10^2 + 2 \times 10 \times 7 + 7^2 = 100 + 140 + 49 = 289$. That is, Fig. II + Fig. IV = Fig. I or Fig. III.

The check (or proof) for square root is, of course, the squaring of the result.

Here $17^2 = 289$.

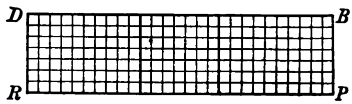


FIG. V

248. This explanation may also be given as follows:

If we let $f =$ the *f*ound part of the root,
and $n =$ the *n*ext figure of the root,
then $(f + n)^2 = f^2 + 2fn + n^2$. (§ 245)

Therefore if we take away f^2 (Fig. II), we shall have $2fn + n^2$ (Fig. IV, where each oblong is $f \times n$, and the small square is n^2).

If we divide this by $2f$, we shall find nearly n, as in Fig. V, where we divided by 2×10 to find 7.

Since the entire explanation of square root depends on this fact, teachers are advised to see that this is clearly understood, both from the figure and from the formula, before proceeding.

$$
\begin{array}{rrcll}
 & \underline{17} & & & \\
 & 289 & \text{contains} & f^2 + 2fn + n^2 & \text{(Fig. I)} \\
f^2 = & \underline{100} & & & \text{(Fig. II)} \\
2f = 20 & 189 & \text{“} & 2fn + n^2 & \text{(Fig. IV)} \\
\underline{2f + n = 27} & \underline{189} & = & \text{“} \quad \text{“} &
\end{array}
$$

The greatest square of *tens* in 289 is 100, for 20^2 is greater than 289.

This is f^2, and therefore $f = 10$, for 10 is evidently the square root of 100.

189 contains $2fn + n^2$, because f^2 has been subtracted from a number containing $f^2 + 2fn + n^2$.

Dividing this by $2f$ we approximate n, and $n = 7$. I.e., $189 \div 20 = 7$ (nearly), for 8 and 9 would be found to be too large, as on p. 234.

But $2f + n$ multiplied by n equals $2fn + n^2$ (Fig. V), thus completing the square.

Therefore $289 = f^2 + 2fn + n^2 = 10^2 + 2 \times 10 \times 7 + 7^2 = 17^2$.

249. The square root of common fractions. It is easy to find the square root of common fractions. For since $(\frac{2}{3})^2 = \frac{4}{9}$, therefore $\frac{2}{3} = \sqrt{\frac{4}{9}}$. That is,

To extract the square root of a common fraction take the square root of each term.

WRITTEN EXERCISE

1. $\sqrt{3249}$.	2. $\sqrt{3721}$.	3. $\sqrt{3969}$.	4. $\sqrt{5041}$.
5. $\sqrt{1681}$.	6. $\sqrt{2209}$.	7. $\sqrt{2809}$.	8. $\sqrt{3481}$.
9. $\sqrt{4489}$.	10. $\sqrt{5329}$.	11. $\sqrt{5929}$.	12. $\sqrt{6241}$.

Find the sides of the squares, given the areas:

13. 6724 sq. ft.	14. 9409 sq. ft.	15. 7225 sq. in.
16. 7569 sq. ft.	17. 7921 sq. rd.	18. 8281 sq. in.
19. 9025 sq. ft.	20. 6889 sq. yd.	21. 9801 sq. yd.

Find the value of the following:

22. $\sqrt{\frac{144}{529}}$.	23. $\sqrt{\frac{961}{1024}}$.	24. $\sqrt{\frac{1089}{1156}}$.	25. $\sqrt{\frac{1936}{5041}}$.
26. $\sqrt{\frac{3025}{3249}}$.	27. $\sqrt{\frac{361}{8281}}$.	28. $\sqrt{\frac{441}{9801}}$.	29. $\sqrt{\frac{289}{9409}}$.

250. Number of figures in the square root. It is easy to tell in advance the number of figures in the square root of a perfect square. For

The square of units has 2 figures or 1 figure, since

$$9^2 = 81, \ 1^2 = 1.$$

The square of a number of 2 integral places has 4 or 3 integral places, since

$$99^2 = 9801, \ 10^2 = 100.$$

The square of a number of 3 integral places has 6 or 5 integral places, and so on, since

$$999^2 = 998{,}001, \ 100^2 = 10{,}000.$$

251. *If a square number be separated into periods of two figures each, beginning at the decimal point, the number of periods will equal the number of figures in the root.*

Thus, $\sqrt{43'69'21}$ has 3 integral places;

$\sqrt{1.23'21}$ " 1 " and 2 decimal places.

252. Square root with decimals. Find $\sqrt{151.29}$.

The greatest square of 10's in 151.29 is 100. This is f^2, and therefore $f = 10$.

Then 51.29 contains $2fn + n^2$. (Why is this?)

Dividing by $2f$ (20), we find $n = 2$.

We have now found $f + n = 12$, the square being $100 + 44 = 144$.

	12.3
	1'51.29
	100.
$2f = 20$	51.29
$2f + n = 22$	44.
$2f = 24$	7.29
$2f + n = 24.3$	7.29

Since 12 has been *found*, let us call this f (for *f*ound). Of course, this is not the same as the first number found; it is larger, because we have found more.

7.29 contains $2fn + n^2$, because we have subtracted $f^2 = 144$.

Dividing by $2f$ (24), we find $n = 0.3$.

$2f + n$ multiplied by n equals $2fn + n^2$, the rest of the square.

253. From the examples solved we see that the following are the steps in extracting square root:

1. *Separate into periods of two figures each, beginning at the decimal point.* (See § 251.)

2. *Find the greatest square in the left-hand period and subtract it, bringing down the next period.*

That is, find f^2, the greatest square of 10's, 100's, or 1000's, etc., and subtract it, leaving as a remainder $2fn + n^2$.

3. *Divide the remainder by twice the part already found.*

That is, divide $2fn + n^2$ by $2f$ to find approximately n.

4. *Add the number thus found to this divisor, and multiply by the same number.*

That is, add n to $2f$, and multiply by n, obtaining $2fn + n^2$.

5. *Subtract this result, bring down the next period, and proceed as in 3 and 4.*

That is, having now subtracted a new f^2, proceed as before.

WRITTEN EXERCISE

1. $\sqrt{3249}$.	2. $\sqrt{3721}$.	3. $\sqrt{3969}$.	4. $\sqrt{5041}$.
5. $\sqrt{1681}$.	6. $\sqrt{2209}$.	7. $\sqrt{2809}$.	8. $\sqrt{3481}$.
9. $\sqrt{4489}$.	10. $\sqrt{5329}$.	11. $\sqrt{5929}$.	12. $\sqrt{6241}$.

Find the sides of the squares, given the areas:

13. 6724 sq. ft.	14. 9409 sq. ft.	15. 7225 sq. in.
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The square of a number of 2 integral places has 4 or 3 integral places, since

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The square of a number of 3 integral places has 6 or 5 integral places, and so on, since

$$999^2 = 998{,}001,\ 100^2 = 10{,}000.$$

251. *If a square number be separated into periods of two figures each, beginning at the decimal point, the number of periods will equal the number of figures in the root.*

Thus, $\sqrt{43'69'21}$ has 3 integral places;

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The greatest square of 10's in 151.29 is 100. This is f^2, and therefore $f = 10$.

Then 51.29 contains $2fn + n^2$. (Why is this?)

Dividing by $2f$ (20), we find $n = 2$.

We have now found $f + n = 12$, the square being $100 + 44 = 144$.

Since 12 has been *found*, let us call this f (for *f*ound). Of course, this is not the same as the first number found; it is larger, because we have found more.

	12.3
	1'51.29
	100.
$2f = 20$	51.29
$2f + n = 22$	44.
$2f = 24$	7.29
$2f + n = 24.3$	7.29

7.29 contains $2fn + n^2$, because we have subtracted $f^2 = 144$.

Dividing by $2f$ (24), we find $n = 0.3$.

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3. *Divide the remainder by twice the part already found.*

That is, divide $2fn + n^2$ by $2f$ to find approximately n.

4. *Add the number thus found to this divisor, and multiply by the same number.*

That is, add n to $2f$, and multiply by n, obtaining $2fn + n^2$.

5. *Subtract this result, bring down the next period, and proceed as in 3 and 4.*

That is, having now subtracted a new f^2, proceed as before.

WRITTEN EXERCISE

1. $\sqrt{3249}$.	2. $\sqrt{3721}$.	3. $\sqrt{3969}$.	4. $\sqrt{5041}$.
5. $\sqrt{1681}$.	6. $\sqrt{2209}$.	7. $\sqrt{2809}$.	8. $\sqrt{3481}$.
9. $\sqrt{4489}$.	10. $\sqrt{5329}$.	11. $\sqrt{5929}$.	12. $\sqrt{6241}$.

Find the sides of the squares, given the areas:

13. 6724 sq. ft.	14. 9409 sq. ft.	15. 7225 sq. in.
16. 7569 sq. ft.	17. 7921 sq. rd.	18. 8281 sq. in.
19. 9025 sq. ft.	20. 6889 sq. yd.	21. 9801 sq. yd.

Find the value of the following:

22. $\sqrt{\frac{144}{529}}$.	23. $\sqrt{\frac{961}{1024}}$.	24. $\sqrt{\frac{1089}{1156}}$.	25. $\sqrt{\frac{1936}{5041}}$.
26. $\sqrt{\frac{3025}{3249}}$.	27. $\sqrt{\frac{361}{8281}}$.	28. $\sqrt{\frac{441}{9801}}$.	29. $\sqrt{\frac{289}{9409}}$.

250. Number of figures in the square root. It is easy to tell in advance the number of figures in the square root of a perfect square. For

The square of units has 2 figures or 1 figure, since

$$9^2 = 81,\ 1^2 = 1.$$

The square of a number of 2 integral places has 4 or 3 integral places, since

$$99^2 = 9801,\ 10^2 = 100.$$

The square of a number of 3 integral places has 6 or 5 integral places, and so on, since

$$999^2 = 998{,}001,\ 100^2 = 10{,}000.$$

251. *If a square number be separated into periods of two figures each, beginning at the decimal point, the number of periods will equal the number of figures in the root.*

Thus, $\sqrt{43'69'21}$ has 3 integral places;

$\sqrt{1.23'21}$ " 1 " and 2 decimal places.

252. Square root with decimals. Find $\sqrt{151.29}$.

The greatest square of 10's in 151.29 is 100. This is f^2, and therefore $f = 10$.

Then 51.29 contains $2fn + n^2$. (Why is this?)

Dividing by $2f$ (20), we find $n = 2$.

We have now found $f + n = 12$, the square being $100 + 44 = 144$.

Since 12 has been *found*, let us call this f (for *f*ound). Of course, this is not the same as the first number found; it is larger, because we have found more.

	12.3
	1'51.29
	100.
$2f = 20$	51.29
$2f + n = 22$	44.
$2f = 24$	7.29
$2f + n = 24.3$	7.29

7.29 contains $2fn + n^2$, because we have subtracted $f^2 = 144$.

Dividing by $2f$ (24), we find $n = 0.3$.

$2f + n$ multiplied by n equals $2fn + n^2$, the rest of the square.

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1. *Separate into periods of two figures each, beginning at the decimal point.* (See § 251.)

2. *Find the greatest square in the left-hand period and subtract it, bringing down the next period.*

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That is, divide $2fn + n^2$ by $2f$ to find approximately n.

4. *Add the number thus found to this divisor, and multiply by the same number.*

That is, add n to $2f$, and multiply by n, obtaining $2fn + n^2$.

5. *Subtract this result, bring down the next period, and proceed as in 3 and 4.*

That is, having now subtracted a new f^2, proceed as before.

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9. $\sqrt{4489}$.	10. $\sqrt{5329}$.	11. $\sqrt{5929}$.	12. $\sqrt{6241}$.

Find the sides of the squares, given the areas:

13. 6724 sq. ft.	14. 9409 sq. ft.	15. 7225 sq. in.
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WRITTEN EXERCISE

Extract the square roots in Exs. 1–17:

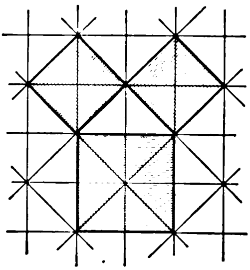
1. 12,321.	2. 54,756.	3. 110.25.
4. 8046.09.	5. 19.4481.	6. 0.2809.
7. 1176.49.	8. 82.2649.	9. 63,001.
10. 21,224,449.	11. 49,112,064.	12. 96,275,344.

In Exs. 13–18 carry the root to two decimal places only.

13. 2. 14. 5. 15. 7. 16. 8. 17. 11.

18. Find the side of a square containing 1 acre (160 sq. rd.).

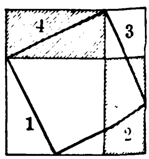
254. Hypotenuse. In a right-angled triangle the side opposite the right angle is called the *hypotenuse.*



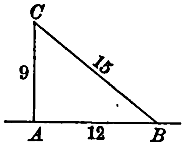
255. Square on the hypotenuse. If a floor is made up of triangular tiles like this, it is easy to mark out a right-angled triangle. In the figure it is seen that the square on the hypotenuse contains 8 small triangles, while each square on a side contains 4 such triangles. Hence

The square on the hypotenuse equals the sum of the squares on the other two sides.

256. This is true for any right-angled triangle. We see that if the 4 triangles, $1 + 2 + 3 + 4$, are taken away from this figure, there remains the square on the hypotenuse. But if we take away the 2 shaded rectangles, which equal the 4 triangles, there remain the squares on the two sides. Therefore the square on the hypotenuse must equal the sum of these two squares.



257. Illustrative problem. If $AB = 12$, and $AC = 9$, how long is BC?



Since $AB^2 + AC^2 = BC^2$,

therefore $12^2 + 9^2 = BC^2$,

or $144 + 81 = 225 = BC^2$,

and $\sqrt{225} = BC$. Therefore $BC = 15$.

WRITTEN EXERCISE

1. How long is the diagonal of a hall 51 ft. by 68 ft.?

2. How long is the diagonal of a square containing 4 sq. ft.? (Two decimals.)

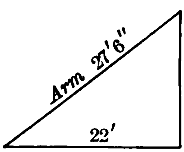
3. The two sides of a right-angled triangle are 20 in. and 30 in. Find the hypotenuse. (Two decimals.)

4. The two sides of a right-angled triangle are 57 in. and 76 in. Find the hypotenuse.

5. What is the direct distance from the cornice of a 100-ft. building to a spot 75 ft. from the foot?

6. Find the length of the hypotenuse when the sides are 321 in. and 428 in.; 40 in. and 75 in.; 72 ft. and 135 ft.

7. A telegraph pole is set perpendicular to the ground, and a wire is fastened to it 18 ft. from the ground, and then to a stake 13 ft. 6 in. from the foot of the pole, so as to hold it in place. How long is the wire?



8. A derrick for hoisting coal has its arm 27 ft. 6 in. long. It swings over an opening 22 ft. from the base of the arm. How far is the top above the opening?

Reversing the procedure in § 255, the square on either side equals the difference of what squares?

Find to two decimal places the hypotenuse of each of the right-angled triangles of which the sides are here given:

9. 35 ft., 26 ft.
10. 81 ft., 35 ft.
11. 10 rd., 13 rd.
12. $42\frac{1}{2}$ ft., $63\frac{1}{4}$ ft.
13. 4.5 in., 7.2 in.
14. 6.25 in., 7.5 in.

It is a good exercise to make up problems like those in Exs. 15–19.

15. A room is 16 ft. long, 12 ft. wide, and 9 ft. high. How far is it from an upper corner diagonally through the room to the opposite lower corner? (Two decimals.)

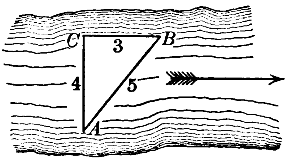
First draw the picture. Then find the hypotenuse on one wall. Then find the hypotenuse required.

16. How long is the diagonal of a cube whose volume is 8 cu. in.? (Two decimals.)

First find the edge. Then proceed as in Ex. 15.

17. A school flag pole is broken by the wind 16 ft. from the ground. The two pieces hold together, and the top of the pole touches the ground 30 ft. from the base. Find the length of the pole.

18. If I start to row directly across a stream, in the direction AC, at the rate of 4 mi. an hour, and if the stream carries me in the direction CB at the rate of 3 mi. an hour, my course will really be AB, the result of these two motions. Suppose I row at the rate of 4.5 mi. per hour, and the stream flows 6 mi., what is my rate of progress?



19. Suppose I walk across the deck of a steamer at the rate of 4 mi. per hour, while the boat moves at the rate of 8 mi. per hour, at what rate am I moving? Answer to two decimal places. (Draw a plan to scale.)

258. Helpful approximations. Because of their frequent use, it is helpful to learn the following approximations:

$\sqrt{2} = 1.414$ $\sqrt{3} = 1.732$ $\sqrt{5} = 2.236$ $\sqrt{10} = 3.162$

259. Illustrative problem. What is the diagonal of a square whose side is 7 in.?

1. The square on the diagonal (hypotenuse) is 49 sq. in. + 49 sq. in. = 2×49 sq. in.
2. Therefore the diagonal = $\sqrt{2 \times 49}$ in.
3. But $\sqrt{2 \times 49} = \sqrt{2} \times 7$
4. $= 1.414 \times 7 = 9.898.$

WRITTEN EXERCISE

1. From § 259 write out a rule for finding the diagonal of a square by multiplying its side by a certain number.

Find the diagonals of squares with sides here given:

2. 19.2 in. 3. 32.8 in. 4. 683 ft. 5. 750 rd.

6. If the diamond of a baseball field is a square 90 ft. on a side, how far is it from the first base directly across to the third?

7. A gate 3 ft. high and 6 ft. wide is to be braced by a stick fastened diagonally across it. How long is the stick?

Notice that $\sqrt{45} = \sqrt{9 \times 5} = 3 \times \sqrt{5}$.

8. How far is it from one lower corner of a room directly to the opposite upper corner, the dimensions of the floor being $12' \times 16'$ and the room being 8 ft. high?

Find the diagonals of squares with areas here given:

9. 81 sq. ft. 10. 256 sq. ft. 11. 6.25 sq. ft.

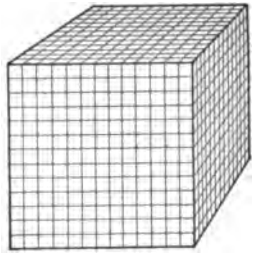
Find the sides of squares with diagonals here given:

12. 14.14 in. 13. 4.242 in. 14. 7.07 ft.

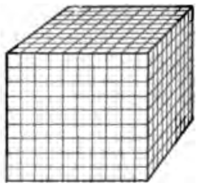
260. How to find a cube root. Find $\sqrt[3]{2197}$.

This subject may be omitted without interfering with the work that follows.

Because 2197 is not so readily factored as the numbers we have considered, we take another method of finding the cube root.



A

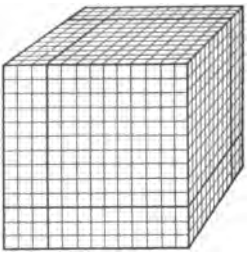


B

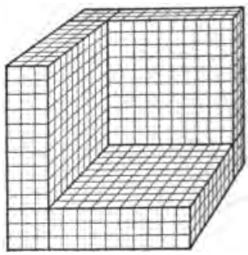
	13
	2197
	1000
300, 99, 399	1197
	1197

Imagine a cube containing 2197 cubic units, as in A, where of course we cannot see all of the small cubes.

The greatest cube of *tens* in 2197, or A, is 1000 (B), for $20^3 = 8000$, and this is greater than 2197.



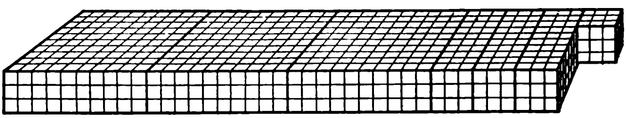
C



D

Taking away 10^3, or 1000, as marked off in C, we have left 1197, as shown in D.

We may now lay D lengthwise, as in E.



E

Now we know that the solids in E have a volume of 1197 cubic units, and that the top surface of the three larger blocks has an area of 3×10^2 units, or 300 square units. The 300 represents nearly the whole top area, the rest being that of the three oblong blocks and the small cube. Therefore if we divide 1197 by 300 (sometimes called the *trial divisor*), we shall have approximately the thickness, or the edge of the small cube. Dividing, $1197 \div 300 = 3$ (nearly).

Now if 3 is the thickness, the area of the top of each oblong piece is 3×10, and the area of the three is $3 \times 3 \times 10$, or 90. The area of the top face of the small cube is 3^2, or 9, making the area of the top of these four pieces $90 + 9$, or 99. Adding the area of the top of the three square blocks, we have $300 + 99$, or 399, as the total top area.

Multiplying 399, the number of square units in the top area, by 3, the number of units of thickness, we have 1197, the number of cubic units, which just completes the cube, D applied to B just making C or A.

261. The cube of $f + n$. From the illustrations in § 260 we see that if we call the first part of the root f and the next part n, the cube of $f + n$ (A) equals the cube of f (B) plus 3 blocks, each with a volume of f^2n, plus 3 other blocks (the oblong ones), each with a volume of fn^2, plus the small cube with a volume n^3. This shows that

$$(f + n)^3 = f^3 + 3f^2n + 3fn^2 + n^3.$$

This we may also see by multiplying $(f + n)^2$ by $f + n$, or $(10 + 3)^2$, which is $10^2 + 2 \times 10 \times 3 + 3^2$, by $10 + 3$, as here shown.

$$\begin{array}{rl}
10^2 + 2 \times 10 \times 3 + 3^2 & \\
10 + 3 & \\
\hline
10^2 \times 3 + 2 \times 10 \times 3^2 + 3^3 & \text{multiplying by } 3 \\
10^3 + 2 \times 10^2 \times 3 + 10 \times 3^2 \qquad & \text{“} \quad \text{“} \ 10 \\
\hline
10^3 + 3 \times 10^2 \times 3 + 3 \times 10 \times 3^2 + 3^3 & = f^3 + 3f^2n + 3fn^2 + n^3
\end{array}$$

WRITTEN EXERCISE

Find the cube root of each of the following:

1. 4913.	2. 6859.	3. 9261.	4. 5832.
5. 13,824.	6. 19,683.	7. 29,791.	8. 46,656.
9. 132,651.	10. 157,464.	11. 226,981.	12. 300,763.
13. 551,368.	14. 753,571.	15. 884,736.	16. 941,192.
17. 592,704.	18. 778,688.	19. 857,375.	20. 970,299.

By the method of § 261, *find the cube of each of the following, proving the result by multiplication:*

21. 12, that is, $10 + 2$.

$$(10+2)^3 = 10^3 + 3 \times 10^2 \times 2 + 3 \times 10 \times 2^2 + 2^3$$
$$= 1000 + 600 + 120 + 8 = 1728.$$

22. 14.	23. 21.	24. 25.	25. 30.
26. 55.	27. 60.	28. 75.	29. 99.

262. Number of figures in the cube root. It is easy to tell in advance the number of figures in the cube root of a perfect cube. For

The cube of units has 3, 2, or 1 figure, since

$$9^3 = 729,\ 4^3 = 64,\ 1^3 = 1.$$

The cube of a number of 2 integral places has 6, 5, or 4 integral places, since

$$99^3 = 970{,}299,\ 40^3 = 64{,}000,\ 10^3 = 1000.$$

The cube of a number of 3 integral places has 9, 8, or 7 integral places, and so on. Therefore

263. *If a cube number be separated into periods of three figures each, beginning at the decimal point, the number of periods will equal the number of figures in the root.*

Thus, $\sqrt[3]{1{,}771{,}561}$ has 3 integral places;
$\sqrt[3]{26.463{,}592}$ " 1 " and 2 decimal places.

264. Cube root by the formula for $(f+n)^3$. 1. Find $\sqrt[3]{2197}$.

If we let $f =$ the *f*ound part of the root,
and $n =$ the *n*ext figure of the root,
then $(f+n)^3 = f^3 + 3f^2n + 3fn^2 + n^3$. (§ 261)

Therefore if we take away f^3 (B on page 242), we shall have $3f^2n + 3fn^2 + n^3$ (D, page 242, where each square block is f^2n, each oblong fn^2, and the small cube is n^3).

If we divide this by $3f^2$, we shall find approximately n (as in E, page 242, where we divided the number of units of volume by the number of units of area of the three squares).

				$\overline{1 \quad 3}$	
				2,197	$\sqrt[3]{2{,}197} = 13$
$3f^2$	$3fn+n^2$	$3f^2+3fn+n^2$	$f^3 =$	1000	
300	99	399		1197	contains $3f^2n+3fn^2+n^3$
				1197	= " " "

The greatest cube of *tens* in 2197 is 1000, for $20^3 = 8000$, and this is greater than 2197. Therefore f^3 is 1000. Since $f^3 = 1000$, f is 10.

The remainder, 1197, contains $3f^2n + 3fn^2 + n^3$, because from a number containing $(f+n)^3$, or $f^3 + 3f^2n + 3fn^2 + n^3$, f^3 has been subtracted.

Dividing this by $3f^2$ (as seen also on page 242), we approximate n, and $n = 3$.

Now the quantity which multiplied by n equals $3f^2n + 3fn^2 + n^3$, and thus, with the f^3 already found, completes the cube of $f+n$, is $3f^2 + 3fn + n^2$. (See also page 242.)

Therefore we add to $3f^2$ the quantity that will make this, that is, $3fn + n^2$. Now $3fn + n^2 = 3 \times 10 \times 3 + 3^2 = 99$, and $300 + 99 = 399$.

Multiplying 399 by 3, the result is 1197, exactly completing the cube.

It is easily seen that 2197 must be the cube of 13, for (§ 261) $13^3 = 10^3 + 3 \times 10^2 \times 3 + 3 \times 10 \times 3^2 + 3^3 = 1000 + 900 + 270 + 9$ $= 1000 + 1179$, which are exactly the numbers above subtracted from 2197, leaving no remainder.

2. Find $\sqrt[3]{1,771,561}$.

$3f^2$	$3fn+n^2$	$3f^2+3fn+n^2$	1 2 1 1,771,561	$\sqrt[3]{1,771,561}=121.$	
			$f^3 =$ 1 000 000		
30,000	6400	36,400	771 561	contains $3f^2n+3fn^2+n^3$	$f=100$
			728 000	= " " "	$n=20$
43,200	361	43,561	43 561	contains " " "	$f=120$
			43 561	= " " "	$n=1$

There cannot be any thousands in the root, for $1000^3 = 1,000,$-$000,000$, and this is larger than the given cube.

The greatest cube of *hundreds* in 1,771,561 is 1,000,000, because 200^3 is larger than the given number. Therefore $f = 100$.

Then the remainder, 771,561, contains $3f^2n + 3fn^2 + n^3$, because f^3, or 1,000,000, has been subtracted from a number containing $f^3 + 3f^2n + 3fn^2 + n^3$.

Because this contains approximately $3f^2n$, if we divide by $3f^2$ (or 30,000) we shall find approximately n. We therefore find that $n = 20$.

	1 2 1
	1,771,561
	1
300	771
64	
364	728
43200	43561
361	
43561	43561

Adding $3fn + n^2$, or $3 \times 100 \times 20 + 20^2$, to $3f^2$, we have 36,400 (as explained on page 245). This multiplied by 20 (that is, by n) equals 728,000, thus completing the cube of 120.

That 1,000,000 and 728,000 are together the cube of 120 is evident from the fact that $120^3 = 100^3 + 3 \times 100^2 \times 20 + 3 \times 100 \times 20^2 + 20^3$ (§ 261) $= 1,000,000 + 600,000 + 120,000 + 8000 = 1,000,000 + 728,000$.

We may now consider 120 as the found part of the root, and proceed as before, finding $n = 1$. The root is therefore 121.

The annexed arrangement shows all of the necessary figures, and is the one recommended after the process is understood. It simply preserves the figures actually used from step to step, not writing the others.

In case there are more figures in the root we may subtract and proceed as before, dividing again by three times the cube of the found part.

265. The following are therefore seen to be the steps in extracting cube root :

1. *The number being already separated into three-figure periods, beginning at the decimal point, find the greatest cube in the left-hand period. This is the cube of the number represented by the first figure.*

2. *Subtract this cube, bringing down but one period.*

3. *Divide this remainder by three times the square of the found part considered as tens (the so-called trial divisor) to find the next figure.*

4. *To three times the product of the two parts add the square of the second, considered as units, and add all this to the trial divisor, thus making the complete divisor.*

5. *Multiply the complete divisor by the second, thus completing the cube of the first two.*

6. *Subtract this, bring down the next period, and proceed as before.*

The subject of cube root being now so commonly postponed until algebra is studied, a more extended explanation is not felt to be necessary.

WRITTEN EXERCISE

Extract the cube root of each of the following:

1. 2197.	2. 3375.	3. 2744.
4. 68,921.	5. 74,088.	6. 91,125.
7. 97,336.	8. 110,592.	9. 140,608.
10. 205,379.	11. 250,047.	12. 4,330,747.
13. 1,295,029.	14. 1,771,561.	15. 2,628,072.
16. 4,096,000.	17. 4,826,809.	18. 50,653,000.
19. 54,010,152.	20. 97,336,000.	21. 99,252.847.
22. 91,733.851.	23. 0.114791256.	24. 34,645,976.

MENSURATION

266. Ratio of circumference to diameter. By measuring several circles, dividing each circumference by its diameter, and taking the average of the results, the circumference will be found to be about $3\frac{1}{7}$ times the diameter.

267. Value of π. It is proved in Geometry that this ratio of circumference to diameter is more nearly 3.1416. The ratio is denoted in mathematics by the Greek letter π (pī).

268. Formula for circumference. Therefore, if $c =$ circumference, $d =$ diameter, and $r =$ radius, we have

$$\frac{c}{d} = \pi,$$

whence $c = \pi d,$

or, because $d = 2\,r,$ $c = \pi \times 2\,r = 2\,\pi r$

$$= 2 \times 3\tfrac{1}{7} \times \text{radius (nearly).}$$

269. Illustrative problems. 1. Required the circumference when the radius is 7 in.

$$c = 2\,\pi r = 2 \times 3\tfrac{1}{7} \times 7 \text{ in.} = 44 \text{ in.}$$

More exactly, $2 \times 3.1416 \times 7$ in. $= 43.9824$ in.

2. Required the diameter when the circumference is 2827.44 in.

Since $\pi d = c,\ d = \dfrac{c}{\pi}$, or $d = \dfrac{2827.44 \text{ in.}}{3.1416} = 900$ in.

ORAL EXERCISE

Using $\pi = 3\frac{1}{7} = \frac{22}{7}$, state the circumferences of circles of diameters as follows:

1. 7 in.	2. 21 in.	3. 14 in.	4. 70 in.
5. 28 in.	6. 35 in.	7. 42 in.	8. 700 in.
9. 77 in.	10. 140 in.	11. 280 in.	12. 350 in.
13. 0.7 in.	14. 0.21 in.	15. 0.14 in.	16. 0.28 in.

WRITTEN EXERCISE

Find the circumference ($\pi = 3\frac{1}{7}$), *given the diameter:*

1. 68.2 in.	2. 48.3 ft.	3. 423 in.
4. 5.11 ft.	5. 53.9 ft.	6. 6.37 in.
7. 4.69 ft.	8. 58.1 in.	9. 6.02 ft.
10. 13 ft. 6 in.	11. 17 ft. 8 in.	12. 64.4 ft.

Find the diameter ($\pi = 3\frac{1}{7}$), *given the circumference:*

13. 132 ft.	14. $58\frac{4}{7}$ ft.	15. $97\frac{3}{7}$ ft.
16. 176 in.	17. 770 ft.	18. $96\frac{4}{5}$ ft.
19. 68.2 in.	20. 3.96 ft.	21. 0.484 ft.

Find the circumference ($\pi = 3.1416$), *given the diameter:*

22. 17 in.	23. 13 in.	24. 2.8 in.
25. 4.37 in.	26. 2.25 ft.	27. 6 ft. 2 in.

Find the diameter ($\pi = 3.1416$), *given the circumference:*

28. 53.4072 in.	29. 84.8232 in.	30. 97.3896 in.
31. 94.2478 ft.	32. 182.2228 ft.	33. 138.2304 ft.

Find the circumference ($\pi = 3\frac{1}{7}$), *given the radius:*

34. 49 in.	35. 77 in.	36. 91 in.
37. 105 in.	38. 15.4 in.	39. $1\frac{3}{4}$ in.

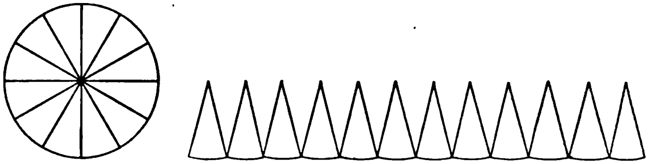
40. What is the diameter of a water tank whose circumference is 74.8 ft.? ($\pi = 3\frac{1}{7}$.)

41. What is the circumference of a rod whose diameter as measured by the calipers is 2.1 in.? ($\pi = 3\frac{1}{7}$.)

42. What is the circumference of a wire whose diameter as measured by the calipers is 0.63 in.? ($\pi = 3\frac{1}{7}$.)

43. What radius must be used in drawing a pattern for a wheel that shall be $91\frac{1}{7}$ in. in circumference? ($\pi = 3\frac{1}{7}$.)

270. Area of a circle. A circle can be separated into figures which are nearly triangles. The height is the radius, and the sum of the bases is the circumference. If these



were exact triangles the area would be $\frac{1}{2} \times r \times c$, or $\frac{1}{2}rc$. It is proved in Geometry that this is the true area.

271. Given the radius, to find the area. If $a =$ area, $c =$ circumference, and $r =$ radius,

$a = \frac{1}{2}rc$, or, because c is the same as $2\pi r$, we may write

$$a = \frac{1}{2}r \times 2\pi r, \text{ or } a = \frac{r \times \cancel{2}\pi r}{\cancel{2}} = \pi r^2.$$

272. *The area of a circle is π times the square on the radius.*

273. Illustrative problem. Required the area of a circle whose radius is 5 in.

$a = \pi r^2 = \pi \times 25$ sq. in. $= 3.1416 \times 25$ sq. in. $= 78.54$ sq. in.

In the rest of the problems involving π, use the value $3\frac{1}{7}$ unless otherwise directed.

ORAL EXERCISE

State the area, given the radius as follows:

1. 7 in. 2. 70 in. 3. $\frac{1}{22}$ in. 4. $\frac{7}{22}$ in.

5. If you take a radius half as long, the area will be what part as great?

6. If you double the length of the radius, the area will be how many times as large?

274. Illustrative problem. Required the radius, the area being 50.2656 sq. in.

1. Since $a = \pi r^2$, $r^2 = \frac{a}{\pi}$, by dividing these equals by π.
2. 50.2656 sq. in. $\div$ 3.1416 = 16 sq. in.
3. Since $\sqrt{16} = 4$, the radius is 4 in.

WRITTEN EXERCISE

1. What is the area of the cross section of a water pipe that is 2 in. in diameter?

2. How many square feet in the base of a water tank that is 42 ft. in diameter?

3. A horse tethered by a rope 21 ft. long can graze over how many square feet of ground?

4. A horse tethered by a rope can graze over 3850 sq. ft. of ground. How long is the rope?

5. How long is the equator cn a globe 12.67 in. in diameter? What is the area of the equator circle cut from such a globe?

6. A school flag pole has a circumference of 24.2 in. at the base. What is the diameter? the radius? the area of a cross section?

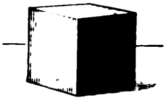
7. What is the area of the cross section of a circular iron beam whose circumference is 31.416 in.? (Use $\pi = 3.1416$.) Suppose the circumference were 53.4072 in.?

8. A boy has a ball tied to a string 1 yd. 6 in. long. As he swings it around, how long is the circumference traveled by the ball? What is the area of the circle inclosed?

9. A tinsmith wishes to make a pattern for the bottom of a pail, the area being 154 sq. in., and to allow $\frac{1}{4}$ in. all around for soldering. What radius should he use in drawing the circle?

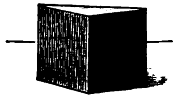
ORAL EXERCISE

1. A represents a cube 1 in. on an edge. What is its volume? Suppose it were 2 in. on an edge?



A

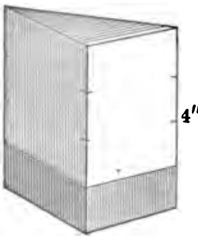
2. B represents half of a cube that was 1 in. on an edge. What is its volume? Suppose the original cube were 3 in. on an edge?



B

3. What is the volume of a prism 1 in. high, with a base ½ sq. in.? 1 sq. in.? 3 sq. in.?

4. Suppose the area of the base in C is 5 sq. in., what is the volume of the lower shaded part that is 1 in. high? What is the total volume?



C

275. Volume of a prism. We see that, if b is the area of the base of a prism, h is the height, and v is the volume, $v = bh$.

Pupils should be led to read their own rules from all such formulas. Thus,

276. *The volume of a prism equals the area of the base multiplied by the height.*

This is to be understood with the meaning given in § 38.

State the volumes of prisms with bases and altitudes as follows:

5. 9 sq. in., 4 in.
6. 17 sq. in., 7 in.
7. 16 sq. in., 5 in.
8. 440 sq. in., 25 in.
9. 480 sq. in., 12½ in.
10. 900 sq. in., 50 in.
11. 330 sq. in., 33⅓ in.
12. 480 sq. in., 16⅔ in.
13. 8000 sq. in., 75 in.
14. 500 sq. ft., 10½ ft.

WRITTEN EXERCISE

Find the volumes of prisms with bases and altitudes as follows:

1. 375 sq. in., 29 in.
2. 1.28 sq. ft., 3.2 in.
3. 67.9 sq. in., 4.8 in.
4. 42.6 sq. in., 7.3 in.
5. 61.3 sq. ft., 2.91 ft.
6. 62.8 sq. in., 3.17 in.

Find the altitudes of prisms with volumes and bases as follows:

7. 243 cu. in., 29 sq. in.
8. 783 cu. in., 87 sq. in.
9. 140 cu. in., $17\frac{1}{2}$ sq. in.
10. $178\frac{1}{4}$ cu. in., $19\frac{1}{4}$ sq. in.

Find the volumes of rectangular solids as follows:

11. 42 in. by 6.8 in. by 3.5 in.
12. 2.9 in. by 3.8 in. by 6.4 in.
13. 3.91 ft. by 4.27 ft. by 6.8 ft.
14. 2.8 ft. by 3.25 ft. by 4.75 ft.
15. 6.5 ft. by 4.75 ft. by 3.25 ft.
16. 21.3 in. by 29.2 in. by 3.7 in.

17. The area of one face of a cube is 3721 sq. in. What is the volume of the cube?

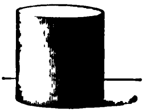
18. A prism with a square base of 127.69 sq. in. is twice as high as it is wide. Find its volume.

19. A prism has a square base, a volume of 4851 cu. in., and a height of 11 in. Required the lengths of the edges.

20. A prism has a square base, a volume of $1687\frac{1}{2}$ cu. in., and an altitude that is half its width. Find its altitude.

If the altitude equaled the width, $1687\frac{1}{2}$ would be the cube of the edge. As it is, $1687\frac{1}{2}$ is therefore half the cube of the side of the base.

277. Volume of a cylinder. The same reasoning as that employed on page 252 evidently gives the volume of a cylinder. Hence we may say that



$$v = bh.$$

278. *The volume of a cylinder equals the area of the base multiplied by the height.*

279. If the cylinder is the common one with a circular base, the area of the circle is πr^2 (§ 271). Therefore, with such a cylinder,

$$v = \pi r^2 h.$$

ORAL EXERCISE

State the volumes, given bases and altitudes as follows:

1. 0.48 sq. in., $\frac{7}{8}$ in.
2. 1.2 sq. ft., $\frac{1}{12}$ ft.
3. 300 sq. in., $33\frac{1}{3}$ in.
4. 150 sq. in., 50 in.
5. 600 sq. in., $66\frac{2}{3}$ in.
6. 80 sq. in., $12\frac{1}{2}$ in.

WRITTEN EXERCISE

1. Find the volume of a water tank 40 ft. high and 40 ft. in diameter.

2. How many cubic feet in a log 16 ft. long, the average diameter being 1 ft. 11 in.?

3. How many gallons (231 cu. in.) in a cylindrical tank 32 ft. high and 30 ft. in diameter?

4. Find the number of cubic feet in a boiler 15 ft. long and 3 ft. 6 in. in diameter, internal measure.

5. Find the number of cubic inches in a pipe 18 ft. long, 4 in. in external diameter, the metal being $\frac{1}{4}$ in. thick.

6. What is the volume of a cylinder with radius 7 in. and height 17 in.? with radius 9 in. and height 45 in.?

ORAL EXERCISE

1. If we slit the curved surface of the cylinder shown on page 254, and spread it out flat, what will it become? How may we find the area of this surface? Try it.

2. What is the area of the curved surface of a cylinder 6 in. high and 8 in. around? 9 in. high and 9 in. around?

3. How many square feet of tin are needed for a pipe 8 ft. long and 6 in. around, allowing 1 sq. ft. for overlapping?

280. Curved surface of a cylinder. We see that *the area of the curved surface of a cylinder equals the product of the height and the circumference.*

That is, if c = circumference, h = height, and a = area,

$$a = ch.$$

Or, if r = radius of the base,

$$a = 2\pi r \times h.$$

State the area of the curved surfaces of cylinders with heights and circumferences as follows:

4. 25 in., 80 in.
5. $33\frac{1}{3}$ in., 69 in.
6. 75 in., 40 in.
7. 50 in., 106 in.
8. $66\frac{2}{3}$ in., 48 in.
9. $12\frac{1}{2}$ in., 24 in.
10. $16\frac{2}{3}$ in., 36 in.
11. $33\frac{1}{3}$ in., 33 in.

12. How many square inches in the curved surface of a wire 1 in. around and 200 ft. long?

13. A tin cup is 7 in. around and 3 in. high, both measures including allowance for soldering. How many square inches of tin are needed for the curved surface?

14. A tin water pipe has a circumference of 9 in. and a length of 10 ft., both measures including allowance for soldering. How many square inches of tin are needed?

WRITTEN EXERCISE

1. How many square inches of surface on a wire 10 ft. long and $\frac{7}{8}$ in. in circumference?

2. If the height and diameter of a solid cylinder are both 7 in., what is the total area of surface?

To the cylindrical surface add the areas of the circles forming the top and bottom.

3. How many square inches of tin are needed for a cylindrical cup 4 in. high and 4 in. in diameter?

4. How many square feet in the curved surface of a water tank that is 40 ft. high and 127.3 ft. around?

5. Write a rule for finding the circumference of a circle, given the radius; also for finding the area of a circle.

6. How many square feet of surface on the outside of a smokestack 30 ft. high and 2 ft. in exterior diameter?

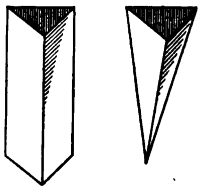
7. The curved surface of a granite cylindrical shaft, 3 ft. 6 in. in diameter and 22 ft. 3 in. long, is to be polished. How many square feet of surface are to be polished?

8. In a certain factory there is a room heated by 210 ft. of steam pipe, 2 in. in diameter. Required the radiating surface. (That is, the area of the curved surface which radiates the heat.)

9. A large suspension bridge has 4 cables, each 1872 ft. long and 1 ft. 2 in. in diameter. In letting the contract for painting these cables it is necessary to know their surface. Compute it.

10. In making metal bedsteads some iron rods 0.75 in. in diameter are covered with thin rolled brass. Suppose a shop needs 6000 ft. of such rods, how many square feet of rolled brass will be needed, not allowing for waste?

281. Volume of a pyramid. If we construct a hollow prism and a hollow pyramid of the same base and height, filling the former with sand, the latter can be filled three times with the same amount. Therefore



$$v = \frac{1}{3}\,bh.$$

282. *The volume of a pyramid equals one third the product of its base and height.*

ORAL EXERCISE

State the volumes of pyramids with bases and altitudes as follows:

1. 75 sq. ft., 7 ft.
2. 21 sq. ft., 9 ft.
3. 33 sq. ft., 6 ft.
4. 120 sq. ft., 7 ft.
5. 70 sq. in., 9 in.
6. 63 sq. ft., 12 ft.
7. 39 sq. in., 10 in.
8. 38 sq. in., 15 in.
9. 450 sq. in., 20 in.
10. 480 sq. in., 10 in.

WRITTEN EXERCISE

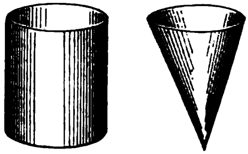
Find the volumes of pyramids with bases and altitudes as follows:

1. 129.3 sq. in., 3.72 in.
2. 702.39 sq. in., 16.8 in.
3. 202.11 sq. ft., 8.92 ft.
4. 187.02 sq. ft., 10.7 ft.

Find the bases of pyramids with volumes and altitudes as follows:

5. 89.6 cu. in., 12.8 in.
6. 123.2 cu. in., 17.6 in.
7. 178.2 cu. in., 19.8 in.
8. 140.4 cu. in., 23.4 in.
9. The Great Pyramid in Egypt is $480\frac{3}{4}$ ft. high. Its base is a square, 764 ft. on a side. What is its volume? If it weighs 168 lb. per cubic foot, what is its weight?

283. Volume of a cone. In the same way that we found the volume of a pyramid, we may find that of a cone.



The volume of a cone equals one third the product of its base and height.

284. If the base is circular, as is commonly the case,

$$v = \tfrac{1}{3}\pi r^2 h.$$

It is desirable that these solids should be constructed by the class.

285. Illustrative problem. What is the volume of a cone 5 in. high, the radius of the base being 2 in.?

1. The area of the base $= \pi r^2 = 3.1416 \times 4$ sq. in.
2. The volume $= \frac{1}{3} \times 5 \times 3.1416 \times 4$ cu. in. $= 20.944$ cu. in.

WRITTEN EXERCISE

Find the volumes of cones, given the radii and heights as in Exs. 1–4, using $\pi = 3\frac{1}{7}$ in all the exercises:

1. $r = 7.7$ in., $h = 8.2$ in.
2. $r = 4.2$ in., $h = 7.3$ in.
3. $r = 6.3$ in., $h = 9.8$ in.
4. $r = 11.9$ in., $h = 5.7$ in.

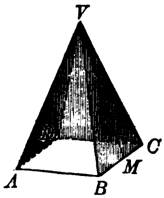
5. What is the volume of a cone whose altitude is 18 in. and the circumference of whose base is $25\frac{1}{7}$ in.?

6. A water tank is 40 ft. high and 132 ft. in circumference. What is its capacity in cubic feet? in gallons?

7. What is the volume of a cone whose altitude equals the diameter of the base, the circumference of the base being 44 in.?

8. The interior diameter of a water pipe is 4 in. The water flows through at the rate of 2 ft. per second. How much water flows through in a minute?

286. Regular pyramid. If the sides and angles of the base of a pyramid are respectively equal, and the vertex is exactly over the center of the base, the pyramid is said to be *regular.*



287. Slant height. The altitude of any triangle forming a side of a regular pyramid is called the *slant height* of the pyramid.

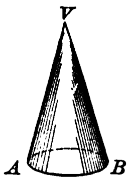
VM is the slant height of the pyramid shown.

288. Surface of a regular pyramid. Since the lateral (side) surface of a regular pyramid is made up of triangles, each of which equals half the base times the altitude (page 62), therefore

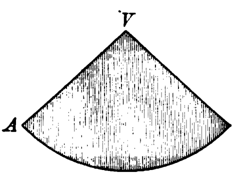
The lateral area of a regular pyramid equals the perimeter of the base multiplied by half of the slant height.

For example, if *AB* in the above pyramid is 5 ft., and *VM* is 8 ft., the lateral area is $4 \times 5 \times 4$ sq. ft., or 80 sq. ft.

289. Surface of a cone. If we should slit the surface of this common form of cone, often called a *right circular cone*, and flatten it out, as here shown, we should have part of a circle. In the same way that we found the area of a circle (§ 270) we find that



The lateral area of a right circular cone equals the circumference of the base multiplied by half of the slant height.



For example, if the circumference is 10 ft. and the slant height is 8 ft., the lateral area is 10×4 sq. ft., or 40 sq. ft.

In the exercises on page 260 it is understood that regular pyramids and right circular cones are given unless the contrary is stated.

WRITTEN EXERCISE

Find the lateral surfaces of regular pyramids with the following perimeters of bases and slant heights:

1. 16 in., 4 in.
2. 32 in., 17 in.
3. 48 in., 15 in.
4. 3 ft. 4 in., 7 in.
5. 2 ft. 8 in., 11 in.
6. 6 ft. 7 in., 15 in.

Also of right circular cones with the following circumferences and slant heights:

7. 17.2 in., $5\frac{3}{8}$ in.
8. 16.5 in., $4\frac{7}{8}$ in.
9. 14.9 in., 2.8 in.
10. 6 ft. 8 in., 1 yd.
11. 3 ft. 4 in., 17 in.
12. 2 ft. 7 in., 19 in.

13. A cone has a lateral area of 220 sq. in. The radius of the base is 7 in. Find the slant height.

14. A pyramid has a lateral area of 200 sq. in. The slant height is 8 in. Find the perimeter of the base.

15. A conical spire has a slant height of 68 ft. The perimeter of the base is 60 ft. What is the lateral surface?

16. What is the entire area of a cone, including the base, the slant height of which is 7 ft., the diameter of the base being 7 ft.?

17. The Great Pyramid of Cheops has a square base 764 ft. on a side. The slant height is about 500 ft. About what is the lateral area of the pyramid?

18. How many square feet should be allowed for slating a steeple in the form of a regular pyramid of slant height 40 ft., the perimeter of the base being 48 ft.?

19. A dome is surmounted by a gilded cone whose slant height is 16 ft. 8 in.; the radius of the base being 3 ft. 6 in. How many square feet must be allowed for gilding?

290. Surface of a sphere. If we wind a sphere with cord, and wind a cylinder whose radius equals the radius of the

sphere, and whose height equals the diameter, we find that it takes as much cord for the cylinder as for the sphere.

In the picture, for convenience, only half of each is wound.

Therefore *the surface of a sphere equals the curved surface of a cylinder of the same radius and height.*

291. Formula for area of the sphere. Since the curved surface of the cylinder is $2\pi r \times h$, where $h = 2r$, therefore

$$\text{area of sphere} = 2\pi r \times 2r = 4\pi r^2.$$

292. Illustrative problem. If the earth is a sphere of 4000 mi. radius, what is its area?

1. $4000^2 = 16{,}000{,}000$.
2. $4\pi \times 16{,}000{,}000 = 4 \times 3\frac{1}{7} \times 16{,}000{,}000 = 201{,}142{,}857\frac{1}{7}$.
3. Therefore the area is about 201,143,000 sq. mi.

WRITTEN EXERCISE

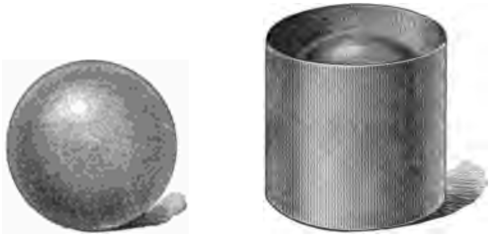
1. If a ball has a radius of $2\frac{1}{4}$ in., what is its area?

2. The sun's diameter is 866,500 mi. Find its area.

3. If a tennis ball is $2\frac{5}{8}$ in. in diameter, what is its area?

4. A gilded ball is to be put on top of a tower. The diameter of the ball is 2′ 6″. How many square inches are to be gilded, making no allowance for the support?

293. Sphere and cylinder compared. If we place this sphere in this cylinder of the same diameter and height, and fill



in the spaces with sand, the sand will fill one third of the cylinder when the sphere is removed. Therefore

A sphere equals two thirds of a cylinder of the same diameter and height.

294. Volume of a sphere. And since the volume of the cylinder is $\pi r^2 \times 2r$, or $2\pi r^3$ (§ 278), and the sphere is $\frac{2}{3}$ as large, or $\frac{2}{3}$ of $2\pi r^3$,

The volume of a sphere is $\frac{4}{3}\pi r^3$.

295. Illustrative problem. What is the volume of a sphere whose radius is 21 in.?

$$\frac{4}{3}\pi r^3 = \frac{4}{3} \times 3\tfrac{1}{7} \times 21^3$$
$$= \frac{4 \times 22 \times \cancel{21} \times 21 \times 21}{\cancel{3} \times \cancel{7}} = 38{,}808.$$

Therefore the volume is 38,808 cu. in.

In the following examples and on pages 263, 264, use $\pi = 3\frac{1}{7}$.

WRITTEN EXERCISE

What are the volumes of spheres with radii as follows?

1. $2\frac{3}{8}''$. 2. $10.5''$. 3. 1000 mi. 4. $7'\,6''$. 5. $9'\,7''$.

Also with diameters as follows?

6. $3'\,6''$. 7. $4'\,8''$. 8. 8000 mi. 9. $9.6'$. 10. $11.4'$.

Find the areas of the squares whose sides are as follows:

11. $\frac{23}{48}$ in. **12.** $\frac{13}{16}$ in. **13.** $\frac{17}{36}$ in. **14.** $\frac{15}{64}$ in.

15. 2.87 ft. **16.** 3.92 ft. **17.** 17.5 rd. **18.** 0.37 in.

Find the areas of the rectangles whose dimensions are:

19. 3.4 ft., 9.2 ft. **20.** 6.4 in., 7.3 in.

21. $1\frac{1}{4}$ yd., $27\frac{1}{2}$ in. **22.** 4.1 ft., 11.3 in.

23. 4.92 in., 0.97 in. **24.** 14 ft. 3 in., 2 ft. 8 in.

Find the areas of the triangles whose bases and altitudes are respectively as follows:

25. 4.7 ft., 3.8 ft. **26.** $6\frac{1}{4}$ yd., 27 in.

27. 0.37 in., 0.42 in. **28.** 6.4 in., 4.9 in.

29. $4\frac{1}{2}$ yd., 3 ft. 9 in. **30.** 6.8 ft., 11.9 in.

Given rectangles with areas and lengths as follows, to find the widths:

31. 306 sq. in., 17 in. **32.** 90.3 sq. in., 43 in.

33. 14.62 sq. in., 4.3 in. **34.** 4.18 sq. in., 2.2 in.

35. 473 sq. in., 3 ft. 7 in. **36.** 850 sq. in., 4 ft. 2 in.

Given triangles with areas and bases as follows, to find the altitudes:

37. 864 sq. in., 27 in. **38.** 14.72 sq. ft., 4.6 ft.

39. 101.4 sq. in., 26 in. **40.** 25.62 sq. in., 6.1 in.

41. 0.0704 sq. in., 0.32 in. **42.** 2911 sq. in., 3 ft. 5 in.

Find the sides of the squares whose areas are given in Exs. 43–51, carrying the results to two decimal places:

43. 2 A. **44.** 171 sq. ft. **45.** 385 sq. ft.

46. 153 sq. in. **47.** 352 sq. rd. **48.** 251 sq. yd.

49. 7.52 sq. in. **50.** 6.04 sq. yd. **51.** 8.51 sq. in.

Find the areas of the circles whose radii are:

52. 14 in. **53.** 15.4 in. **54.** 39.9 in.
55. 64.4 ft. **56.** 71.4 ft. **57.** 92.4 yd.
58. 123.9 rd. **59.** 164.5 rd. **60.** 235.9 yd.

Find the radii of the circles whose areas are:

61. 308 sq. in. **62.** 770 sq. in. **63.** 1386 sq. in.
64. 1694 sq. ft. **65.** 1848 sq. ft. **66.** 2618 sq. ft.
67. 40.04 sq. yd. **68.** 44.77 sq. yd. **69.** 141.68 sq. yd.

Find the volumes and surfaces of the spheres whose radii are as follows:

70. 181 in. **71.** 172 in. **72.** 625 in.
73. 31.8 in. **74.** 27.6 in. **75.** 34.2 in.

Find the volumes of the pyramids whose bases and altitudes are respectively:

76. 21.6 sq. ft., 3 ft. **77.** 16.4 sq. in., 17.1 in.
78. 17.4 sq. in., 12.3 in. **79.** 26.2 sq. ft., 5 ft. 1 in.

Find the volumes of the cylinders whose altitudes and the radii of whose bases are respectively:

80. 2.8 ft., 1.7 ft. **81.** 2.7 in., 7.2 in.
82. 6.2 in., 3.4 in. **83.** 14.2 ft., 8.1 ft.

Find the volumes of the cones whose altitudes and the radii of whose bases are respectively:

84. 6.2 ft., 4.1 ft. **85.** 3.9 ft., 2.4 ft.
86. 3.4 in., 4.2 in. **87.** 6.7 in., 7.8 in.
88. 2.8 in., 3.1 in. **89.** 12.8 ft., 6.1 ft.
90. 4 ft. 7 in., 2 ft. 1 in. **91.** 6 ft. 8 in., 3 ft. 2 in.

92. If a cubic foot of granite weighs 165 lb., what is the weight of a sphere of granite 1 ft. in diameter?

93. How many tons of water will fill a tank 14 ft. by 8 ft. by 8 ft., allowing 1000 oz. to a cubic foot?

94. A bowl is in the form of a hemisphere 4.2 in. in diameter. How many cubic inches does it contain?

95. If a cubic foot of marble weighs 173 lb., what is the weight of a cylindrical marble column 10 ft. high and 16 in. in diameter?

96. How many loads (cubic yards) of earth must be removed in digging a canal 2 mi. 1740 ft. long, 200 ft. wide, and 16 ft. deep?

97. In a cylindrical jar 4 in. high and 3 in. in diameter, half full of water, a marble 1 in. in diameter is dropped. It causes the water to rise how much?

98. What is the weight of a sphere of steel 7 in. in diameter, steel being 7.8 times as heavy as water, and 1 cu. ft. of water weighing 1000 oz.?

99. What is the weight of a sphere of marble 3 ft. in circumference, marble being 2.7 times as heavy as water, 1 cu. ft. of water weighing 1000 oz.?

100. The entire surface of a pyramid is made up of four equal-sided triangles 3 ft. on a side. What is the total area? (Carry the result to hundredths.)

101. The diameter of a sphere and the altitude of a cone are equal, and they have equal radii. The volume of the sphere is how many times that of the cone?

102. Taking the radius of the earth as 4000 mi., and the earth as an exact sphere, what is its volume? How many square miles on its surface? (Carry the answers only to 1000 square or cubic miles.)

MISCELLANEOUS PROBLEMS

296. Nature of the problems. The following list contains certain problems whose interest is rather in the reasoning exercised in solution than in their applicability to the life of to-day.

WRITTEN EXERCISE

1. Divide 91 into two parts having the ratio of 5 : 8.

2. The product of two numbers is 175 and their quotient is 7. What are the numbers?

3. How many bananas are there in a bunch of which 7 are decayed and 95% are not?

4. The sum of two numbers is 783 and their difference is 191. What are the numbers?

5. At 200 lb. per cubic foot, what is the weight of a block of stone $4' \, 3\frac{1}{4}'' \times 1' \, 1'' \times 9.6''$?

6. At $1.95 a rod, how much will it cost to fence a rectangular 2-acre field that is 5 ch. long?

7. If iron weighs 444.56 lb. per cubic foot, find the weight of a 14-ft. bar of $2'' \times 1\frac{1}{2}''$ iron.

8. How much is gained by buying 63 gal. of molasses for $25.20 and selling it at 12¢ a quart?

9. What is the rate of gain when oranges are bought at 40 for $1, and sold at 50¢ a dozen?

10. What is the rate of gain when eggs are bought at 16 for a quarter and sold at 13 for a quarter?

11. What is the height of a block of marble $4' \, 2'' \times 2' \, 1''$, that weighs as much as one $3' \, 4'' \times 2' \, 1'' \times 2' \, 5''$?

12. A man owns 100 A. of coal land, the coal bed being 11 ft. thick. Allowing 35 cu. ft. to the ton, how many tons of coal in the bed?

13. The minute hand of my watch is $\frac{7}{10}$ in. long. Over what area does it pass in a day?

14. If 15 horses eat 11 bu. of oats in 9 da., how long will 44 bu. last 45 horses at the same rate?

15. In the year when Michigan produced 11,135,215 long tons of iron, how many pounds did it produce?

16. If telegraph poles are 198 ft. apart, and a train passes one every 3 sec., what is the train's rate per hour?

17. The pitch of a screw is 16 threads to the inch. How many turns must be made to move the screw $\frac{3}{4}$ in.?

18. A dealer bought hay at $8.66 a ton and sold it at $9.75, gaining $408.75. How many tons did he buy?

19. What will it cost to fill in 3 acres of swamp land, raising the level on an average of 5 ft., at 45¢ a load?

20. A man used a bicycle for a month and sold it 15% below cost, receiving $29.75 for it. How much did he lose?

21. What is the list price of some goods which cost $68.40 after the discounts 10%, 5% have been deducted?

22. At what price must a dealer mark goods that cost him $450 so as to take off 10% and still make 10% profit?

23. What is the shortest distance that can be measured exactly by a yardstick, a fathom rod, or a pole 1 rd. long?

24. A man sold two carriages at $150 each. On one he gained 20%, and on the other he lost 20%. Did he gain or lose on both, and how much?

25. A man bought a horse for $95 and sold it for $114. What was his gain per cent? The purchaser sold the horse for $95. What was his loss per cent?

26. In the year when Michigan produced 7,729,808 barrels of salt, this was 37.6% of the total amount produced in the United States. How many were produced in all?

27. At what distance from the end must a piece of timber 16″ square be sawed so as to cut off 9 cu. ft.?

28. Around a circular flower bed 28 ft. in diameter is a gravel walk 7 ft. wide. What is the area of the walk?

29. If 6 men can do a piece of work in 4 da., how many more must be employed to finish the work one day sooner?

30. A man can mow $\frac{3}{7}$ of a field in $1\frac{3}{4}$ hr. How many hours will it take to mow the whole field at this rate?

31. If 84 men can do a piece of work in 2 mo., how long will it take 12 men to do the same work at the same rate?

32. A train 156 ft. long is traveling 60 mi. an hour. How long will it take it to completely cross a bridge 295 ft. long?

33. One man can do a piece of work in 4 da. which another can do in 5 da. At these rates, how long will it take them to do the work together?

The first does what part in one day? The second does what part in one day? Together they do what part in one day? How many days will it take them?

34. A boy, after doing $\frac{3}{8}$ of a piece of work in 30 days, is assisted by a man, with whom he completes it in 6 days. How long would it take each to do the work alone?

35. If a certain number of men can do a piece of work in 15 da., how long will it take if twice as many men are added to this number?

36. If 19 men can do a piece of work in 76 da. of 7 hr. each, how many men, working at the same rate, will it take to do the work in 133 hr.?

37. An expert typesetter, working by the hour, receives $32.40 a week, working 9 hr. daily except Sunday. How many hours a day must he work to earn $19.20 in 4 da. at the same rate?

38. What two equal numbers produce, when multiplied together, 15,129?

39. The square root of a certain number is 42.3. What is 42.3% of the number?

40. A square is 300 ft. on a side. How long is the diagonal? Answer to two decimal places.

41. Multiply the square root of 13.7641 by the sum of 2.06 and 1.65, and extract the square root of the product.

42. A 51-ft. ladder rests against a building, its foot being 24 ft. from the wall. How high does it reach?

43. How long is a ladder that just reaches the top of a 4-story building, averaging 10 ft. to a story, the foot of the ladder being 9 ft. from the wall?

44. From the top of a 24-ft. flagstaff a rope is stretched to a point 7 ft. from the foot of the staff. How long is the rope?

45. A baseball "diamond" is a square 90 ft. on a side. How far must the second baseman throw from his base to the home plate? Answer to two decimal places.

46. How far must the catcher throw from the home plate to the shortstop, who stands halfway between the second and third bases of a baseball diamond? Answer to two decimal places. (See Ex. 45.)

47. If a locomotive travels 90 mi. in 2 hr., the piston making 162 strokes a minute, how many strokes per minute must be made for the same locomotive to travel 200 mi. in $4\frac{1}{2}$ hr.?

48. A city newspaper office must get out an edition of 126,000 papers. It has two presses, each with a capacity of 300 papers a minute. How long will it take to print the edition?

49. How many steps of 2 ft. 9 in. each would you have to take in a minute in order to walk at the rate of 4 mi. per hour?

50. At the rate of $\frac{2}{3}$ cu. yd. of earth in 15 min., how long would it take a man to excavate a cellar 15 ft. square and $7\frac{1}{2}$ ft. deep?

51. A machine does a certain amount of work in 12 hr. If its capacity be increased 20%, how long will it take it to do the work?

52. Of three partners, one is entitled to $\frac{5}{18}$ and another to $\frac{7}{18}$ of the profits. If they make $4500, what is the share of each of the three?

53. If a certain gun metal is composed of 16% tin and the rest copper, how many pounds of tin will be needed with 46.2 lb. of copper?

54. Two cans have a capacity of 7 pt. and 10 pt. respectively. How would you measure out 16 pt. into a third can by using these alone?

55. A dealer marked a bedroom set 15% above cost, this representing a gain of $9.30. He sold it at a discount of 10%. How much did he gain?

56. A strip of carpet 50 yd. long and 27 in. wide weighs 200 lb. What is the weight of such a carpet covering an assembly room 135 ft. × 150 ft.?

57. If a gallon of water weighs 10 lb., and a cubic foot of water weighs 1000 oz., how many gallons are there in a cistern which contains 16 cu. ft.?

58. A man bought a piece of alloy for $72.25. The number of pounds purchased and the number of dollars per pound were the same. How many pounds did he purchase?

59. What is the number which multiplied by 80% of itself equals 4500?

60. The diagonal of one face of a cube is $\sqrt{162}$ in. Find the surface and the volume of the cube.

61. A horse is tied to a corner post of a square field by a rope 35 ft. long. Over what part of an acre can he feed?

62. If a horse is tethered by a 30-ft. rope, it can graze over how many times as much ground as it could if the rope were only 20 ft. long?

63. What is the area of the largest circle that can be inscribed in a square whose area is 196 sq. in.? of the largest square that can be inscribed in the circle?

64. A locomotive wheel 5 ft. in diameter makes 300 revolutions a minute, and travels at this rate for 4 hr. 10 min. without stopping. How far does it go? (Take $\pi = 3\frac{1}{7}$.)

65. A wheel 7 ft. 8 in. in circumference revolves 3762 times in going a certain distance. Another wheel revolves 3496 times in going the same distance. What is the circumference of the latter?

66. A piece of brickwork 279 ft. long is completed in 21 days. At this rate, how long would it take to complete a piece twice as high and 651 ft. long?

67. A boy buys apples at the rate of 5 for 2 ct., and sells them at the rate of 8 ct. a dozen. How many apples must he buy and sell in order to gain $2?

68. The equatorial diameter of the earth is estimated at 20,926,202 ft., and the polar diameter at 20,854,895 ft. Express each in miles and find the difference.

69. A, B, and C bought a horse for $112 and sold it at a profit of 25%, by which A gained $6 and C twice as much. How much did each of the three pay for the horse?

70. How many acres in a square field whose diagonal is 120 rd.?

71. What is the square root of $\frac{1}{3}$ of the product of the square root and the cube root of 729?

72. If 145 bu. of turnips last 53 oxen a fortnight, how long will 435 bu. last 371 oxen?

73. A man computing the cost of 43 articles multiplied, by mistake, the cost of each by 34 instead of 43 and obtained \$84.32. What was the correct cost of the lot?

74. Two masts of a ship are 45 ft. 6 in. apart, one being 60 ft. high and the other 70 ft. Supposing them perpendicular to the deck, how far is it from the top of one mast directly to the top of the other? (Two decimals.)

75. Add $\frac{3}{7}$, $\frac{4}{5}$, $\frac{3}{8}$, and $\frac{3}{4}$. Reduce the result to a decimal fraction, to three decimal places. Also reduce each of the addends to a decimal fraction carried to four places, and add.

76. One man can do $\frac{1}{5}$ of a piece of work in a day, and another can do $66\frac{2}{3}\%$ as much. How long will it take them, working together, to do the whole work?

77. Two ships, sailing in opposite directions, pass in mid-ocean. One averages 7° 30′ a day, and the other 75% as much. What is their difference in ship time exactly 4 da. after passing?

78. In closing out a line of goods a merchant asked 20% less than a certain article cost, but at an auction sale he succeeded in getting 20% more than this asking price. What per cent did he lose on the cost?

79. If Michigan, one of the largest producers of Portland cement, made 1,602,370 bbl. in a certain year, an increase of 10% on the preceding year, how many barrels were produced in the preceding year?

80. Water is flowing into a tank whose base measure is $4' \, 4'' \times 2' \, 7''$ at the rate of 0.62 cu. ft. in 3 min. How long will it take the water to fill it to a depth of $2' \, 8''$?

81. A 12-in. gun can fire a shell weighing 850 lb. every 30 sec. At this rate, how many pounds of shell could a battle ship fire from four 12-in. guns in 2 min.?

82. A train leaves a city at 5:15 P.M., and reaches a city $109\frac{1}{2}$ mi. distant at 7:51 P.M. Allowing 10 min. for stopping at stations, what is the rate of travel per hour?

83. The greatest known depth of the ocean is 5000 fathoms, and the greatest mountain height is 29,002 ft. Find the difference in level, in miles, to three decimal places.

84. A square park is surrounded by a gravel walk $7' \, 6''$ wide. The park and walk together contain 1.6 acres. Find the cost of making the gravel walk at 10¢ per square yard.

85. A grocer bought 400 bunches of lettuce for $8. He sold all but 43 heads at 5¢ each. Of the 43 heads he sold 30 at 1¢ each and the rest he threw away. How much did he gain?

86. The product of two numbers is 48 less than half the number of feet in a mile, and one of the numbers is the number of cubic inches in a cubic foot. What is the other number?

87. A farmer finds that a bin 8 ft. long, 3 ft. 6 in. wide, and 5 ft. deep holds 112 bu. How many bushels will be contained in a bin 50% longer, twice as wide, and of the same depth?

88. How much must the terms of the fraction $\frac{123}{145}$ be increased to make the fraction equal $\frac{27}{29}$? decreased to make the fraction equal $\frac{60}{71}$? increased to make the fraction equal $\frac{65}{76}$?

89. If 6 men can lay the tiles in a large railway station in 16 da., how many men must be added to the job to complete it 4 da. sooner?

90. How many times will a locomotive drive wheel 5 ft. 10 in. in diameter revolve in going from New York to Chicago, a distance of 983 mi.?

91. A man deposited some money in the bank, and then drew out 15% of the amount, then 20% of the remainder, and then 30% of what was still left, and there then remained $809.20. What was the deposit?

92. If the area of Illinois is 56,650 sq. mi., and if in a certain year 91.5% of the area was included in farms, and 84.5% of this part was improved farm land, how many acres of improved farm land were there?

93. When Adams County, Illinois, had a population of 67,100, Lasalle County had a population 31% larger, and Cook County had 1200% as much as both these counties together. What was then the population of Cook County?

94. When Port Arthur was captured it was found that there was food enough for the 45,000 soldiers and inhabitants for 6 wk. Suppose there had been 9000 more men, for how long would the food have been sufficient?

95. A village has a water tank 42 ft. in diameter. In 7 hr. 30 min., when no water is being pumped in, the water is lowered 6 ft. How many gallons are drawn out on an average each hour of this period? (Use 7.6 gal. to 1 cu. ft.)

96. When the manufactured products of Illinois were a billion and a quarter dollars a year, the only states with a larger product were New York, with 75% more, and Pennsylvania, with 48% more. The manufactures of New York then exceeded those of Pennsylvania by what per cent?

97. A rug-cleaning establishment charges 12¢ a yard for cleaning carpets 27 in. wide. At the same rate per square yard, how much will it charge for cleaning two rugs $7' \, 6'' \times 5' \, 6''$ and $5' \times 8' \, 6''$?

98. A grain of gold can be beaten out into a leaf of 56 sq. in. How many of these leaves, pressed one upon another, would make a pile 1 in. high, 1 cu. in. of gold weighing 10 troy ounces?

99. The quotient of two numbers is the sum of the numbers denoting the length of a mile expressed in feet and the length of a rod expressed in feet. The dividend is 21,186. What is the divisor?

100. A certain university library increases at the rate of 18,000 books a year. Allowing 300 working days to the year, and 10 hours to the day, how often, on an average, does a new book come to the library?

101. If $3\frac{1}{2}$ bbl. of lime are required for plastering 100 sq. yd., how many barrels will be required for a job of plastering equivalent to a wall surface $231' \times 9'$? How many for 6831 sq. ft.?

102. For a cellar floor 153 cu. ft. of concrete are needed. To make this there are used 2 parts of fresh powdered lime, 1 part of Portland cement, 6 parts of gravel. How many cubic feet of each are used?

103. If it takes 1 bbl. of lime and $\frac{5}{8}$ cu. yd. of sand to make the mortar for 1000 bricks, and if $22\frac{1}{2}$ bricks are allowed per cubic foot of wall, how much lime and sand are needed for a wall $30' \times 10' \times 1' \, 6''$?

104. If the more liberal estimate of 23 bricks to 1 cu. ft. is allowed, how many bricks would be needed for a wall $50' \times 100' \times 20''$?

105. A boy who can swim 1 yd. a second swims downstream for 10 minutes, the stream flowing 15 yd. a minute. How far does he go? At the same rates, how long will it take him to swim back to where he started?

106. I paid a bookseller $5.05 for three books, a history, an atlas, and a dictionary. The history and atlas together cost $2.85, and the atlas and dictionary together $3.80. What was the price of each?

107. A tank $8' \times 3' \times 9''$ is filled with pulp for making paper. How long a strip of paper, 2′ 6″ wide, can be made from this pulp, the paper being $\frac{1}{250}$ in. thick, and half the volume of the pulp being lost in drying and rolling?

108. Two persons start at the same time from places $6\frac{1}{2}$ mi. apart, and go towards each other. One walks at the rate of 3 mi. an hour, and the other goes in an automobile at the rate of 10 mi. an hour. When and where will they meet?

109. A druggist has his clerk prepare 200 boxes of salve, the material costing $4.90, the boxes $2.20, the labels $2, and the time of the clerk being worth $3. He sells the salve at 15¢ per box. How much does he gain by the transaction?

110. Two water filters discharge into the same jar, one at the rate of 16 qt. an hour, the other at the rate of $3\frac{1}{2}$ qt. every 15 min. What is the capacity of the smallest jar that can be filled in an integral number of minutes by either filter?

111. A man wishes to go from one corner of a rectangular field 60 rd. long and 11 rd. wide, to the diagonally opposite corner. How many steps will he save by going directly across the field, instead of going along the two sides, allowing 2 ft. 6 in. to a step?

112. An ice company has an ice house 120 ft. long, 52 ft. wide, and 28 ft. high, after making allowance for packing. How many tons of ice will it hold, allowing $58\frac{1}{8}$ lb. to the cubic foot?

113. How many croquet balls 3 in. in diameter can be placed in a cubical can whose interior height is 2 ft. 3 in.? How many quarts of water can be poured in after the balls are in place?

114. At the time that Illinois had 11,427 miles of railway track it had more than any other state. If the weight of a rail averaged 80 lb. to the yard, how many tons of steel were used for the Illinois roads?

115. Two men rent an automobile for fifteen weeks during the summer season. One uses it 7 wk., and the other 8 wk. If they pay $345 rent, what is the share of each?

116. Three men own lots having frontages of 35 ft., 40 ft., and 50 ft., respectively, on the same street. They are assessed $150 for street improvement. If assessed according to the frontage, what should each pay?

117. Two houses rent for $300 a year, the rent being paid monthly in advance in one case, and at the end of each quarter (3 mo.) in the other. What is the difference in the amount of each rent in 2 yr., allowing 6% simple interest?

118. Three men own adjoining land fronting on the same street, the frontages being 561 ft., 357 ft., and 459 ft. respectively. They agree to cut each piece into building lots, and they wish all the lots to have the same frontage. What is the greatest possible frontage of each, and how many lots will each man have?

119. At what time between 1 and 2 o'clock will the hands of a clock be together?

Looking at the clock, it is evident that they are together at 12. We may also think of the hour hand as 60 minute spaces ahead of the minute hand at that time, so that the minute hand must gain 60 minute spaces to overtake the hour hand again. But in 1 hr. it evidently *gains* only 55 minute spaces, for at 1 o'clock the hour hand is 5 minute spaces ahead. Since it gains 55 in 1 hr., to gain 60 it will take $\frac{60}{55}$ hr., or $1\frac{1}{11}$ hr., or 1 hr. 5 min. $27\frac{3}{11}$ sec. (Draw the face of a clock to illustrate this.)

At what time between 2 and 3 o'clock will they be together?

120. At what time will the hands of a clock be together between 4 and 5 o'clock? between 7 and 8 o'clock?

121. At what times between 2 and 3 o'clock will the hour and minute hands be at right angles?

From noon the minute hand would have to gain on the hour hand 120 minute spaces for the two to be together, or 120 + 15 or 120 + 45 for them to be at right angles.

122. At what times between 5 and 6 o'clock will the hour and minute hands of a clock be at right angles? together?

123. At what times between 9 and 10 o'clock will the hour and minute hands of a clock make with each other an angle of 120°?

124. If a clock is 2 min. 30 sec. fast at noon, January 1, and gains 3 min. 30 sec. a week, what time will it indicate at noon on January 31?

125. Earth, Mars, and Saturn were in the same straight line on the same side of the sun in 1875. Taking the period of revolution of Mars about the sun as $1\frac{7}{8}$ yr., and that of Saturn as $29\frac{1}{2}$ yr., in what year will they again be in the same relative positions and in the same direction from the sun?

126. If the cotton crop of Mississippi decreased 5% in a certain year, amounting to 1,347,100 bales, how many bales were produced before the decrease?

127. The world's production of copper in a certain year was 575,000 long tons, of which the United States produced 308,000. Of the United States, the largest producers were Michigan, 192 million pounds, and Montana, 27% more than Michigan. What per cent of the world's production was that of the United States? of Michigan? of Montana?

128. What is the present worth of (or sum which put at interest will amount to) $230.25, due in 3 yr. 9 mo. 18 da., money being worth 6%? If we call the difference between the present worth and the given sum (or future worth) the true discount, as was formerly the custom, find the true discount in this case.

129. What is the present worth (see Ex. 128) of $151.33, due in 4 mo. 15 da., money being worth 6%?

130. What is the present worth and true discount (see Ex. 128) of $161.50, due in 1 yr. 3 mo. 10 da., money being worth 6%?

131. The following are some of our greatest coal-producing states, with the number of tons produced by each in a certain year, and the value of the coal at the mines. Find the value per ton in each case.

Alabama	11,700,000 T.	$14,375,000
Colorado	7,652,000 T.	$9,182,400
Illinois	34,955,000 T.	$36,353,200
Indiana	11,191,700 T.	$12,310,870
Kentucky	7,075,000 T.	$7,145,750
Ohio	24,574,000 T.	$29,488,800
Pennsylvania	103,000,000 T.	$115,360,000
West Virginia	26,883,000 T.	$28,227,150

TABLES FOR REFERENCE

Length

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.).
320 rods, or 5280 feet = 1 mile (mi.).
4 inches = 1 hand.
6 feet = 1 fathom.
120 fathoms = 1 cable length.
1.15 common miles = 1 knot (nautical mile).

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
$30\frac{1}{4}$ square yards = 1 square rod (sq. rd).
160 square rods = 1 acre (A.).
640 acres = 1 square mile (sq. mi.).

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).
128 cubic feet = 1 cord (cd.).
$24\frac{3}{4}$ cubic feet = 1 perch (stone, etc.).

Weight

16 ounces (oz.) = 1 pound (lb.).
2000 pounds = 1 ton (T.).
100 pounds = 1 hundredweight (cwt.).
112 pounds = 1 long hundredweight.
2240 pounds = 1 long ton.

Troy Weight

24 grains (gr.) = 1 pennyweight (pwt. or dwt.).
20 pennyweights = 1 ounce (oz.).
12 oz. = 1 pound (lb.).
The avoirdupois pound contains 7000 gr., the Troy pound 5760 gr.
3.2 grains = 1 carat (diamond weight).
But "16 carats fine" means $\frac{16}{24}$ pure gold.

Apothecaries' Weight

20 grains (gr.) = 1 scruple (sc. or ℈).
3 scruples = 1 dram (dr. or ʒ).
8 drams = 1 ounce (oz. or ℥).
12 ounces = 1 pound (lb.).

Liquid Measure

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).
231 cubic inches = 1 gallon.
31.5 gallons = 1 barrel (bbl.) (varies).
2 barrels = 1 hogshead (varies).
16 fluid ounces = 1 pint (apothecaries').
57.75 cubic inches = 1 liquid quart.

Dry Measure

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.)
2150.42 cubic inches = 1 bushel.
67.2 cubic inches = 1 dry quart.

Time

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
12 months (mo.) = 1 year (yr.).

Value

10 mills = 1 cent (ct. or ¢).
10 cents = 1 dime (d.).
10 dimes = 1 dollar ($).
10 dollars = 1 eagle (E.).

Angles and Arcs

60 seconds (60″) = 1 minute (1′).
60 minutes = 1 degree (1°).
90 degrees = 1 right angle.
360 degrees = 1 circumference.

Surveyors' Table of Length

7.92 inches = 1 link (li.).
100 links = 4 rods = 1 chain (ch.).
80 chains = 5280 ft. = 1 mile.

City property is usually measured by feet and decimal fractions of a foot; farm property, by rods or chains.

Surveyors' Table of Square Measure

16 square rods (sq. rd.) = 1 square chain (sq. ch.).
10 square chains = 1 acre (A.).
640 acres = 1 square mile (sq. mi.).
36 square miles = 1 township (tp.).

A square rod was formerly called a *perch*.
A quarter of an acre (40 sq. rd.) was formerly called a *rood*.

Counting

12 units = 1 dozen.
12 dozen, or 144 = 1 gross (gr.).
12 gross, or 1728 = 1 great gross.
20 units = 1 score.

Paper

24 sheets = 1 quire.
20 quires = 480 sheets = 1 ream.

Now, for convenience in counting, 500 sheets are more often called a *ream;* and the word *quire* is used only for folded note paper, other paper being usually sold by the pound.

Metric Length

A myriameter	=	10,000	meters.
A **kilometer** (km. or Km.)	=	1000	"
A hektometer (Hm.)	=	100	"
A dekameter (Dm.)	=	10	"
Meter (m.)			
A decimeter (dm.)	=	0.1	of a meter.
A **centimeter** (cm.)	=	0.01	"
A **millimeter** (mm.)	=	0.001	"

1 meter = nearly 39.37 inches.
1 kilometer = nearly 0.6 mile.

Metric Square Measure

A square myriameter	=	100,000,000	square meters.
" **kilometer** (km^2.)	=	1,000,000	" "
" hektometer	=	10,000	" "
" dekameter	=	100	" "
Square meter (m^2.)			
A square decimeter (dm^2.)	=	0.01	of a square meter.
" centimeter (cm^2.)	=	0.0001	" "
" millimeter (mm^2.)	=	0.000001	" "

The abbreviation sq. m. is often used for m^2., sq. dm. for dm^2., etc.
A square dekameter is called an *are* (land measure).

100 ares = 1 hektare (about 2.47 acres).

Metric Cubic Measure

A cubic myriameter	=	10^{12}	cubic meters.
" kilometer	=	10^9	" "
" hektometer	=	1,000,000	" "
" dekameter	=	1000	" "
Cubic meter (m^3.)			
A cubic **decimeter** (dm^3.)	=	0.001	of a cubic meter.
" centimeter (cm^3.)	=	0.000001	" "
" millimeter (mm^3.)	=	0.000000001	" "

A cubic meter is called a *stere* (in wood measure).

Metric Capacity

A **hektoliter** (hl. or Hl.)	=	100 liters.
A dekaliter (Dl.)	=	10 "
Liter (l.)		
A deciliter (dl.)	=	0.1 of a liter.
A centiliter (cl.)	=	0.01 "
A milliliter (ml.)	=	0.001 "

1 liter = 1 cubic decimeter = nearly 1 quart.

A liter of water weighs approximately 1 kilogram.

Metric Weight

A **metric ton** (t. or M.t.)	=	1,000,000 grams.
A quintal (q.)	=	100,000 "
A myriagram (Mg.)	=	10,000 "
A **kilogram** (kg. or Kg.)	=	1000 "
A hektogram (Hg.)	=	100 "
A dekagram (Dg.)	=	10 "
Gram (g.)		
A decigram (dg.)	=	0.1 of a gram.
A centigram (cg.)	=	0.01 "
A milligram (mg.)	=	0.001 "

The metric ton is nearly the weight of 1 m^3. of water at its greatest density; the kilogram, of 1 dm^3.; and the gram, of 1 cm^3.

A kilogram is about $2\frac{1}{5}$ lb.; a metric ton about 2205 lb.

English Money

12 pence (d.) = 1 shilling (s.) = $0.243 +.
20 shillings = 1 pound (£) = $4.8665.

French Money

100 centimes (c.) = 1 franc (fr.) = $0.193.

This system is used in several European countries.

German Money

100 pfennigs (pf.) = 1 mark (M.) = $0.238.

DEFINITIONS OF THE MOST COMMON TERMS USED IN ARITHMETIC

Abstract number. A number that does not refer to any particular kind of object or measure is called an *abstract number.* All *numbers* are, however, essentially abstract, whether the kind of thing numbered is mentioned or not.

Addends. Numbers to be added are called *addends.*

Addition. The process of finding a number that equals two or more numbers taken together is called *addition.*

Aliquot part. A number which is contained in a given number an integral number of times is called an *aliquot part* of that number. Thus $12\frac{1}{2}$ is an aliquot part of 100.

Amount of note. The sum of the principal (or face) and interest of a note is called the *amount.*

Analysis. In arithmetic the process of reasoning to find out a truth is called *analysis.*

Antecedent. The first term of a ratio is called the *antecedent.*

Arc. A portion of a circumference is called an *arc.*

Area. The number of square units contained in a surface is called its *area.*

Arithmetic. The science that treats of numbers and their applications is called *arithmetic.*

Assets. The property of an individual or corporation is called the *assets.*

Average. The sum of n given numbers, divided by n, is called their *average.* Thus the average of three numbers is $\frac{1}{3}$ of their sum.

Bank. An establishment for the transaction of the business of loaning sums of money, receiving money on deposit, and exchange, is called a *bank.*

Bank discount. The interest paid in advance on money borrowed at a bank is called *bank discount.*

Base. The number of which some per cent is to be found is called the *base.* The line on which a figure appears to stand is called the *base.*

Bill. A written statement of goods sold or of work done is called a *bill.*

Bill of exchange. A foreign draft is often called a *bill of exchange.*

Bond. A written or printed promise to pay a certain sum at a specified time, signed by the maker and bearing his seal, is called a *bond.*

Cancellation. Dividing both terms of a fraction by the same factor is called *cancellation.* The factor is then said to be *canceled.*

Capacity. The measure of a receptacle, indicating the number of units it contains, is called the *capacity.*

Capital. The money contributed by the stockholders to maintain a corporation is called its *capital* or *capital stock.*

Check. An order on a bank to pay money is called a *check.* An operation that tends to verify another one is called a *check.*

Circle. A figure bounded by a curve, every point of which is equidistant from a point within called the *center,* is called a *circle.*

Circumference. The bounding line of a circle is called its *circumference.* In advanced mathematics *circle* is used for *circumference.*

Commission. The percentage charged by an agent for transacting business is called *commission.*

Common denominator. A denominator common to two or more fractions is called a *common denominator* of the fractions.

Common divisor. A factor that is common to two or more numbers is called a *common divisor* or *common factor.*

Common factor. *See* Common Divisor.

Common fraction. A fraction having both terms fully expressed by figures is called a *common fraction.*

Common multiple. A multiple of each of two or more numbers is called a *common multiple* of the numbers.

Complex fraction. A fraction that has a fraction in either numerator or denominator, or both, is called a *complex fraction.*

Composite number. A number not prime is called a *composite number.*

Compound fraction. An expressed multiplication of an integer or a fraction by a fraction is called a *compound fraction;* for example, $\frac{2}{3}$ of 7 or $\frac{2}{3}$ of $\frac{4}{5}$.

Compound interest. If interest is not paid when due, but is added to the principal to form a new principal, and interest is reckoned on this amount, the obligation is said to draw *compound interest.*

Compound numbers. Denominate numbers in which two or more units are expressed are called *compound numbers.*

Concrete number. A number that refers to some particular kind of object or measure is called a *concrete number.*

Consequent. The second term of a ratio is called the *consequent.*

Consignee. The person, firm, or corporation to which goods are sent (consigned) is called a *consignee.*

Consignor. The one who sends (consigns) goods to another is called a *consignor.*

Corporation. A number of individuals organized by law to act as one body is called a *corporation;* also called a *company.*

Coupon. A note attached to a bond, promising to pay the regular interest, to be cut off and presented for payment when due, is called a *coupon.*

Creditor. A person to whom another owes money is called his *creditor.*

Cube. A regular solid with six equal squares as faces is called a *cube.* The third power of a number is called its *cube.*

Cube root. One of the three equal factors of a number is called its *cube root.*

Customhouse. The place where government duties on imported goods are received is called a *customhouse.*

Debtor. A person who owes money to another is called his *debtor.*

Decimal fraction. A fraction whose denominator is not written, but is 10 or some power of 10, is called a *decimal fraction.*

Decimal point. A period written before tenths in a decimal fraction is called a *decimal point.*

Decimal system. A system of numbers based upon the powers of ten is called a *decimal system;* for example, our common system of numbers and the metric system.

Denominate numbers. Concrete numbers denoting measure (including weight) are called *denominate numbers.*

Denominator. The number which shows into how many equal parts a unit has been divided is called the *denominator* of the fraction, — in a common fraction, the number below the line.

Difference. The number which added to one number makes another is called the *difference* between them. It is the result in taking one number from another.

Discount. A deduction made on a price or amount is called a *discount.* The difference between the face or par value and a less market value is also called a *discount,* as when stocks sell at 10% discount. Interest paid in advance on a note is called *discount.*

Dividend. The number to be divided is called the *dividend.* It is the given product of the quotient and divisor.

Dividends. The shares of the earnings divided at intervals among the stockholders of a corporation are called *dividends*, a stockholder being said to receive a *dividend.*

Division. The operation by which, given the product of two numbers and one of them, the other is found, is called *division.*

Divisor. The number by which the dividend is divided is called the *divisor.*

Draft. A check drawn by one bank on another, or by one individual on another, is called a *draft.*

Drawee. A person to whose order a draft is payable is called the *drawee.*

Drawer. A person who signs a draft is called the *drawer.*

Duties. Government taxes on imported goods are called *duties.* Some governments levy taxes on exported goods, and they are also called duties.

Equation. The expressed equality of two quantities is called an *equation.*

Equilateral triangle. A triangle whose three sides are equal is called an *equilateral triangle.*

Even number. A number that contains the factor 2 is called an *even number.*

Evolution. The process of finding a root of a number is called *evolution.*

Exact interest. Interest reckoned on the basis of 365 days to the year is called *exact interest.*

Exchange. The payment of money by means of checks, money orders, or drafts, is called *exchange.*

Exponent. A small figure placed at the right and a little above a number is called an *exponent.* If it is an integer, it shows how many times the other number is taken as a factor; thus, $3^2 = 3 \times 3 = 9$. There are also fractional exponents, where the denominator indicates a root and the numerator a power; thus, $8^{\frac{2}{3}}$ means $\sqrt[3]{8^2} = \sqrt[3]{64} = 4$.

Extremes and means. The first and last terms of a proportion are called the *extremes;* the second and third, the *means.*

Face of note. The principal mentioned in a note is called the *face of the note.*

Factors. The numbers which multiplied together make another number are called its *factors.* (This definition is limited to integers, except in speaking of roots.) In some lines of business agents are called *factors.*

Fraction. One or more of the equal parts of a unit is called a *fraction.* In general, any indicated division in the form $\frac{a}{b}$ is called a *fraction.*

Greatest common divisor. The greatest factor common to two or more numbers is called their *greatest common divisor.* It is also called *greatest common measure.*

Improper fraction. A fraction whose numerator equals or exceeds the denominator is called an *improper fraction.*

Index. A figure written in a radical sign to indicate what root is to be extracted is called an *index* of the root; thus, $\sqrt[3]{8}$ means the cube root of 8. In square root the index is not written, as $\sqrt{4}$.

Indorse. To write one's name on the back of a note or other document is to *indorse* it.

Insurance. An agreement to compensate any one for some specified loss is called *insurance.*

Integer. *See* Number.

Integral number. An *integer* is also called an *integral* or *whole number.*

Interest. Money paid for the use of money is called *interest.*

Involution. The process of finding a power of a number is called *involution.*

Isosceles triangle. A triangle two of whose sides are equal is called an *isosceles triangle.*

Latitude. The degree measure of the distance of a place from the equator is called its *latitude.*

Least common denominator. The least denominator common to several fractions is called their *least common denominator.*

Least common multiple. The least of all the common multiples of two or more numbers is called their *least common multiple.*

Length. The number of linear units a figure contains is called its *length.*

Liability. An obligation to pay money is called a *liability.*

Like numbers. Numbers referring to the same unit are called *like numbers;* thus, 2 ft. and 3 ft.

Longitude. The degree measure of the distance of a place from a prime (or standard) meridian (usually the one through Greenwich, England) is called its *longitude.*

Lowest terms. If the terms of a fraction are prime to each other the fraction is said to be reduced to its *lowest terms.*

Maker of a note. The one who signs a note is called the *maker.*

Maturity. The date on which a note or bond is due is called the date of its *maturity*.

Mean proportional. If the two means of a proportion are equal, either is called the *mean proportional* between the extremes.

Means. *See* Extremes.

Minuend. The number from which we subtract is called the *minuend*.

Mixed decimal. A number composed of an integer and a decimal fraction is called a *mixed decimal*.

Mixed number. The sum of a whole number and a fraction is called a *mixed number*.

Multiple. The product of two abstract integers is called a *multiple* of either.

Multiplicand. The number multiplied is called the *multiplicand*.

Multiplication. The process of taking one number as many times as there are units in another is called *multiplication* (by an integer).

Multiplier. The number by which we multiply is called the *multiplier*.

Mutually prime numbers. Numbers that have no common factor except one are said to be *prime to each other*, or *mutually prime*.

Net proceeds. The money that remains after all discounts and charges are paid is called the *net proceeds* of any sum.

Notation. The writing of numbers by means of symbols is called *notation*.

Note. *See* Promissory Note.

Number. A unit or a collection of units is called a *whole number* or an *integer*. *See also* Fraction.

Numeration. The naming of numbers is called *numeration*.

Numerator. The number which shows how many parts have been taken to make a fraction is called the *numerator*, — in a common fraction, the number above the line.

Obligation. A sum which a person is obliged to pay is called an *obligation*.

Odd number. A number that is not even is called an *odd number*.

Order. In writing numbers the place occupied by a figure is called its *order*. Thus we speak of units of the second order, meaning the tens.

Par value. The face or nominal value of stock is called its *par value*. In railroad stocks this is usually $100 a share.

Partial payment. A part payment on a note is also called a *partial payment*.

Payee. The person to whom the sum mentioned in a note, check, draft, or other similar document is payable is called the *payee.*

Per cent. Another name for hundredths is *per cent.* It originally meant "by the hundred."

Percentage. The result found by taking a certain per cent of the base is called the *percentage.* That part of arithmetic that deals with per cent is often called *percentage.*

Perimeter. The length of the boundary of a plane figure is called the *perimeter.*

Periods. Groups of three figures each, marked off by a separatrix, are called *periods.*

Perpendicular. A line that makes a right angle with another is said to be *perpendicular* to it.

Place value. The value of a period or of a figure, depending upon its position, is called its *place value.*

Policy. A written contract of insurance is called a *policy.*

Poll tax. A tax on a person (usually limited to male citizens over twenty-one years of age), without regard to the amount of property owned, is called a *poll tax.* (The old name for head was "poll.")

Power. The product of two or more equal factors is called a *power.*

Premium. Money paid for insurance for a certain time is called a *premium.* The excess of the market value above the par value of stock or other evidences of value is called a *premium.*

Present worth. The present value of a debt due in the future is called the *present worth.*

Prime factors. Factors that are prime numbers are called *prime factors.*

Prime number. A number that has no factors except itself and one is called a *prime number.*

Principal. A sum of money that draws interest is called the *principal.*

Proceeds. The face (or amount, if it draws interest) of a note less the discount is called the *proceeds.*

Product. The result of multiplying two or more numbers together is called the *product.*

Promissory note. A paper signed by a debtor, agreeing to pay money, is called a *promissory note.*

Proper fraction. A fraction whose numerator is less than the denominator is called a *proper fraction.*

Proportion. An expression of equality of two ratios is called a *proportion.*

Quotient. The number found by division is called the *quotient.*

Rate. The number of hundredths of the base to be taken is called the *rate;* thus the rate of interest may be 6%, the *rate per cent* of interest being then 6. The amount of tax on a dollar is often called the *rate of taxation.* The premium on each $100 of fire insurance or each $1000 of life insurance is often called the *rate* of insurance.

Ratio. The relation of one quantity to another of the same kind, as expressed by division of the first by the second, is called their *ratio.*

Reciprocal. The *reciprocal* of a fraction is the fraction with its terms interchanged. Unity divided by any number is called the *reciprocal* of the latter.

Reduction. Changing the form of a number without changing the value is called *reducing* the number. (This process is used in dealing with fractions and compound numbers.)

Reduction ascending. Reduction of compound numbers to a higher denomination is called *reduction ascending.*

Reduction descending. Reduction of compound numbers to a lower denomination is called *reduction descending.*

Remainder. The part of the dividend remaining when the division is not exact is called the *remainder.* The difference in subtraction is also called the *remainder.*

Root. One of the equal factors of a number is called a *root.* *See also* Square Root.

Scalene triangle. A triangle whose three sides are unequal is called a *scalene triangle.*

Separatrix. The comma used in separating periods in large numbers is called a *separatrix.*

Share. One of a specified number of equal parts of the capital stock of a company is called a *share.*

Similar fractions. If several fractions have the same denominator, they are said to be *similar fractions.*

Simple fraction. A common fraction whose terms are integral is called a *simple fraction.*

Solid. A magnitude that has length, breadth, and thickness is called a *solid.*

Square. A plane figure with four equal sides and four equal angles is called a *square.*

Square root. One of the two equal factors of a number is called its *square root.*

Stockholder. One who has stock in a corporation or company is called a *stockholder.*

Stocks. The shares of corporations generally are called *stocks.*

Subtraction. The operation of finding the difference between two numbers is called *subtraction.*

Subtrahend. The number which is subtracted is called the *subtrahend.*

Sum. The result in addition is called the *sum.*

Surface. A magnitude that has length and breadth, but not thickness, is called a *surface.*

Tax. Money assessed by a government for its support is called a *tax.*

Terms of a fraction. The numerator and denominator together are called the *terms of the fraction.*

Triangle. A plane figure formed by three straight lines is called a *triangle.*

Unit. Any one thing is called a *unit.* The number one is called *unity.*

Unit fraction. A common fraction whose numerator is 1 is called a *unit fraction.*

Volume. The number of cubic units a solid contains is called its *volume.*

Whole number. *See* Number.

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6. 1,000,958,326.0001, 1,001,000,988.7551, 1,000,705,999.8731.

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3. $3\frac{1}{12}$ ft., 14 yd., $2\frac{1}{4}$ lb., 3 mi., 2 T., 25 gal., 144 ft.
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2. 204 in., 258 in., 108 in., 792 in.
3. 3 ft., 12 ft., 144 ft., 162 ft., 9.9 ft.
4. 5000 lb., 6500 lb., 9 lb., 7520 lb., 427 lb., 99 lb.
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2. 358 ft. 6 in.
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6. $2\frac{1}{8}$, $2.125 = 2\frac{1}{8}$.
7. $\frac{12}{6}=\frac{24}{12}=\frac{36}{18}=2$.
8. $3\frac{1}{120}$. Common fractions.
9. 3.175. Decimals.
10. $31x + 23y$, 3123, 32 ft. 11 in., 32 lb. 7 oz.
11. $250.56.
12. $317.66.
13. $338.45.
14. $443.10.
15. $1244.53.
16. $1245.30.
17. $1831.
18. $793.50.
19. $2421.38.
20. $3703.20.
21. $2601.
22. $3314.63.
23. $8858.80.
24. $14,585.48.
25. $23,135.89.
26. $18,237.87.
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2. $83.56.
3. $200.43.
4. $297.04.
5. $103.59.
6. $331.85.
7. $681.35.
8. $228.95.
9. $308.25.
10. $1806.55.
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12. $3987.20.
13. $88.87.
14. 88 mi. 5267 ft.
15. 88 T. 1987 lb.
16. $3078.
17. 30 mi. 5258 ft.
18. 30 T. 1978 lb.
19. $1,092,855,499.
20. $10,219,191,055.
21. $1,142,901,501.75.
22. 219.
23. 2.19.
24. 21 ft. 11 in.
25. 21 lb. 15 oz.
26. 2 ft.
27. 1999 lb.
28. 11 mo.
29. 8 yr. 5 mo. 18 da.
30. 10 yr. 9 mo. 19 da.
33. $1438\frac{5}{7}$.
34. $6924\frac{19}{35}$.
35. $2303\frac{1}{21}$.
36. $1244\frac{7}{15}$.
37. $4304\frac{9}{10}$.
38. $6597\frac{23}{24}$.
39. 12 ft. $7\frac{1}{6}$ in.
40. 102 lb. $10\frac{5}{8}$ oz.
41. 228 gal. $2\frac{1}{24}$ qt.
42. $1894\frac{7}{60}$.
43. 58 sq. ft. $54\frac{5}{6}$ sq. in.
44. 78 cu. ft. $1421\frac{21}{40}$ cu. in.
45. 195 da. 23 hr. 1 min. 53.25 sec.
46. 103 da. 6 hr. 2 min. $33\frac{3}{4}$ sec.
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50. 664,005, 1,714,885.

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3. 12,168, $121.68, 1215 ft. 6 in., 117,468.
4. 10,700, 104 sq. ft. 124 sq. in., 100,700.
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6. $1132.80, 1150 gal. 2 qt., 1160 yd. 1 ft.
7. $2,551,900.50, 255,042 da. 7 hr. 30 min.
8. 1,080,000, 10,746 sq. ft. 76 sq. in., 10,642,500.
9. 351,155.
10. 1,183,896.
11. 1,244,078.
12. 1,394,196.
13. 4,700,094.
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15. 2,713,634.
16. 5,921,013.
17. $56,962.75.
18. $179,762.50.
19. $158,397.76.
20. $274,533.
21. $224,820.80.
22. $248,718.25.
23. $409,149.78.
24. $706,362.93.
25. 470 ft. 4 in.
26. 952 ft.
27. 1562 ft. 4 in.
28. 5615 ft. 8 in.
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30. 380 lb. 15 oz.
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32. 558 lb.
33. 1188 ft.

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2. 247.
3. 696.
4. $36\frac{3}{4}$.
5. $11\frac{1}{6}$.
6. $15\frac{3}{5}$.
7. $9\frac{1}{6}$.
8. $30\frac{1}{4}$.
9. $33\frac{13}{40}$.
12. 105.3.
13. 147.84.
14. 246.33.
15. 810.46.
16. 788.42.
17. 0.9625.
18. 9.35.
19. 65.4481.
20. 5.214.
21. 1.17304.
22. 20.9934.
23. 5601.76.
24. 1064.45.
25. 4.784.
26. 8.27891.
27. 15.50625.
28. 4.59375.
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33. 3.67536.
34. 5.18616.
35. 1015.6708.
36. $35.062.
37. $53.865.
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39. $591.2136.
40. 4823.
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43. 222.3056.
44. 2.118924.
45. 4.017263.
46. 58037.58.
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48. $\frac{1}{10}$.
49. $\frac{2}{5}$.
50. $\frac{3}{4}$.
51. $\frac{1}{6}$.
52. $\frac{143}{192}$.
53. $4424.76.
54. $5266.93.
55. $5645.75.
56. $12,952.14.
57. $53,609.20.
58. $82,084.86.
59. 335 ft. 10 in.
60. 905 lb. $7\frac{1}{2}$ oz.
61. 1610 yd. 21 in.
62. $17266\frac{1}{12}$.
63. $49,063\frac{5}{24}$.
64. $531,701\frac{7}{25}$.
65. $49,366\frac{1}{16}$.
66. $109,222\frac{2}{9}$.
67. $32,781\frac{27}{128}$.
68. $23.01.
69. $47.84.
70. $95.90.
71. $42.19.
72. $225.47.
73. 1211 ft. 9 in.
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76. $406\frac{2}{3}$.

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2. 23.	17. 33.	29. $248\frac{84}{121}$.	41. 23.
3. 23.	18. $189\frac{1}{2}$.	30. 92.	42. 32.
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8. 42.	20. 43.	32. 99.	44. 52.
9. 37.	21. 209.	33. 84.	45. 3.25 lb.
10. 49.	22. 313.	34. $22\frac{1}{2}$.	46. $129.
11. 62.	23. 21.	35. 13.	47. $146.25.
12. $114\frac{1}{7}$.	24. $211\frac{10}{49}$.	36. 23.	48. 0.75 A.
13. 37.	25. 309.	37. 41.	49. 13.
14. 146.	26. 129.	38. 63.	50. 4.
15. 42.	27. 32.	39. 91.	

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1. 81 gal. 3 qt.	6. 12.	11. $30\frac{1022}{1131}$.
2. 39 yd. $4\frac{2}{11}$ in.	7. 23 hr. 42 min. 15 sec.	12. 1 hr. 26 min. 39 sec.
3. 9.	8. 211 cu. ft. $1179\frac{3}{23}$ cu. in.	13. 2 yd. 2 ft. 11 in.
4. 8.	9. 4.	14. 62 lb. 8 oz.
5. 12.	10. 14 lb. 9 oz.	15. 42 min. 37 sec.

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1. $1\frac{1}{6}$.	5. $\frac{3}{4}$.	9. $3\frac{1}{3}$.	13. $\frac{1}{2}$.	17. $\frac{35}{276}$.	21. $\frac{1859}{5491}$.
2. $4\frac{1}{2}$.	6. $\frac{16}{105}$.	10. $1\frac{1}{55}$.	14. $\frac{1}{3}$.	18. $1\frac{214}{3757}$.	22. $\frac{984}{1127}$.
3. $\frac{10}{21}$.	7. $\frac{39}{55}$.	11. $\frac{11}{27}$.	15. $3\frac{3}{5}$.	19. $1\frac{35}{14472}$.	23. 6.
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1. $6\frac{6}{13}$.	7. $6\frac{41}{62}$.	13. $3\frac{4}{7}$.	19. $2\frac{125}{128}$.	25. $24\frac{1}{34}$.	31. 6.
2. $8\frac{35}{89}$.	8. $4\frac{44}{71}$.	14. $4\frac{13}{20}$.	20. $10\frac{2}{131}$.	26. $\frac{78}{149}$.	32. $8\frac{78}{231}$.
3. $10\frac{18}{23}$.	9. $3\frac{32}{39}$.	15. $8\frac{152}{293}$.	21. $11\frac{77}{83}$.	27. $4\frac{87}{368}$.	33. 3.
4. $6\frac{21}{25}$.	10. $4\frac{3}{13}$.	16. $4\frac{1}{37}$.	22. $11\frac{53}{77}$.	28. 2.	34. 7.
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1. $392.20.	**6.** $2772.96.	**11.** $20,735.76.	**16.** $934.81.
2. $1028.92.	**7.** $2122.51.	**12.** $19,521.82.	**17.** 6,218,295.
3. $1291.59.	**8.** $2880.56.	**13.** 255 ft.	**18.** 9,799,016.
4. $2286.32.	**9.** $30,328.04.	**14.** 77 lb. 5 oz.	**19.** 21,264,047.
5. $2649.64.	**10.** $30,670.29.	**15.** 17 yr. 5 mo. 6 da.	**20.** $52,962,410.

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1. $93.64.	**15.** $12.40.	**29.** 3 yr. 277 da.
2. $61.89.	**16.** $196.42.	**30.** 4° 41′ 39″.
3. $109.31.	**17.** $279.89.	**31.** 6 mi. 4755 ft.
4. $330.28.	**18.** $178.87.	**32.** 64 ft. 6 in.
5. $100.09.	**19.** $188.33.	**33.** 12° 47′ 33″.
6. $278.78.	**20.** $278.40.	**34.** 47 gal. 2 qt.
7. $121.49.	**21.** $123.60.	**35.** 2 yr. 3 mo.
8. $239.83.	**22.** $737.98.	**36.** 45° 40′ 49″.
9. $732.84.	**23.** $857.04.	**37.** $64.21.
10. $106.26.	**24.** $1993.79.	**38.** $237.12.
11. $572.93.	**25.** 7 ft. 11 in.	**39.** $161.41.
12. $573.99.	**26.** 11 lb. 12 oz.	**40.** 277,896,484 bu.
13. $386.49.	**27.** 11 yd. 29 in.	**41.** 426,408,355 A.
14. $588.89.	**28.** 2 mo. 23 da.	**42.** 201,926.

43. $7,399,509,782. **45.** $7,422,907.

44. 1,098,861, 2,837,488, 3,975,171. **46.** 635.83 mi., 927.14 mi.

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1. 604,000, 659,375, 861,500, $6,170,625.

2. 33,700, 240,900, 103,400, $1,446,400.

3. $856, $409, $5.31, $4410.50.

4. 100, $5.57, $2381.11, $5388.56.

5. $27.40, $22.44, $9522.08, $45,594.14.

6. 1, $24.86, $316.90, $24.72.

7. 2961, 6714, $8829, 5652 ft., 13,284, $211,329.

8. 67,353, 41,382, $53,207, 43,692 ft., $344,157.

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6. 139.14.	9. 548.04.	12. 654.96.	15. 266.76.	18. 319.104.

19. 537.12, \$1505.68, \$5424.84, \$17,107.72.
20. 137.6, \$546.40, \$2211.20, \$2540.
21. 2091.6, \$988.80, \$1462.50, \$8289.60.
22. 149 yd. 23. 294 yd. 24. 149,707.88.

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1. \$36.55.	11. $1\frac{5}{8}$.	21. $1383\frac{1}{3}$.	31. 259.2.
2. $1\frac{5}{6}$.	12. 79 yd.	22. \$9.05.	32. 21.44.
3. $187\frac{1}{2}$ ft.	13. \$41.40.	23. $\frac{13}{16}$.	33. 7008 yd.
4. \$15.30.	14. $1\frac{7}{12}$.	24. $46\frac{7}{8}$ ft.	34. 142 gross.
5. $1\frac{9}{32}$.	15. $85\frac{1}{3}$ mi.	25. 825.	35. 549.
6. $59\frac{3}{4}$ ft.	16. \$13.46.	26. 804.	36. 36 gross.
7. \$20.55.	17. $1\frac{13}{15}$.	27. 2478.	37. 72.
8. $4\frac{19}{20}$.	18. $175\frac{4}{5}$ yd.	28. 11,860.	38. 48 mi.
9. $222\frac{3}{4}$ lb.	19. $214\frac{1}{5}$.	29. 8175.	39. 10 mi.
10. \$23.80.	20. $2058\frac{1}{3}$.	30. 100.8.	

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1. 1521 ft., 18,252 in.	5. $3\frac{14}{27}$ cu. yd., 164,160 cu. in.
2. 98 ft., 1176 in.	6. $820\frac{3}{4}$ sq. yd., $7386\frac{3}{4}$ sq. ft.
3. 3 A. 80 sq. rd.	7. 79 sq. ft., 11,376 sq. in.
4. 3106 sq. in.	8. $35\frac{5}{9}$ cu. yd., 960 cu. ft., 1,658,880 cu. in.

9. 4.8675 T., 9735 lb., 155,760 oz.
10. 19.713 T., 19 T. 1426 lb., 630,816 oz.
11. 6.875 gal., 27.5 qt., 55 pt.

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1. 80 ch., 320 rd., 8000 li.	7. 1416 li., 14.16 ch.	13. 5 ch. 28 li.
2. 1 mi. 62 ch., 568 rd.	8. 63,360 in.	14. 8 mi. 98 rd.
3. $4\frac{1}{4}$ ch., 425 li., 3366 in.	9. 14 ch. 1 rd.	15. 121 ch. 2 rd.
4. 39.6 ft., 660 ft., 82.5 ft.	10. 58 ch. 71 li.	16. 476 ch. 48 li.
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4. 108.8 A. 9. 8.84 A. 14. 8.632 A. 19. 146.77841 A.
5. 6.05 A. 10. 1.813 A. 15. 19.60875 A. 20. 18.63555 A.
21. 4840 sq. yd., 43,560 sq. ft. 22. $272\frac{1}{4}$ sq. ft., 39,204 sq. in.
23. 102,400 sq. rd., 3,097,600 sq. yd., 27,878,400 sq. ft.
24. 3.05 sq. mi., 1952 A. 26. 3421.68 sq. ft.
25. 7.8 A., 78 sq. ch., 1248 sq. rd. 27. 380 sq. rd.
28. $x = 15$ ch., $y = 8$ ch., 100 ch., 48 A.
29. 22,440 sq. ft. 30. $3825. 31. The second, $140.
32. $x = 60$ ch., $y = 15$ ch., 190 ch., 140 A.
33. No, 400 sq. ch., 300 sq. ch. 35. $23,383.50.
34. $5400. 36. $9618.75.

Pages 58, 59

1. 20 sq. ft. 6. 41 ch. 11. $13,612.50.
2. $7.6\frac{8}{17}$ ch. 7. 18.6875 sq. in. 12. 16 ft. 4 in.
3. 68 ch. 8. 16.8 ft. 13. 32 ft. 6 in.
4. $12\frac{1}{3}$ ft. 9. 16 ft. 6 in. 14. $30\frac{1}{3}$ in.
5. 71 ft. 10. 32 ft. 15. $91\frac{103}{192}$ sq. ft.
16. 13 yd. 8 in. 17. $36\frac{3}{4}$ sq. yd.
18. 117 rd., 8307 sq. rd., 95 rd., 8835 sq. rd.
19. $118\frac{3}{4}$ rd., 95 rd., $317\frac{1}{2}$ rd., 290 rd. 20. 98 ft.

Page 60

1. 102.24 A., 204.52 A., 109.2 A., 415.96 A.
2. $20,174.06. 6. $571.20.
3. 1.596 A., 414.364 A. 7. 363 ft., 966 ft.
4. $1105. 8. 42,060 sq. ft.
5. $371.70. 9. 200 sq. rd.

Page 61

1. 121.68 sq. ft. 2. $39\frac{3}{4}$ sq. ft. 3. $16\frac{1}{16}$ sq. ft. 4. 7 yd. 5. 32 in.
6. 1181.5 sq. ft., 26.13 sq. in., 43,974.4925 sq. ft.

Page 62

1. 688 sq. ft.
2. 22.785 sq. ft.
3. 34.36$\frac{1}{2}$ sq. in.
4. 791.56$\frac{1}{4}$ sq. ft.
5. 27.405 sq. in.
6. 1 sq. yd. 108 sq. in.
7. 7614 sq. in.
8. 640 sq. in.
9. 600 sq. rd., 3$\frac{3}{4}$ A.
10. 25 rd., 20 in.

Page 64

1. 38$\frac{37}{86}$ sq. rd.
2. 154,721 sq. ft.
3. 17,589$\frac{1}{2}$ sq. ft.
4. 52,274 sq. in.
5. 1031$\frac{1}{3}$ sq. ft.
6. 4264$\frac{1}{8}$ sq. ft.
7. 17$\frac{1}{2}$ sq. yd.
8. 3 ft. 2 in.
9. 4056.25 sq. ft.
10. 6084.375 sq. ft.
11. 9731.25 sq. ft.
12. 4461.875 sq. ft.
13. 1623.75 sq. ft.
14. 1082.5 sq. ft.
15. 2706.25 sq. ft.
16. 4330 sq. ft.
17. 2706.25 sq. ft.
18. 2165 sq. ft.

Page 66

1. 160 A.
2. 160 A.
3. 80 A.
4. 80 A.
5. 80 A.
6. 80 A.
7. 40 A.
8. 40 A.
9. 40 A.
10. $10,400.
11. $3000.
12. 160 A.
13. 320 A.
14. 480 rd.
15. 2 mi.
16. 3 A., $14.25.

Pages 68, 69

1. 8$\frac{3}{4}$ cd., 24 od.
2. 112 loads.
3. 33\frac{1}{3}$, 41\frac{2}{3}$.
4. 7486$\frac{1}{3}$ cu. ft.
5. 129,600 cu. ft.
6. 111\frac{1}{9}$.
7. 1904 cu. ft., 65,280 lb.
8. 6000 cu. ft.
9. 60 cu. ft., 65 cu. ft., 103,680 cu. in., 112,320 cu. in.
10. 54$\frac{1}{6}$ bu.
11. 7, 3.
12. 457$\frac{1}{3}$ cu. ft.
13. 16$\frac{41}{49}$ T.
14. 12 $\frac{123}{196}$ T.
15. 7$\frac{1}{2}$ T., 5$\frac{1}{2}$ T.
16. 300 cu. ft., 8.

Page 70

1. 13,858 cu. yd.
2. 19$\frac{1}{2}$ cu. yd., 234 cu. yd.
3. 9$\frac{3}{4}$ hr., 39 min.
4. 710$\frac{2}{3}$, 78$\frac{26}{27}$, 21,320 ft.
5. 5.19675 da.
6. $2.67.
7. 6,880,346$\frac{2}{11}$.
8. $3, $3.50.

Pages 73, 74

1. 112,000 cu. ft.
2. 6 ft.
3. 15 in.
4. 112.5 sq. ft.
5. $43\frac{1}{3}$ ft.
6. $12\frac{1}{2}$ ft.
7. 27.5 ft.
8. 3.9 in.
9. 7.2 in.
10. 11 ft.
11. 12 ft.
12. 9.5 ft. × 12 ft. × 12 ft.
13. $30.
14. 9 ft. × 14.3 ft. × 16.4 ft.
15. $68\frac{68}{215}$ bu.
16. 160.
17. $3280.50.
18. $1615\frac{61}{77}$ gal.
19. $26.58.
20. $7\frac{123}{275}$ T.
21. $59\frac{65}{77}$ gal.
22. $691.20.
23. $46.91.
24. $291.55.

Page 75

1. First, 2 to right, 4 up.
2. First, 2 to left, 4 up.
3. First, 2 to left, 3 down.
4. First, 3 to right, 2 down.

Page 77

1. 1 hr. 48 min. 17 sec.
2. 5 hr. 2 min. 2 sec.
3. 11 hr. 8 min. 10 sec. A.M.
4. 105°, 9 P.M.
5. 4 A.M., 8 P.M.
6. 8 hr. 22 min. 42 sec. A.M., 8 hr. 2 min. 42 sec. A.M.

Pages 78, 79

1. 30° 47′ 30″.
2. 105° W.
3. 2° 5′ 40.5″.
4. 22° 45′ W.
5. 135° E.
6. 150° E.
7. 37° 30′ W.
8. Fast, 3 hr. 30 min. 3 sec.
9. 12 hr. 9 min. $38\frac{2}{15}$ sec. P.M.
10. 7 : 30 A.M.
11. 46° 53′ 45″ W.
12. 110° 11′ 12.2″ W.
13. 17° 26′ 15″ E.

Page 82

1. 6 hr. 9 min. 21 sec. P.M.
2. 9 P.M., 6 A.M., 4 A.M.
3. 4 hr. 20 min. 54 sec.
4. 9 min. $33\frac{1}{3}$ sec.
5. 2 : 30 P.M., 1 : 30 P.M.
6. 1 hr. 9 min. 21 sec. P.M.
7. Standard, 9 min. 42.72 sec.
8. 6 min. 25.1 sec.
9. 9 hr., 9 hr. 3 min. 17.62 sec.
10. 9 P.M. Dec. 31, 10 P.M. Dec. 31, 11 P.M. Dec. 31, 5 A.M. Jan. 1, 6 A.M. Jan. 1.

Page 83

1. 4 min. $6\frac{3}{10}$ sec., 5 min. $\frac{4}{5}$ sec.
2. 44 min. 13.9 sec., 5 hr. 49 min. $28\frac{1}{3}$ sec.
3. 3 hr. 14 min. $43\frac{12}{25}$ sec., 5 hr. 4 min. $20\frac{1}{5}$ sec.
4. 54 min. $33\frac{1}{10}$ sec., 2 hr. 19 min. $15\frac{22}{25}$ sec.
5. 11 A.M., 10 hr. 50 min. 17.28 sec. A.M.
6. 11 hr. 54 min. 59.2 sec. A.M., 10 hr. 54 min. 59.2 sec. A.M.
7. 32 min. $32\frac{8}{15}$ sec.
8. 1 hr. 1 min. 50 sec.
9. 29 min. $35\frac{1}{3}$ sec.
10. 1 hr. 36 min. $40\frac{1}{3}$ sec.
11. 1 hr. 10 min. 49 sec.
12. 33 min. $18\frac{2}{3}$ sec.
13. 1 hr. 41 min. $4\frac{2}{3}$ sec.
14. 2 hr. 11 min. $13\frac{4}{15}$ sec.
15. 25 min. $9\frac{4}{15}$ sec.
16. 3 hr. 13 min. $5\frac{2}{15}$ sec.
17. 4 hr. 13 min. 18 sec.
18. 36 min. 55 sec.
19. 46° 2′.
20. 151° 1′.
21. 79° 16′ 15″.
22. 107° 19′ 30″.
23. 124° 46′ 45″.
24. 87° 43′ 45″.

Page 86

1. \$560.80.
2. \$246.24.
3. $57\frac{1}{7}\%$, $62\frac{1}{2}\%$, $27\frac{3}{11}\%$, $26\frac{2}{3}\%$, $15\frac{5}{8}\%$, $41\frac{2}{3}\%$.
4. \$308.50, \$1543.
5. \$26.25.
6. \$579.39, \$255.75, \$325.50.
7. \$142.50.
8. 1533.
9. $\frac{12}{25}$.
10. $\frac{2}{5}$.
11. $\frac{17}{20}$.
12. $\frac{2}{7}$.
13. $\frac{67}{400}$.
14. $\frac{9}{40}$.
15. $\frac{5}{9}$.
16. $\frac{3}{8}$.
17. $\frac{7}{9}$.
18. $\frac{3}{11}$.
19. $\frac{1}{7}$.
20. $\frac{89}{500}$.
21. $\frac{259}{800}$.
22. $\frac{279}{1000}$.
23. $\frac{473}{700}$.
24. $\frac{341}{640}$.
25. 17.
26. \$37.07.
27. \$486.
28. \$42.06.

Page 88

1. 292 da.
2. \$55.80.
3. \$25.69, \$51.38, \$12.85, \$64.23.
4. 3933.
5. 4 da.
6. \$252.
7. \$161.20, \$145.08.
8. \$656.08.
9. 235.

Page 90

1. $33\frac{1}{3}\%$.
2. $11\frac{1}{9}\%$.
3. 20%.
4. 15%.
5. $16\frac{2}{3}\%$.
6. 12,626.12 sq. mi.
7. 90%.
8. 15%.

Page 91

1. $13\frac{1}{48}\%$.
2. $20\frac{5}{6}\%$.
3. $26\frac{34}{39}\%$.
4. $5\frac{13}{15}\%$.
5. 11.98+%.
6. $21\frac{3}{7}\%$.
7. $18\frac{58}{169}\%$.

Page 92

1. 15.
2. 50%.
3. $0.21.
4. $21\frac{3}{7}\%$.
5. 25%.
6. $20.
7. The first, $23\frac{4}{7}\frac{1}{3}\%$.
8. 15.
9. $135.20.

Page 95

1. $42.50, $17.
2. $35.75, $28.60.
3. $29.25, $17.55.
4. $324, $113.40.
5. $175.
6. $225.
7. 400 ft.
8. 28,200.
9. 11%.
10. $1650.

Page 96

1. $3162.50, $3080.
2. $3080, $3220.
3. $371, $367.50, $362.25, $365.75.
4. 238.
5. $891, $907.50, $924, $899.25.
6. $13,175.
7. $12\frac{1}{2}\%$.
8. $8308.75.

Page 97

1. $3\frac{1}{3}\%$.
2. 60%.
3. $12.50.
4. $31\frac{1}{4}$ sq. mi.
5. $8.40.

Page 98

1. 60%, $166\frac{2}{3}\%$, $37\frac{1}{2}\%$, $62\frac{1}{2}\%$.
2. 22,500, 50,000.
3. $175.
4. $124.08.
5. $62.50, $92.50.

Page 100

1. $48.13.
2. $593.75.
3. $88.73.
4. $38.96.
5. $21.95.
6. $25.08.
7. $105.08.
8. $268.54, $219.71, $292.95.
9. $261.25.
10. $372.75.
11. $546.75.

Page 101

1. $21.98.
2. $58.04.
3. $43.90.
4. $28.27.
5. $9.19.
6. $37.08.
7. $18.85.
8. $45.10.
9. $33.98.
10. $106.91.
11. $78.07.
12. $36.28.
13. $622.71.

Page 104

1. $12.80.
2. $7.
3. $7.88.
4. $23.18.
5. $15.40.
6. $35.25.
7. $90.30.
8. $46.43.
9. $221.58.
10. $252.62.
11. $271.25.
12. $17.73.
13. $33.16.
14. $84.40.
15. $187.43.
16. $380.67.
17. $1933.29.
18. $2560.16.
19. $345.14.
20. $466.61.
21. $408.50.
22. $147.33.
23. $263.18.
24. The second plan, $4.20.

Pages 105, 106

1. 123 da.
2. 158 da.
3. 110 da.
4. 224 da.
5. 122 da.
6. 127 da.
7. $4.90.
8. $11.59.
9. $18.19.
10. $8.22.
11. $7.48.
12. $4.45.
13. $10.27.
14. $8.99.
15. $7.01.
16. $1003.96.
17. $152.44.
18. $76.73.
19. $455.89.
20. $283.07.
21. $896.15.
22. $645.40.
23. $696.18.
24. $844.80.
25. $1042.66.
26. $2098.68.
27. $7.64.
28. $471.59.

Page 107

1. $37.50.
2. $52.90.
3. $3.80.
4. $80.60.
5. $7.37.
6. $364.50.
7. $5.38.
8. $18.75.
9. $10.08, same.
10. $12.01, $11.63, $11.88.

Page 108

1. $61.15.
2. $584.40, $596.28, $578.46, $566.58.
3. $15.33, $15.12.
4. $1.82, $3.95.
5. $50 – $49.32 = $0.68.
6. $6.42, $6.51.
7. $34.19, $34.67.
8. $2.37, $2.41.
9. $2.49, $2.53.
10. $4.75, $4.81.
11. $46.23, $46.88.
12. $34.72, $35.20.
13. $17.17, $17.41.
14. $40.11, $40 at 60 da., $40.67 at 61 da.
15. $13.95, $14.15.

Page 109

1. 5%.
2. 6%.
3. 4%.
4. 5%.
5. 4%.
6. 4%.
7. 6%.
8. 4%.
9. 4%.
10. $3\frac{1}{2}\%$.
11. 3%.
12. $5\frac{1}{2}\%$.
13. 3%.
14. $3\frac{1}{2}\%$.
15. 2%.
16. $4\frac{1}{2}\%$.
17. 4%.
18. 4%.
19. 6%.
20. 5%.

Page 110

1. 2 yr. 6 mo.
2. 2 yr. 6 mo.
3. 2 yr. 7 mo.
4. 1 yr. 8 mo.
5. 1 yr. 9 mo.
6. 1 yr. 6 mo.
7. 1 yr. 5 mo. 3 da.
8. 2 yr.
9. 1 yr. 2 da.
10. 4 mo.
11. 16 yr. 8 mo.
12. 2 yr. 6 mo.
13. 11 yr.
14. 14 yr.

Pages 112, 113

1. $41.33.
2. $126.
3. $2.68.
4. $24.59.
5. $56.11.
6. $105.75.
7. $102.62.
8. $1022.71.
9. 5%.
10. 4%.
11. $3\frac{1}{2}\%$.
12. $3\frac{1}{2}\%$.
13. 1 yr. 2 mo. $10\frac{10}{77}$ da.
14. 2 yr. 4 mo. 28 da.
15. 5 yr.
16. 1 yr. 6 mo. 15 da.
17. 3 yr. 6 mo.
18. $840.
19. $30.
20. $55 less.
21. The first, $3.13.
22. $3\frac{13}{14}\%$.
23. $405.
24. $1035.
25. $3327.50 loss.
26. $5302.50.
27. Net income $190 in each case.
28. $19\frac{3}{7}\%$.

Page 114

1. $4\frac{1}{6}$ A., 1000.
2. $266\frac{2}{3}$ lb., $8\frac{8}{9}$ lb.
3. 7200.
4. 1920, 40.
5. 10 da., $18.
6. 1500.

Pages 115, 116

1. $12\frac{1}{2}\%$.
2. $440, $17\frac{1}{6}\%$.
3. $11\frac{7}{13}\%$, gain.
4. $37\frac{1}{2}\%$, $56\frac{1}{4}\%$.
5. $16.31, gain.
6. $13.40, $13.10, $13.20, $13.35.
7. $0.25.
8. $3.59.
9. $38\frac{8}{9}\%$.
10. $59.40, $30\frac{5}{9}\%$.
11. $0.29.
12. $25.50, $10\frac{10}{19}\%$.
13. $2, $4\frac{1}{6}\%$.
14. $6, 25%.
15. $1.44 gain, $8\frac{16}{23}\%$.
16. $2.88 gain.
17. $42, $3.99.

Page 118

1. 32.83.
2. 457.104.
3. 19.032.
4. 217.98.
5. 65.1.
6. 2.38.
7. 3.65.
8. 8.37.
9. 0.51.
10. 1.1.
11. 2.1.
12. 8.
13. 9.775.
14. $1.34\frac{5}{7}$.
15. $0.9\frac{1}{7}$.

Page 120

1. 187, 341.
2. 169, 533.
3. 189, 261.
4. 476, 574.
5. 6.51, 6.72.
6. 4.41, 11.18
7. 33, 55, 77.
8. 65, 91, 117.
9. 147, 231, 273.
10. 583, 689, 901.
11. 8.19, 10.01, 15.47.
12. .891, 1.701, 1.863.
13. 12.07, 13.49, 14.91.
14. 260, 130.
15. $1600, $1200.
16. $28.75, $40.
17. $13.20, $12.70.

Page 121

1. $22.6\frac{64}{73}$.
2. 4.135.
3. $15.7\frac{9}{11}$.
4. 0.24.
5. $2.003\frac{9}{11}$.
6. $1.281\frac{7}{9}$.
7. $2.3\frac{75}{107}$.
8. $77\frac{13}{19}$.
9. 1.5.
10. 3120.
11. $106.7\frac{48}{97}$ in.
12. $1.4\frac{89}{171}$.
13. 5473.

Pages 122, 123

1. 46 bbl.
2. $2142.
3. 302 ft.
4. $101\frac{1}{2}$ ft.
5. 4 hr.
6. 4.
7. $21\frac{13}{15}$ sec.
8. 12 min. 40 sec.
9. 75 mi.
10. 3.
11. 5.
12. 8 min. 23.2 sec.
13. 29 min. $59\frac{1}{3}$ sec.
14. $14.70.
15. $4.

Pages 126, 127

1. 15 bbl.
2. $2\frac{2}{3}$ bu.
3. $992.79.
4. $7.20.
5. $5\frac{5}{6}$.
6. $132.50.
7. 50.
8. $\$9.37\frac{1}{2}$.
9. $5\frac{1}{3}$ da.
10. 6, 2.
11. $31\frac{139}{267}$ da.
12. 0.75 mo.
13. 180 T.
14. 5 da.
15. $4\frac{3}{8}$ da.
16. $48, $35.
17. 10.
18. 40.
19. $17.50.
20. 14.
21. 1350.

Pages 128-130

1. $1\frac{163}{1280}$ A.
2. 21 ft.
3. \$3.
4. \$4.55.
5. \$0.82$\frac{1}{2}$.
6. \$8.05.
7. \$30.37$\frac{1}{2}$.
8. \$11.25.
9. \$26.25.
10. \$10.
11. \$138.
12. $242\frac{3}{16}$ lb.
13. 6300.
14. 12,675,000.
15. 2,281,500.
16. 20.24 oz.
17. 12 lb. $9\frac{3}{5}$ oz.
18. 19 lb.
19. 84%.
20. 6,445,312$\frac{1}{2}$ lb., 2,352,539,062$\frac{1}{2}$ lb.
21. 189 bbl., 891 bu.
22. 2,038,405,600 bu.
23. \$17.50.
24. \$450,000,000, \$45,000,000.
25. 34 bu., 25¢, \$8.50.
26. \$1100.
27. 15 bu., \$58,500,000.
28. $10\frac{1}{3}$ T.
29. 30 min.

Pages 132-137

1. $\frac{100}{241}$%, $99\frac{141}{241}$%.
2. 3.125 lb., 3.125 lb., 243.75 lb.
3. 299.04 lb.
4. 139.32 lb.
5. 138,750.
6. 28,224.
7. 80 lb., 300 lb., 60 lb., 225 lb.
8. 875 lb., 950 lb., 175 lb.
9. 1075 lb., 800 lb., 125 lb.
10. \$12.07$\frac{1}{2}$.
11. 262.5 lb., 37.5 lb., 112.5 lb., 337.5 lb., 700 lb., 100 lb., 300 lb., 900 lb., 1050 lb., 150 lb., 450 lb., 1350 lb.
12. \$18.82.
13. \$18.72.
14. \$16.86.
15. \$60.50.
16. $269\frac{3}{13}$%.
17. $137\frac{1}{27}$%.
18. 96 bu., $18\frac{2}{7}$ bu.
19. 37.625 lb.
20. 43.895$\frac{5}{6}$ lb.
21. \$10.53$\frac{1}{2}$.
22. 21,000 lb.
23. $3\frac{1}{8}$%.
24. 66 lb.
25. \$48.
26. \$0.54, the second.
27. A. 152 lb., 44.08 lb., \$12.34. B. 185 lb., 62.9 lb., \$17.61.
C. 277 lb., 74.79 lb., \$20.94. D. 114 lb., 34.2 lb., \$9.58.
E. 212 lb., 55.12 lb., \$15.43. F. 421 lb., 147.35 lb., \$41.26.
G. 521 lb., 166.72 lb., \$46.68.

		a	*b*	*c*	*d*
28.	1.	672 lb.	168 lb.	196 lb.	\$45.08.
	2.	672 lb.	168 lb.	196 lb.	\$45.08.
	3.	528 lb.	132 lb.	154 lb.	\$35.42.
	4.	408 lb.	102 lb.	119 lb.	\$27.37.
	5.	528 lb.	132 lb.	154 lb.	\$35.42.

29. 30.72 lb., 35.84 lb.
30. 4%, 34 lb., $39\frac{2}{3}$ lb., \$9.92.
31. 5 lb.
32. 500 lb.
33. $181\frac{9}{11}$ lb.

Pages 139, 140

1. $1200.
2. $1150.
3. $83\frac{1}{3}$%.
4. $1350.
5. 300 da.
6. $2.
7. $1690.
8. $9 decrease.
9. $4, second plan.
10. $5.16, second place.
11. $270, $324, $450, $600.
12. $4.60, $4.37, $4.83, 6.13\frac{1}{3}$, operating department.
13. $25.

Pages 141, 142

1. $91.25.
2. $465.
3. $375.
4. $231.
5. $168.30.
6. $911.10.
7. $107.18.
8. $1852.50.
9. $73.20.
10. $167.

Page 144

1. $3000, $3038.77.
2. $4720, $4764.06.
3. $3920, $3937.02.
4. $2520, $2532.40.
5. $3080, $3095.15.
6. $1933.75, $1940.25.
7. $713.07, $717.77.
8. $12,400, $12653.19.
9. $502.09, $507.42.
10. $312.69, $314.40.
11. $225.23.
12. $324.73.
13. $731.59.
14. $765.41.
15. $900.94.
16. $541.21.
17. $1093.08.
18. $1126.16.
19. $2143.72.
20. $3152.83.
21. $9.46 more at simple interest.
22. $35.13.
23. $1380.86.

Page 146

1. $509.
2. $1176.92.
3. $524.23.
4. $2462.35.
5. $930.75.
6. $211.44.
7. 715.23.
8. $1668.
9. $246.80.
10. $272.

Page 148

1. $155.25.
2. $156.
3. $159.23.
4. $517.43.
5. $810.40.
6. $848.74.

Page 151

1. $4.38, $870.62.
2. $12.19, $962.81.
3. $2.13, $422.87.
4. $21.40, $2546.60.
5. $12.50, $1237.50.
6. $18.75, $1481.25.
7. $32.06, $2532.94.
8. $44.69, $3205.31.
9. $15.94, $4234.06.
10. $1.63, $323.87.
11. $3.56, $423.94.
12. $0.99, $236.51.
13. $6.25, discount.
14. $3.38, discount.
15. $18.75, discount.
16. $1500.
17. $22.50, $1477.50; $15, $1485.
18. $700, discount = $3.50.

Page 153

1. $0.75, $300.25 ; $0.80, $300.20.
2. $6.86, $799.81 ; $6.99, $799.68.
3. $2.78, $552.72 ; $2.87, $552.63.
4. $1.39, $331.74 ; $1.43, $331.70.
5. $5.03, $999.97 ; $5.19, $999.81.
6. $21.59, $1252.54 ; $21.77, $1252.36.

The second set of answers in Exs. 1–6 are the results obtained when both the first and last days of the discount period are counted.

7. $31.25, $3718.75.
8. $35.21, $4189.79.
9. $99.75, $9400.25.
10. $65.63, $7434.37.
11. $54.57, $4170.43.
12. $86.41, $5488.59.
13. $34.38, $7465.62.
14. $26.13, $4723.87.
15. $0.21, $42.29.
16. $0.14, $27.46.
17. $0.18, $35.32.
18. $0.14, $28.61.
19. $1.26, $124.24.
20. $2.75, $272.50.
21. $5.63, $369.87.
22. $6.76, $443.99.
23. $1.91, $455.59.
24. $1.23, $295.27.
25. $3.60, $284.
26. $4.69, $370.71.
27. $31.97, $2718.03.
28. $38.07, $3236.93.
29. $45.82, $4714.18.
30. $26.42, $2718.58.
31. $6.37, $1538.63.
32. $5.05, $1569.95.
33. $1250.
34. $1747.07.

Pages 155, 156

1. $393.75.
2. $337.28.
3. $102.52.
4. $651.97.
5. $191.96.
6. $178.85.
7. $211.08.
8. $476.18.
9. $453.49.
10. $375.58.
11. $60.20.
12. $408.34.
13. $297.09.

Page 157

1. $189.95.
2. $106.42.
3. $558.99.

Page 159

1. $1.34, $0.67, $2.68.
2. $5.67.
3. $5.42, $1.17.
4. $5.87, $8.80.
5. $1.03, $0.20.
6. $6.43, $12.86.
7. $3.74, $11.22.

Page 161

1. $65.87.
2. $540.
3. $95.42.
4. $62.04.
5. $121.87.
6. $345.
7. $3317.28.
8. $3375.25.
9. $304.77.

Pages 162, 163

1. $187.70.
2. $29.30.
3. $1.39.
4. $1.35.
5. $63.07.
6. $4.40.
7. $4.65.
8. $481.41.

Pages 165, 166

1. $31.05, $56.25, $65.70.
2. $440, $670, $330.
3. $810, $975, $615.
4. $75, $54, $66.
5. $210, $315, $560.
6. $110, $88, $66.
7. $65.10, $36.
8. $240, $315.
9. $11.10, $14.40.
10. $5446.45, $3853.55.
11. $175, $2325.
12. $3925, $2475.
13. $3900, $900, $2400.
14. $288, $189, $378.
15. $8.16, $6.36, $10.20.

Page 169

1. $300.60.
2. $3203.20, $2501.25.
3. Add 15¢, 25¢, 10¢, 30¢, 30¢.
4. $2751.10.
5. 0.3%.
6. $2750, $2.75.
7. $3750, $1.50.
8. $250.25, $150.15, $100.10, $350.35.
9. Draft, 10¢.
10. Registered letter, 2¢.

Pages 170, 171

1. $749.25.
2. $549.45.
3. $149.70.
4. $\frac{4}{15}$%.
5. $\frac{1}{5}$%.
6. $74.85.
8. $\frac{1}{20}$%.
9. $249.75.
10. $99.90.
11. $0.75.

Page 173

1. $3756.70
2. $3496.50.
3. $751.50.
4. $6753.38.
5. $2452.45.
6. $17,465.
7. $5161.
8. $3408.50.
9. Discount, $\frac{1}{8}$%.
10. Discount, $\frac{1}{5}$%.
11. $249.75.
12. $2447.50.
13. $379.75.
14. $3240.

Page 175

1. 720d.
2. 528d.
3. 786d.
4. 44d.
5. 219d.
6. 1214d.
7. 8.66$\frac{2}{3}$s.
8. 9.25s.
9. 66.75s.
10. £2.1.
11. £3.2.
12. £6.225.
13. 2500 c.
14. 3700 c.
15. 3530 c.
16. 2.75 fr.
17. 4.75 fr.
18. 12.75 fr.
19. 3.5 M.
20. 4.8 M.
21. 92.75 M.
22. 17,500 pf.
23. 20,000 pf.
24. 1075 pf.
25. $365.25.
26. $331.16.
27. $80.36.
28. £8.
29. £5.
30. £9.
31. $17.85.
32. $29.75.
33. $892.50.
34. 40 M.
35. 60 M.
36. 300 M.
37. $16.41.
38. $44.39.
39. $144.75.

Page 177

1. $433.50.
2. $58.75.
3. $176.02, $145.51.
4. £15.
5. 650 fr.
6. 322.12 M.
7. $610.73.
8. $111.
9. Demand draft $1.37 better.
10. $130.69.

Page 180

1. 0.2753 m.
2. 4.764 m.
3. 29.38 m.
4. 4.862 km.
5. 0.12758 km.
6. 0.628341 km.
7. 78.33$\frac{1}{3}$ km.
8. 44.16$\frac{2}{3}$ km.
9. 3.249 km.
10. 450.6 mi.
11. 171.6 mi.
12. 20.94 mi.
13. 3.168 mi.
14. 1.7856 mi.
15. 0.88686 mi.
16. 55$\frac{1}{4}$ ft.
17. 208 ft.
18. 351 ft.
19. 224.055 ft.
20. 97.045 ft.
21. 264.212 ft.
22. 1850.39 in.
23. 127.5588 in.
24. 184.2516 in.
25. 1358.265 in.
26. 1.114171 in.
27. 118.11 in.
28. 2.7559 in.
29. 659.75 ft.
30. 121.875 ft.
31. 125.4 mi.
32. 106.8 mi.
33. 61 fr. 20 c.
34. 27.9$\frac{3}{7}$ mi.

Page 181

1. 7,500,000,000,000 cm^2.
2. 370,000 cm^2.
3. 42.96 cm^2.
4. 62,500 cm^2.
5. 80,583,750 A., using 2$\frac{1}{2}$ A. = 1 ha.

Page 182

1. 19,750 m^3.
2. 0.00042765384 m^3.
3. 1161.63 m^3.
4. 1 m^3.
5. 100 m^3.
6. 1 m^3.

Page 184

1. 170 kg.
2. 10 kg.
3. 3.275 kg.
4. 7012$\frac{5}{11}$ kg.
5. 22 kg.
6. 7.275 kg.
7. 1001$\frac{9}{11}$ kg.
8. 3 kg.
9. 79 kg.
10. 77 lb.
11. 200 5-ct. pieces.
12. 0.0352 oz., 0.0022 lb.
13. 55 lb.
14. 11$\frac{1}{4}$ t., 86.7 t.
15. 30.94 t., 8.4 t.
16. 28.2906 kg.
17. 0.6075 t.
18. 3174.8706 g.
19. 521.1 g., 0.5211 kg.
20. 165 lb.

Page 186

1. 7.224 kg.
2. 5.81 kg., 12.782 lb.
3. 11,811 in., 984.25 ft., 0.3 km.
4. 337.75 g., 5201.35 grains.
5. 2.25 kg., 4.95 lb.
6. 2929.5 g.
7. 5138 g.
8. 29.625 kg.
9. 160.2 kg.
10. 0.06 ha., 0.124 A.
11. 0.77 lb., 12.32 oz., 0.35 kg.
12. 30,000 kg., 1875 lb.

Page 189

1. $1052.18.

Pages 190, 191

1. 42$\frac{2}{23}$%.
2. 22$\frac{58}{61}$%, 18$\frac{2}{61}$%, 28$\frac{17}{61}$%.
3. 4$\frac{433}{488}$%.
4. 2$\frac{8}{11}$%.
5. 97$\frac{3}{11}$%.
6. 3566$\frac{2}{3}$%.
7. 32%.
8. 40$\frac{8}{11}$%.
9. $92,049,300.
10. 11$\frac{1}{10}$%.
11. $229,926,355.40.
12. $53,590,000, $179,410,000.
13. $9,180,000.

Page 192

1. 0.0086$\frac{22}{23}$%.
2. $7.80.
3. 9.1%.
4. $0.08, $1,826,512.
5. $2000, $296.
6. 507,500 mi., 475,020,000 mi.
7. $514.89.

Pages 193, 194

1. $3,220,000, $3,590,000.
2. 474 lb.
3. 459 lb.
4. 4,000,000 lb., 31,120,000 lb.
5. 24 hr.
6. 15,000.
7. 23, $4\frac{8}{23}$%.
8. 23 knots.
9. 10.1 in.
10. 130,206 lb.
11. 72 hr.
12. 20, 20.
13. 27,200 T.
14. $696, $8352.
15. 10,420$\frac{3}{4}$ ft.

Page 196

1. $1957.50, *ad valorem*.
2. $1050, specific.
3. $127.84.
4. $131.49.
5. $1620.
6. $2830.
7. $144.75, $36.19, $180.94.
8. $1905.88, $762.35, $2668.23.
9. $790.75.

Page 197

1. $900.
2. $250.
3. $1125.
4. $62.50.
5. $39.
6. $216.
7. $37.50.
8. $80.
9. $197.
10. $75.
11. $278.50.
12. $15.
13. $64.
14. $262.50.
15. $139.50.
16. $27.

Page 199

1. $40.63.
2. $1125.
3. $0.0055.
4. $64.75.
5. $0.0075.
6. $55.60.
7. $0.004.
8. $0.003.
9. $0.0055.
10. $0.003.
11. $0.0045.
12. $0.006.
13. $2,750,000.
14. $3,500,000.
15. $8,250,000.
16. $9,125,000.
17. $19,035.
18. $47,625.
19. $39,875.
20. $83,070.

Page 200

The second answer is the net tax plus the collector's 1%, calling a fraction of a cent one cent.

1. $35.75, $36.11.
2. $48.13, $48.62.
3. $26.81, $27.08.
4. $42.21, $42.65.
5. $66, $66.66.
6. $90.75, $91.66.
7. $134.20, $135.55.
8. $129.25, $130.55.
9. $171.05, $172.77.
10. $127.88, $129.16.
11. $103.82, $104.86.
12. $248.88, $251.37.

Pages 203, 204

1. \$23.75.
2. \$30.80.
3. \$43.13.
4. \$160.
5. \$75.
6. \$3500.
7. \$2052.
8. \$202.13.
9. \$1.54, \$0.51.
10. \$1.20, \$0.40.
11. \$10,000.
12. \$10,500, \$16,800.
13. \$6050.
14. \$31.25, \$1500.
15. 3-year policy, \$10.50.
16. \$5880.
17. \$2972.98.
18. \$7785.
19. \$25,000.
20. \$1200, \$3.
21. $16\frac{2}{3}$ yr.
22. \$280.

Page 205

1. \$9000.
2. \$44,900.
3. \$123.35.
4. \$102, \$163.20.
5. \$54, \$162, \$108.
6. \$16.92, \$25.38, \$84.60.

Pages 208, 209

1. \$136.95.
2. \$410.40, \$4104.
3. \$2739.
4. \$171, \$3420.
5. \$243.75, \$4875.
6. \$66, \$82.25.
7. \$263.60, \$6590.
8. \$4076.
9. \$2739, \$3834.60.
10. \$24.60.
11. \$951.50.
12. \$1912, \$922.
13. \$402.
14. \$1072.80.
15. \$331.20.
16. \$1917.

Page 211

1. 15,000.
2. \$1,250,000.
3. \$500.
4. \$270.
5. \$135.
6. $1\frac{1}{4}\%$.
7. \$2,750,000.
8. \$12,500, \$250.
9. $2\frac{3}{4}\%$.
10. 5%.

Page 213

1. \$9553.13.
2. \$3415.63.
3. \$8512.50.
4. \$7843.75.
5. \$16,387.50.
6. \$43,125.
7. \$10,730.
8. \$31,937.50.
9. \$29,553.13.
10. \$44,468.75.
11. \$20,667.50.
12. \$17,085.
13. $68\frac{1}{8}$.
14. 40.

Pages 214, 215

1. $150.
2. $306.25.
3. $2.50 loss.
4. $16,467.50.
5. $31.25 gain.
6. $125 loss.
7. $275 loss.
8. $75 gain.
9. Neither gain nor loss.
10. $131.25 loss.
11. $12.50 gain.
12. $50 gain.
13. $106.25 loss.
14. Neither gain nor loss.
15. $18.75 gain.
16. $93.75 gain.
17. $68.75 gain.
18. $87.50 loss.
19. $287. 50 loss.
20. $100 loss.
21. $124\frac{7}{8}$.
22. $142\frac{3}{8}$.
23. $24,393.75.
24. 250.
25. 200.
26. $272.

Pages 217, 218

1. $7218.75, $7706.25, $5146.88, $7500.
2. 4%.
3. 5% stock @ $139\frac{7}{8}$.
4. Same.
5. $3\frac{1}{2}$%.
6. $3\frac{49}{117}$%.
7. $5\frac{1}{2}$% note.
8. 7% stock @ $149\frac{7}{8}$.
9. 5% bond @ 121, $\frac{100}{11737}$%.
10. The same.
11. The same.
12. The same.
13. 5% bond @ 108, $\frac{5}{351}$%.
14. 6% bond @ 140, $\frac{10}{819}$%.
15. $3\frac{1}{2}$% bond @ 107, $\frac{25}{2461}$%.
16. 5% bond @ 111, $\frac{1}{222}$%.
17. The same.
18. 4% bond @ 86, $\frac{20}{559}$%.
19. The same.
20. 5% stock @ 95, $\frac{5}{19}$%.
21. 5% stock @ 114, $\frac{50}{7809}$%.
22. $1487.50, stock; $1103.75, at interest.
23. $83.79 gained by stock.

Page 219

1. $2760.
2. $84.38.
3. $33.75.
4. $318.75.

Page 220

1. $13,732.50.
2. $5283.75.
3. $184,650.
4. $7000.
5. $147,000.
6. $150.
7. $93.75 gain.
8. $53,345.
9. $540 gain.

Pages 221-223

1. 120.
2. Wage earners, \$285; others, \$1511.
3. Materials, \$181,170,000; labor, \$56,430,000; salaries, \$5,940,000; miscellaneous, \$17,820,000; profit, \$35,640,000.
4. White pine, 543,950 M; hemlock, 247,940 M; spruce, 106,260 M; yellow pine, 703,340 M; oak, 323,840 M; other woods, 604,670 M.
5. \$10,373,000.
6. \$3,542,000.
7. $29\frac{1}{31}\%$.
8. \$172,000,000.
9. \$222,000,000.
10. \$98,000,000.
11. 31%.
12. 376,441 doz.
13. Decreased, \$30.
14. Population, $8\frac{4}{7}\%$; ice, $34\frac{26}{411}\%$.
15. \$786,600,000.
16. \$255,280,000.
17. 10¢.
18. Canada, 103,105,926 bu.; U.S., 687,372,840 bu.; both, 790,478,766 bu.
19. 42,500,000 cu. ft.
20. 21,900,000 bbl.
21. $4\frac{6}{11}\%$, $4\frac{16}{21}\%$, $4\frac{8}{23}\%$, $4\frac{1}{6}\%$.

Page 224

1. \$112, \$109.20.
2. \$4, \$4.30.
3. 1.775%.
4. \$3,258,048.60 @ \$4.10.
5. 672,000,000 lb., 436,800,000 lb., 60.6 + %, 39.4 – %.
6. 5,717,333 T.
7. 322.

Page 225

1. \$0.47+.
2. $1126\frac{1}{4}$ lb.
3. 730 lb.
4. 275,968 T.
5. $2\frac{3}{4}$ in.
6. 290,400 long T.
7. \$2.47 or \$2.52, depending on how cross pieces run.

Page 226

1. 2437.
2. 255,000,000 cu. ft.
3. 1750 cu. ft.
4. $944 : 5775 =$ nearly $1 : 6.12$.
5. 5.
6. 1.44 – m.
7. 3 ft. $11\frac{1}{12}$ in.
8. \$761,400.
9. \$25,698,336.

Page 227

1. 691.15 mi.
2. 2850 passengers, 342 crew.
3. $41\frac{81}{109}\%$.
4. 2590.
5. 5600 T., 12,544,000 lb.
6. 15 times, $3\frac{3}{4}$ times.

Pages 228, 229

1. $113\frac{7}{16}$ T., $22{,}687\frac{1}{2}$ T.
2. 1.8°.
3. $39\frac{1}{5}$° F., $16\frac{2}{3}$° C.
4. 29.871 in., 29.433 in.
5. 4,515,720 T., 1,517,340 T., 1,052,700 T., 3,121,800 T., 2,403,060 T., 1,996,500 T., 2,983,860 T., 3,397,680 T.
6. 1,517,450 mi.
7. 6.94375 lb.
8. 80%.
9. 1.765 – mi.
10. 19,488 ft.
11. 44 min.
12. 787.9872 T.
13. 1 mi., 100 mi.
14. 13,392 lb.

Page 231

1. 25.	6. 48.	11. 6.4.	16. 2.1 in.	21. 0.35 ft.	26. 48.4 in.
2. 18.	7. 36.	12. 121.	17. 66 in.	22. 324 ft.	27. 576 in.
3. 22.	8. 33.	13. 2.2 in.	18. 125 in.	23. 440 in.	28. 512 in.
4. 27.	9. 8.1.	14. 1.4 ft.	19. 77 yd.	24. 420 ft.	29. 520 ft.
5. 24.	10. 3.5.	15. 1.5 ft.	20. 3.3 yd.	25. 560 ft.	30. 1440 in.

31. 96 rd., 32 rd.
32. 52 rd., 104 rd., 2.
33. 13 rd.
34. 68 rd., 204 rd., 3.

Page 232

1. 11. 2. 9. 3. 8. 4. 12. 5. 25. 6. 14. 7. 16. 8. 18. 9. 22 in.

Page 236

1. 57.	6. 47.	11. 77.	16. 87 ft.	21. 99 yd.	26. $\frac{55}{57}$.
2. 61.	7. 53.	12. 79.	17. 89 rd.	22. $\frac{12}{23}$.	27. $\frac{19}{91}$.
3. 63.	8. 59.	13. 82 ft.	18. 91 in.	23. $\frac{31}{32}$.	28. $\frac{7}{33}$.
4. 71.	9. 67.	14. 97 ft.	19. 95 ft.	24. $\frac{33}{34}$.	29. $\frac{17}{97}$.
5. 41.	10. 73.	15. 85 in.	20. 83 yd.	25. $\frac{44}{71}$.	

Page 238

1. 111.	4. 89.7.	7. 34.3.	10. 4607.	13. 1.41.	16. 2.828.
2. 234.	5. 4.41.	8. 9.07.	11. 7008.	14. 2.236.	17. 3.316.
3. 10.5.	6. 0.53.	9. 251.	12. 9812.	15. 2.645.	18. 12.649+rd.

Pages 239, 240

1. 85 ft.
2. 2.82 ft.
3. 36.05 in.
4. 95 in.
5. 125 ft.
6. 535 in., 85 in., 153 ft.
7. 22 ft. 6 in.
8. 16 ft. 6 in.
9. 43.60 ft.
10. 88.24 ft.
11. 16.40 rd.
12. 76.20 ft.
13. 8.49 in.
14. 9.76 in.
15. 21.93 ft.
16. 3.46 in.
17. 50 ft.
18. 7.5 mi.
19. 8.94 mi.

Page 241

2. 27.1488 in.
3. 46.3792 in.
4. 965.762 ft.
5. 1060.500 rd.
6. 127.260 ft.
7. 6.708 ft.
8. 21.54 ft.
9. 12.726 ft.
10. 22.624 ft.
11. 3.535 ft.
12. 10 in.
13. 3 in.
14. 5 ft.

Page 244

1. 17.
2. 19.
3. 21.
4. 18.
5. 24.
6. 27.
7. 31.
8. 36.
9. 51.
10. 54.
11. 61.
12. 67.
13. 82.
14. 91.
15. 96.
16. 98.
17. 84.
18. 92.
19. 95.
20. 99.
22. 2744.
23. 9261.
24. 15,625.
25. 27,000.
26. 166,375.
27. 216,000.
28. 421,875.
29. 970,299.

Page 247

1. 13.
2. 15.
3. 14.
4. 41.
5. 42.
6. 45.
7. 46.
8. 48.
9. 52.
10. 59.
11. 63.
12. 163.
13. 109.
14. 121.
15. 138.
16. 160.
17. 169.
18. 370.
19. 378.
20. 460.
21. 46.3.
22. 45.1.
23. 0.486.
24. 326.

Page 249

1. $214.3\frac{4}{7}$ in.
2. 151.8 ft.
3. $1329\frac{3}{7}$ in.
4. 16.06 ft.
5. 169.4 ft.
6. 20.02 in.
7. 14.74 ft.
8. 182.6 in.
9. 18.92 ft.
10. $42.4\frac{2}{7}$ ft.
11. $55\frac{1}{21}$ ft.
12. 202.4 ft.
13. 42 ft.
14. $18\frac{6}{11}$ ft.
15. 31 ft.
16. 56 in.
17. 245 ft.
18. $30\frac{4}{5}$ ft.
19. 21.7 in.
20. 1.26 ft.
21. 0.154 ft.
22. 53.4072 in.
23. 40.8408 in.
24. 8.79648 in.
25. 13.728792 in.
26. 7.0686 ft.
27. 19.3732 ft.
28. 17 in.
29. 27 in.
30. 31 in.
31. 29.9999+ ft.
32. 58+ ft.
33. 44 ft.
34. 308 in.
35. 484 in.
36. 572 in.
37. 660 in.
38. 96.8 in.
39. 11 in.
40. 23.8 ft.
41. 6.6 in.
42. 1.98 in.
43. 14.5 in.

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1. $3\frac{1}{7}$ sq. in.
2. 1386 sq. ft.
3. 1386 sq. ft.
4. 35 ft.
5. 39.82 in., 126.12985 sq. in.
6. 7.7 in., 3.85 in., 46.585 sq. in.
7. 78.54 sq. in., 226.9806 sq. in.
8. 22 ft., 38.5 sq. ft.
9. $7\frac{1}{4}$ in.

Page 253

1. 10,875 cu. in.
2. $0.341\frac{1}{3}$ cu. ft.
3. 325.92 cu. in.
4. 310.98 cu. in.
5. 178.383 cu. ft.
6. 199.076 cu. in.
7. $8\frac{11}{25}$ in.
8. 9 in.
9. 8 in.
10. $9\frac{30}{77}$.
11. 999.6 cu. in.
12. 70.528 cu. in.
13. 113.53076 cu. in.
14. 43.225 cu. ft.
15. 100.34375 cu. ft.
16. 2301.252 cu. in.
17. 226,981 cu. in.
18. 2885.794 cu. in.
19. 21 in., 11 in.
20. 7.5 in.

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1. $50,285\frac{5}{7}$ cu. ft.
2. $46\frac{23}{126}$ cu. ft.
3. $169,273\frac{23}{45}$ gal.
4. $144\frac{3}{8}$ cu. ft.
5. 2079 cu. in.
6. 2618 cu. in., $11,455\frac{5}{7}$ cu. in.

Page 256

1. 105 sq. in.
2. 231 sq. in.
3. $62\frac{6}{7}$ sq. in.
4. 5092 sq. ft.
6. $188\frac{4}{7}$ sq. ft.
7. $244\frac{3}{4}$ sq. ft.
8. 110 sq. ft.
9. 27,456 sq. ft.
10. $1178\frac{4}{7}$ sq. ft.

Page 257

1. 160.332 cu. in.
2. 3933.384 cu. in.
3. 600.9404 cu. ft.
4. 667.038 cu. ft.
5. 21 sq. in.
6. 21 sq. in.
7. 27 sq. in.
8. 18 sq. in.
9. 93,537,284 cu. ft., 15,714,263,712 lb.

Page 258

1. $509.329\frac{1}{3}$ cu. in., 1527.988 cu. in.
2. 134.904 cu. in., 404.712 cu. in.
3. 407.484 cu. in., 1222.452 cu. in.
4. 845.614 cu. in., 2536.842 cu. in.
5. $301\frac{5}{7}$ cu. in.
6. 55,440 cu. ft., 415,800 gal.
7. $718\frac{2}{3}$ cu. in.
8. $18,102\frac{6}{7}$ cu. in. $= 10\frac{10}{21}$ cu. ft.

Page 260

1. 32 sq. in.
2. 272 sq. in.
3. 360 sq. in.
4. 140 sq. in.
5. 176 sq. in.
6. $592\frac{1}{2}$ sq. in.
7. $46.2\frac{1}{4}$ sq. in.
8. $40.21\frac{7}{8}$ sq. in.
9. 20.86 sq. in.
10. 10 sq. ft.
11. 340 sq. in.
12. $294\frac{1}{2}$ sq. in.
13. 10 in.
14. 50 in.
15. 2040 sq. ft.
16. $115\frac{1}{2}$ sq. ft.
17. 764,000 sq. ft.
18. 960 sq. ft.
19. $183\frac{1}{3}$ sq. ft.

Page 261

1. $63\frac{9}{14}$ sq. in.
2. $2,359,727,071,428\frac{4}{7}$ sq. mi.; or, say, 2,359,727,000,000 sq. mi.
3. $17\frac{163}{224}$ sq. in.
4. $2828\frac{4}{7}$ sq. in.

Pages 262-265

1. $56\frac{185}{1344}$ cu. in.
2. 4851 cu. in.
3. $4,190,476,190\frac{10}{21}$ cu. mi.
4. $1767\frac{6}{7}$ cu. ft.
5. $3688\frac{857}{4536}$ cu. ft.
6. $22\frac{11}{24}$ cu. ft.
7. $53\frac{19}{81}$ cu. ft.
8. $268,190,476,190\frac{10}{21}$ cu. mi.
9. $463.433\frac{1}{7}$ cu. in.
10. $776.046\frac{6}{7}$ cu. in.
11. $\frac{529}{2304}$ sq. in.
12. $\frac{169}{256}$ sq. in.
13. $\frac{289}{1296}$ sq. in.
14. $\frac{225}{4096}$ sq. in.
15. 8.2369 sq. ft.
16. 15.3664 sq. ft.
17. 306.25 sq. rd.
18. 0.1369 sq. in.
19. 31.28 sq. ft.
20. 46.72 sq. in.
21. $8\frac{19}{32}$ sq. ft.
22. $3.86\frac{1}{12}$ sq. ft.
23. 4.7724 sq. in.
24. 38 sq. ft.
25. 8.93 sq. ft.
26. $3037\frac{1}{2}$ sq. in.
27. 0.0777 sq. in.
28. 15.68 sq. in.
29. 3645 sq. in.
30. $3.37\frac{1}{6}$ sq. ft.
31. 18 in.
32. 2.1 in.
33. 3.4 in.
34. 1.9 in.
35. 11 in.
36. 17 in.
37. 64 in.
38. 6.4 ft.
39. 7.8 in.
40. 8.4 in.
41. 0.44 in.
42. 11 ft. 10 in.
43. 17.88 rd.
44. 13.076 ft.
45. 19.62 ft.
46. 12.369 in.
47. 18.76 rd.
48. 15.84 yd.
49. 2.74 in.
50. 2.457 yd.
51. 2.917 in.
52. 616 sq. in.
53. 745.36 sq. in.
54. 5003.46 sq. in.
55. 13,034.56 sq. ft.
56. 16,022.16 sq. ft.
57. 26,832.96 sq. yd.
58. 48,246.66 sq. rd.
59. 85,046.50 sq. rd.
60. 174,896.26 sq. yd.
61. 9.899 in.
62. 15.65 in.
63. 21 in.
64. 23.216 ft.
65. 24.249 ft.
66. 28.86 ft.
67. 3.569 yd.
68. 3.77 yd.
69. 6.71 yd.

70. $24,848,438\frac{10}{21}$ cu. in., $411,852\frac{4}{7}$ sq. in.
71. $21,323,020\frac{4}{21}$ cu. in., $371,913\frac{1}{7}$ sq. in.
72. $1,023,065,476\frac{4}{21}$ cu. in., $4,910,714\frac{4}{7}$ sq. in.
73. 134,754.95 cu. in., $12,712.73\frac{1}{7}$ sq. in.
74. 88,102.985 cu. in., 9576.41 sq. in.
75. 167,626.12 cu. in., $14,704.04\frac{4}{7}$ sq. in.
76. 21.6 cu. ft.
77. 93.48 cu. in.
78. 71.34 cu. in.
79. $44.3\frac{17}{18}$ cu. ft.
80. 25.432 cu. ft.
81. 439.899 cu. in.
82. $225.254\frac{6}{7}$ cu. in.
83. $2928.080\frac{4}{7}$ cu. ft.
84. $109.184\frac{20}{21}$ cu. ft.
85. $23.533\frac{5}{7}$ cu. ft.
86. 62.83 cu. in.
87. $427.038\frac{6}{7}$ cu. in.
88. $28.189\frac{1}{3}$ cu. in.
89. 498.968 cu. ft.
90. $20\frac{15245}{18144}$ cu. ft.
91. $70\frac{20}{567}$ cu. ft.
92. $86\frac{3}{7}$ lb.
93. 28 T.
94. 19.404 cu. in.
95. $2416\frac{22}{63}$ lb.
96. $1,457,777\frac{7}{9}$.
97. $\frac{2}{27}$ in.
98. $810\frac{215}{216}$ oz.
99. $1230\frac{15}{242}$ oz.
100. 15.59 sq. ft.
101. 2.
102. 268,190,476,000 cu. mi., 201,143,000 sq. mi.

Pages 266-280

1. 35, 56.
2. 5, 35.
3. 140.
4. 487, 296.
5. $740\frac{5}{13}$ lb.
6. $140.40.
7. $129.66\frac{1}{3}$ lb.
8. $5.04.
9. $66\frac{2}{3}$%.
10. $23\frac{1}{13}$%.
11. 1′ $11\frac{1}{5}$″.
12. $1,369,028\frac{4}{7}$ T.
13. $36\frac{24}{25}$ sq. in.
14. 12 da.
15. 24,942,881,600 lb.
16. 45 mi.
17. 12.
18. 375 T.
19. $10,890.
20. $5.25.
21. $80.
22. $550.
23. 66 ft.
24. $12.50 loss.
25. 20%, $16\frac{2}{3}$%.
26. 20,558,000 bbl.
27. $5\frac{1}{16}$′.
28. 769.692 sq. ft.
29. 2.
30. $6\frac{1}{3}$ hr.
31. 1 yr. 2 mo.
32. $5\frac{1}{3}$ sec.
33. $2\frac{2}{9}$ da.
34. 80 da., $10\frac{10}{11}$ da.
35. 5 da.
36. 76.
37. 8 hr.
38. 123 and 123.
39. 756.86967.
40. 424.26+ ft.
41. 3.71.
42. 45 ft.
43. 41 ft.
44. 25 ft.
45. 127.27+ ft.
46. 100.62 ft.
47. 160.
48. 3 hr. 30 min.
49. 128.
50. $70\frac{5}{16}$ hr.
51. 10 hr.
52. $1250, $1750, $1500.
53. 8.8 lb.
55. $2.17.
56. 6 T.
57. 100 gal.
58. $8\frac{1}{2}$ lb.
59. 75.
60. 486 sq. in., 729 cu. in.
61. 0.0220+ A.
62. $2\frac{1}{4}$.

63. 154 sq. in., 98 sq. in.
64. 1,178,571$\frac{3}{7}$ ft.
65. 8 ft. 3 in.
66. 98 da.
67. 750.
68. 13$\frac{389}{1760}$ mi.
69. \$24, \$40, \$48.
70. 45 A.
71. 9.
72. 6 da.
73. \$106.64.
74. 46.58+ ft.
75. 2.3535+.
76. 1$\frac{1}{5}$ da.
77. 3 hr. 30 min.
78. 4%.
79. 1,456,700 bbl.
80. 2 hr. 24$\frac{4}{5}$ min.
81. 13,600 lb.
82. 45 mi.
83. 11.175– mi.
84. \$85.50.
85. \$10.15.
86. 1$\frac{1}{2}$.
87. 336 bu.
88. + 174, – 23, + 7.
89. 2.
90. 283,104.
91. \$1700.
92. 28,032,232.8 A.
93. 1,860,012.
94. 5 wk.
95. 8426.88 gal.
96. 18$\frac{9}{37}$%.
97. \$1.49.
98. 268,800.
99. 4.
100. 10 min.
101. 8.085 bbl., 26.565 bbl.
102. 34 cu. ft., 17 cu. ft., 102 cu. ft.
103. 10$\frac{1}{5}$ bbl., 6$\frac{21}{64}$ cu. yd.
104. 191,666$\frac{2}{3}$.
105. 750 yd., 16$\frac{2}{3}$ min.
106. \$1.25, \$1.60, \$2.20.
107. 10,800 ft.
108. $\frac{1}{2}$ hr., 1$\frac{1}{2}$ mi. from first place.
109. \$17.90.
110. $\frac{4}{15}$ qt., $\frac{7}{30}$ qt.; therefore $\frac{56}{30}$ qt., or 1$\frac{13}{15}$ qt.
111. 66 steps.
112. 5077$\frac{4}{5}$ T.
113. 729, 162$\frac{162}{539}$ qt.
114. 1,608,921.6 T.
115. \$161, \$184.
116. \$42, \$48, \$60.
117. \$6.
118. 51 ft., 11, 7, 9.
119. 1 hr. 5 min. 27$\frac{3}{11}$ sec.
120. 4 hr. 21 min. 49$\frac{1}{11}$ sec., 7 hr. 38 min. 10$\frac{10}{11}$ sec.
121. 2 hr. 27 min. 16$\frac{4}{11}$ sec., 3 hr.
122. 5 hr. 10 min. 54$\frac{6}{11}$ sec., 5 hr. 43 min. 38$\frac{2}{11}$ sec., 5 hr. 27 min. 16$\frac{4}{11}$ sec.
123. 9 hr. 5 min. 27$\frac{3}{11}$ sec., 9 hr. 27 min. 16$\frac{4}{11}$ sec.
124. 17 min. 30 sec. P.M.
125. The year 2878.
126. 1,418,000 bales.
127. 53$\frac{13}{23}$%, 14$\frac{146}{161}$%, 18$\frac{150}{161}$%.
128. \$187.50, \$42.75.
129. \$148.
130. \$150, \$11.50.
131. \$1.228+, \$1.20, \$1.04, \$1.10, \$1.01, \$1.20, \$1.12, \$1.05.